Equations Summary from Inside Cover of Morrison, 2013

Mechanical Energy Balance

$$\frac{\Delta p}{\rho} + \frac{\Delta\langle v\rangle^2}{2\alpha} + g\Delta z + F_{friction} = -\frac{W_{s,by\,fluid}}{m} \qquad \begin{cases}\alpha_{laminar} = 0.5\\ \alpha_{turbulent} \approx 1\end{cases}$$

$$F_{friction} = \left[4f\frac{L}{D} + \sum_{fittings_i} n_i K_{f,i}\right]\frac{\langle v\rangle^2}{2}$$

Fanning Friction Factor (pipe flow)

$$f = \frac{F_{drag}}{\frac{1}{2}\rho\langle v\rangle^2\,(2\pi RL)} = \frac{\Delta p D}{2L\rho\langle v\rangle^2}$$

Note this is <u>correct</u>; there is an error on the inside cover

Drag Coefficient (sphere drop)

$$C_D = \frac{F_{drag}}{\frac{1}{2}\rho v_\infty^2\,(\pi R^2)} = \frac{4gD\left(\rho_{body} - \rho\right)}{3\rho v_\infty^2}$$

Momentum balance on a CV (Reynolds transport theorem)

$$\frac{d\mathbf{P}}{dt} + \iint_{CS} (\hat{n}\cdot\underline{v})\rho\underline{v}\,dS = \sum_{on\,CV} \underline{f}$$

Hydrostatic Pressure

$$p_{bottom} = p_{top} + \rho g h$$

Hagen-Poiseuille Equation (steady, laminar tube flow, incompressible)

$$Q = \frac{\pi(p_0 - p_L)R^4}{8\mu L}$$

Prandtl Equation (steady, turbulent tube flow)

$$\frac{1}{\sqrt{f}} = -4.0\log\left(\frac{4.67}{\mathrm{Re}\sqrt{f}}\right) + 2.28$$

Stokes-Einstein-Sutherland Equation (steady, slow flow around a sphere)

$$F_{drag} = 6\pi R\mu v_\infty$$

Macroscopic Momentum Balance on a CV

$$\frac{d\mathbf{P}}{dt} + \sum_{i=1}^{\#streams}\left[\frac{\rho A\cos(\theta)\langle v\rangle^2}{\beta}\hat{v}\right]\Bigg|_{A_i} = \sum_{i=1}^{\#streams}\left[-pA\hat{n}\right]_{A_i} + \underline{R} + M_{CV}\underline{g} \qquad \begin{cases}\beta_{laminar} = 0.75\\ \beta_{turbulent} \approx 1\end{cases}$$

Navier-Stokes equation (microscopic momentum balance, incompressible, Newtonian fluids)

$$\rho\left(\frac{\partial\underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \mu\nabla^2\underline{v} + \rho\underline{g}$$

Continuity equation (microscopic mass balance, incompressible fluids)

$$\nabla\cdot\underline{v} = 0$$

Total stress tensor

$$\tilde{\underline{\underline{\Pi}}} = -p\underline{\underline{I}} + \tilde{\underline{\underline{\tau}}}$$

$$\begin{pmatrix}\tilde{\Pi}_{11} & \tilde{\Pi}_{12} & \tilde{\Pi}_{13}\\ \tilde{\Pi}_{21} & \tilde{\Pi}_{22} & \tilde{\Pi}_{23}\\ \tilde{\Pi}_{31} & \tilde{\Pi}_{32} & \tilde{\Pi}_{33}\end{pmatrix}_{123} = \begin{pmatrix}\tilde{\tau}_{11} - p & \tilde{\tau}_{12} & \tilde{\tau}_{13}\\ \tilde{\tau}_{21} & \tilde{\tau}_{22} - p & \tilde{\tau}_{23}\\ \tilde{\tau}_{31} & \tilde{\tau}_{32} & \tilde{\tau}_{33} - p\end{pmatrix}_{123}$$

