Question:

The Hagen Poiseuille Eqn is sometimes written with gravity + sometimes without. Which is correct or which should I use?

They are both correct.

For a vertical tube w/ z-Direction pointing downward we obtained:

$$Q = \frac{\pi R^4 (L\rho g + P_0 - P_L)}{8\mu L}$$

(Lecture 6)

For a horizontal tube Geankoplis gets (4th edition, p85):

$$\langle v \rangle = \frac{Q}{\pi R^2} = \frac{(P_0 - P_L) R^2}{8 \mu L} \qquad 29-11$$

Note that there is no z-component of gravity in the horizontal case; if we cross out g in the lecture notes, the equations are the same.

We can define an equivalent pressure $\mathcal{P}$ that incorporates the effect of gravity:

$$\mathcal{P} \equiv P + \rho g h$$ (See Bird, Stewart, + Lightfoot, 1960, p 45)

where h is the distance upward from a chosen reference plane.

For the horizontal pipe

$$\frac{\partial \mathcal{P}}{\partial z} = \frac{\partial P}{\partial z} + \frac{\partial}{\partial z}\left(\cancel{\rho g h}\right)$$

constant

$$\boxed{\frac{\partial \mathcal{P}}{\partial z} = \frac{\partial P}{\partial z}}$$

The two pressures work the same in the microscopic balance.

For the vertical pipe with z pointing down, $h = -z$.

$$\mathcal{P} = P - \rho g z$$

$$\frac{\partial \mathcal{P}}{\partial z} = \underbrace{\frac{\partial P}{\partial z} - \rho g}$$

these two terms appear in the microscopic balance.

z-component: (after simplification)

$$0 = -\frac{\partial P}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \rho g\right.$$

$$= -\frac{\partial \mathcal{P}}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right]\right.$$

Now the microscopic balance looks like there is no gravity term. Equivalent pressure is a way of combining pressure + gravity.

For the Hagen–Poiseuille eqn:

$$\mathcal{P}_0 = P_0$$

$$\mathcal{P}_L = P_L - \rho g L$$

$$\mathcal{P}_0 - \mathcal{P}_L = P_0 - P_L + \rho g L$$

$$\boxed{Q = \frac{\pi R^4 (\mathcal{P}_0 - \mathcal{P}_L)}{8 \mu L}}$$

This version is good for both horizontal + vertical pipes (and other angles too!)

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