Dynamic pressure

$$\mathcal{P} \equiv p + \rho g h$$

Newtonian constitutive equation

$$\tilde{\underline{\underline{\tau}}} = \mu\left(\nabla\underline{v} + (\nabla\underline{v})^T\right)$$

$$= \mu\begin{pmatrix}2\frac{\partial v_1}{\partial x_1} & \frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2} & \frac{\partial v_3}{\partial x_1} + \frac{\partial v_1}{\partial x_3}\\ \frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2} & 2\frac{\partial v_2}{\partial x_2} & \frac{\partial v_2}{\partial x_3} + \frac{\partial v_3}{\partial x_2}\\ \frac{\partial v_3}{\partial x_1} + \frac{\partial v_1}{\partial x_3} & \frac{\partial v_2}{\partial x_3} + \frac{\partial v_3}{\partial x_2} & 2\frac{\partial v_3}{\partial x_3}\end{pmatrix}_{123}$$

Total molecular fluid force on a finite surface $\mathcal{S}$

$$\underline{\mathcal{F}} = \iint_{\mathcal{S}}\left[\hat{n}\cdot\tilde{\underline{\underline{\Pi}}}\right]_{\text{at surface}} dS$$

Stationary fluid

$$\left[\hat{n}\cdot\tilde{\underline{\underline{\Pi}}}\right] = -p\hat{n}$$

Moving fluid

$$\left[\hat{n}\cdot\tilde{\underline{\underline{\Pi}}}\right] = -p\hat{n} + \hat{n}\cdot\tilde{\underline{\underline{\tau}}}$$

Total fluid torque on a finite surface $\mathcal{S}$

$$\underline{\mathcal{T}} = \iint_{\mathcal{S}}\left[\underline{R}\times\left(\hat{n}\cdot\tilde{\underline{\underline{\Pi}}}\right)\right]_{\text{at surface}} dS$$

Total flow rate out through a finite surface $\mathcal{S}$

$$Q = \dot{V} = \iint_{\mathcal{S}}\left[\hat{n}\cdot\underline{v}\right]_{\text{at surface}} dS$$

Average velocity across a finite surface $\mathcal{S}$

$$\langle v\rangle = \frac{Q}{\mathcal{S}}$$

Coordinate system	surface differential dS
Cartesian (top, $\hat{n} = \hat{e}_z$)	$dS = dxdy$
Cartesian (side a, $\hat{n} = \hat{e}_y$)	$dS = dxdz$
Cartesian (side b, $\hat{n} = \hat{e}_x$)	$dS = dydz$
cylindrical (top, $\hat{n} = \hat{e}_z$)	$dS = rdrd\theta$
cylindrical (side, $\hat{n} = \hat{e}_r$)	$dS = Rd\theta dz$
spherical, ($\hat{n} = \hat{e}_r$)	$dS = R^2 \sin\theta d\theta d\phi$

Coordinate system	volume differential dV
Cartesian	$dV = dxdydz$
cylindrical	$dV = rdrd\theta dz$
spherical	$dV = r^2 \sin\theta drd\theta d\phi$

Coordinate system	coordinates	basis vectors
spherical	$x = r\sin\theta\cos\phi$	$\hat{e}_r = (\sin\theta\cos\phi\hat{e}_x) + (\sin\theta\sin\phi\hat{e}_y) + \cos\theta\hat{e}_z$
	$y = r\sin\theta\sin\phi$	$\hat{e}_\theta = (\cos\theta\cos\phi)\hat{e}_x + (\cos\theta\sin\phi)\hat{e}_y + (-\sin\theta)\hat{e}_z$
	$z = r\cos\theta$	$\hat{e}_\phi = (-\sin\phi)\hat{e}_x + \cos\phi\hat{e}_y$
cylindrical	$x = r\cos\theta$	$\hat{e}_r = \cos\theta\hat{e}_x + \sin\theta\hat{e}_y$
	$y = r\sin\theta$	$\hat{e}_\theta = (-\sin\theta)\hat{e}_x + \cos\theta\hat{e}_y$
	$z = z$	$\hat{e}_z = \hat{e}_z$

Divergence Theorem $$\iint_S \hat{n}\cdot \underline{F}\, dS = \iiint_V \nabla\cdot\underline{F}\, dV$$

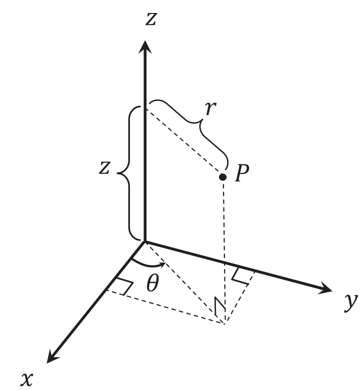
Stokes Theorem $$\oint_C \hat{t}\cdot\underline{F}\, dl = \iint_S \hat{n}\cdot(\nabla\times\underline{F})\, dS$$

Vector identities:

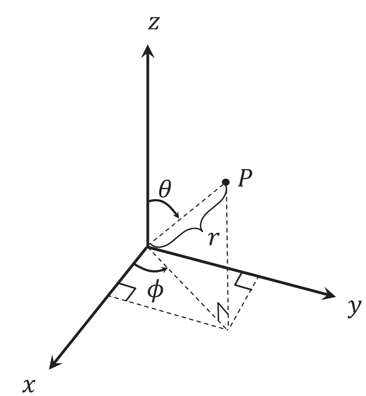
$$\begin{aligned}
\nabla\cdot\nabla\times\underline{F} &= 0 \quad \text{(Divergence of curl} = 0)\\
\nabla\times\nabla f &= 0 \quad \text{(Curl of gradient} = 0)\\
\nabla(fg) &= f\nabla g + g\nabla f\\
\underline{F}\cdot\nabla\underline{F} &= \frac{1}{2}\nabla\left(\underline{F}^2\right) - \underline{F}\times(\nabla\times\underline{F})\\
\nabla\cdot(f\underline{F}) &= f\nabla\cdot\underline{F} + \underline{F}\cdot\nabla f\\
\nabla\times\nabla\times\underline{F} &= \nabla(\nabla\cdot\underline{F}) - \nabla^2\underline{F}\\
\nabla\cdot(\underline{F}\times\underline{G}) &= \underline{G}\cdot(\nabla\times\underline{F}) - \underline{F}\cdot(\nabla\times\underline{G})
\end{aligned}$$

The equations in F. A. Morrison, ***An Introduction to Fluid Mechanics*** (Cambridge, 2013) assume the following definitions of the cylindrical and spherical coordinate systems.

Cylindrical Coordinate System: Note that the θ-coordinate swings around the z-axis



Spherical Coordinate System: Note that the θ-coordinate swings down from the z-axis; this is different from its definition in the cylindrical system above.



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FACTORS FOR UNIT CONVERSIONS

Quantity	Equivalent Values
Mass	1 kg = 1000 g = 0.001 metric ton = 2.20462 lb_m = 35.27392 oz
	1 lb_m = 16 oz = 5 x 10^{-4} ton = 453.593 g = 0.453593 kg
Length	1 m = 100 cm = 1000 mm = 10^6 microns (μm) = 10^{10} angstroms (Å)
	= 39.3701 in = 3.28084 ft = 1.09361 yd = 0.000621371 mile
	1 ft = 12 in. = 1/3 yd = 0.3048 m = 30.48 cm
Volume	1 m^3 = 1000 liters = 10^6 cm^3 = 10^6 ml
	= 35.31467 ft^3 = 219.969 imperial gallons = 264.172 gal
	= 1056.69 qt
	1 ft^3 = 1728 in^3 = 7.48052 gal = 0.028317 m^3 = 28.3168 liters
	= 28,316.8 cm^3
Force	1 N = 1 kg·m/s^2 = 10^5 dynes = 10^5 g·cm/s^2 = 0.22481 lb_f
	1 lb_f = 32.174 lb_m·ft/s^2 = 4.4482 N = 4.4482 x 10^5 dynes
Pressure	1 atm = 1.01325 x 10^5 N/m^2 (Pa) = 101.325 kPa = 1.01325 bars
	= 1.01325 x 10^6 dynes/cm^2
	= 760 mm Hg at 0˚ C (torr) = 10.333 m H_2O at 4˚ C
	= 14.696 lb_f/in^2 (psi) = 33.9 ft H_2O at 4˚C
	100 kPa = 1 bar
Energy	1 J = 1 N·m = 10^7 ergs = 10^7 dyne·cm
	= 2.778 x 10^{-7} kW·h = 0.23901 cal
	= 0.7376 ft·lb_f = 9.47817 x 10^{-4} Btu
Power	1 W = 1 J/s = 0.23885 cal/s = 0.7376 ft·lb_f/s = 9.47817 x 10^{-4} Btu/s = 3.4121 Btu/h
	= 1.341 x 10^{-3} hp (horsepower)
Viscosity	1 Pa·s = 1 N·s/m^2 = 1 kg/m·s
	= 10 poise = 10 dynes·s/cm^2 = 10 g/cm·s
	= 10^3 cp (centipoise)
	= 0.67197 lb_m/ft·s = 2419.088 lb_m/ft·h
Density	1 kg/m^3 = 10^{-3} g/cm^3
	= 0.06243 lb_m/ft^3
	10^3 kg/m^3 = 1 g/cm^3 = 62.428 lb_m/ft^3
Volumetric Flow	1 m^3/s= 35.31467 ft^3/s=15,850.32 gal/min (gpm)
	1 gpm = 6.30902 x 10^{-5} m^3/s=2.228009 x 10^{-3} ft^3/s=3.7854 liter/min
	1 liter/min=0.26417 gpm

Temperature $T(^oC) = \frac{5}{9}\left[T(^oF) - 32\right]$

$T(^oF) = \frac{9}{5}T(^oC) + 32 = 1.8T(^oC) + 32$

Absolute Temperature T(K) = T(˚C) + 273.15

T(˚R) = T(˚F) + 459.67

Temperature Interval (ΔT) 1 C˚ = 1 K = 1.8 F˚ = 1.8 R˚

1F˚ = 1 R˚ = (5/9) C˚ = (5/9) K

USEFUL QUANTITIES

SG = ρ(20˚C)/ρ_{water} (4˚C)

ρ_{water}(4˚C) = 1000 kg/m^3 = 62.43 lb$_m$/ft^3 = 1.000 g/cm^3

ρ_{water}(25˚C) = 997.08 kg/m^3 = 62.25 lb$_m$/ft^3 = 0.99709 g/cm^3

g = 9.8066 m/s^2 = 980.66 cm/s^2 = 32.174 ft/s^2

μ_{water} (25˚C) = 8.937 x 10^{-4} Pa·s = 8.937 x 10^{-4} kg/m·s

= 0.8937 cp =0.8937 x 10^{-2} g/cm·s = 6.005 x 10^{-4} lb$_m$/ft·s

Composition of air:

N_2	78.03%
O_2	20.99%
Ar	0.94%
CO_2	0.03%
H_2, He, Ne, Kr, Xe	0.01%
	100.00%

M_{air} = 29 g/mol = 29 kg/kmol = 29 lb$_m$/lbmole

$\hat{C}_{p,water}$ (25˚C) = 4.182 kJ/kg K = 0.9989 cal/g˚C = 0.9997 Btu/lb$_m$˚F

R = 8.314 m^3·Pa/mol·K = 0.08314 liter·bar/mol·K = 0.08206 liter·atm/mol·K

= 62.36 liter·mm Hg/mol·K = 0.7302 ft^3·atm/lbmole·˚R

= 10.73 ft^3·psia/lbmole·˚R

= 8.314 J/mol·K

= 1.987 cal/mol·K = 1.987 Btu/lbmole·˚R