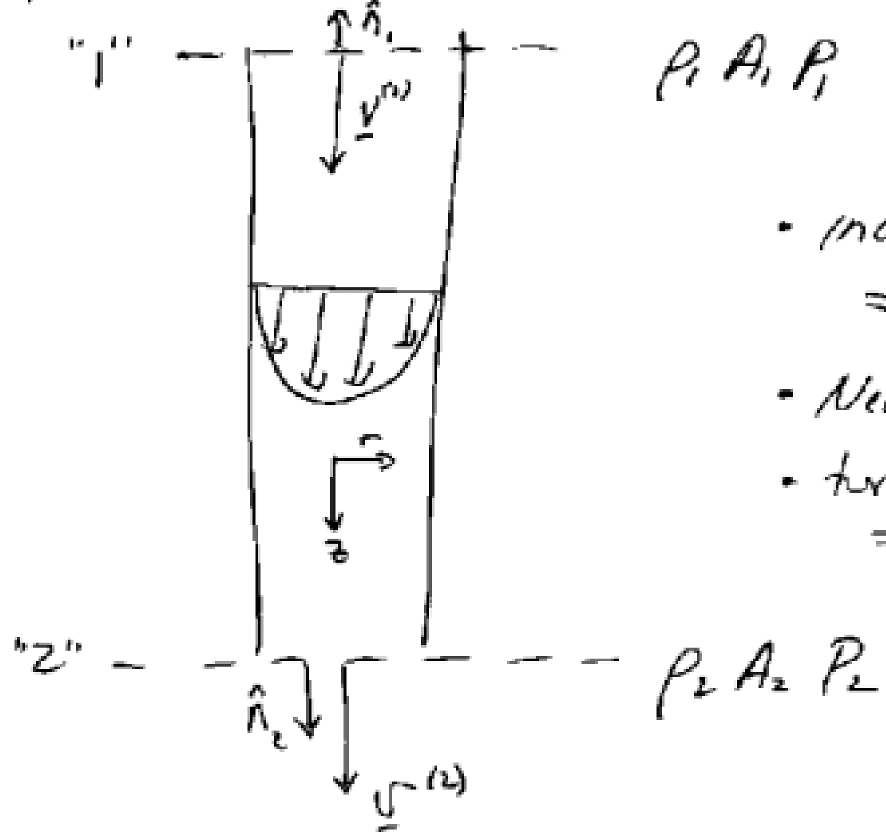
(pipe flow)

9-29-03

USE MACRO MOMENTUM BALANCE TO CALC FORCE ON WALLS IN STEADY TURBULENT FLOW IN PIPE



- incompressible $\Rightarrow \rho_1 = \rho_2$
- Newtonian
- turbulent $\Rightarrow \beta_1 = \beta_2 = 1$

MACRO MASS BAL:

$$M_1 = M_2$$

$$\rho_1 A_1 \langle v^{(1)} \rangle = \rho_2 A_2 \langle v^{(2)} \rangle$$

note: $A_1 = A_2$, $\rho_1 = \rho_2$

$$\boxed{\langle v^{(1)} \rangle = \langle v^{(2)} \rangle}$$

(pipe flow)

9-29-03

MACRO
MOMENTUM
BAL:

$$-\left\{ \hat{v}_1 \left(\frac{m_1 \cos\theta_1 \langle v^{(1)} \rangle}{\beta_1} \right) + \hat{v}_2 \left(\frac{m_2 \cos\theta_2 \langle v^{(2)} \rangle}{\beta_2} \right) \right. + \sum \underline{F}_m = 0$$

$$\hat{v}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{r\theta z} \qquad \hat{v}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{r\theta z}$$

$$\hat{n}_1 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}_{r\theta z} \qquad n_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{r\theta z}$$

$$\cos\theta_1 = \hat{n}_1 \cdot \hat{v}_1 = -1$$

$$\cos\theta_2 = \hat{n}_2 \cdot \hat{v}_2 = 1$$

Forces = pressure, gravity, wall forces

(pipe flow)

ASSEMBLE:

9-29-03

$$-\left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{r\theta z} \rho A \langle v \rangle (-1) \langle v \rangle + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{r\theta z} \rho A \langle v \rangle (1) \langle v \rangle \right\}$$

$$+ \begin{pmatrix} 0 \\ 0 \\ P_1 A \end{pmatrix}_{r\theta z} + \begin{pmatrix} 0 \\ 0 \\ -P_2 A \end{pmatrix}_{r\theta z} + \begin{pmatrix} 0 \\ 0 \\ M_{tot} g \end{pmatrix}_{r\theta z}$$

$$+ \underbrace{\begin{pmatrix} F_r \\ F_\theta \\ F_z \end{pmatrix}_{r\theta z}}_{\text{force on fluid} \; = \; -F_{walls}}$$

r-component: $\boxed{0 = F_r}$

θ-component: $\boxed{0 = F_\theta}$

(Pipe flow)

9-29-03

z-component:

$$\rho A\langle v\rangle^2 - \rho A\langle v\rangle^2 + P_1 A - P_2 A + M_{tot} g + F_z = 0$$

$$F_z = -(P_1 - P_2)A - M_{tot} g$$

$$F_{walls} = -\underline{F} = \begin{pmatrix} 0 \\ 0 \\ (P_1 - P_2)A + M_{tot} g \end{pmatrix}_{r\theta z}$$

Before we neglected gravity and found

$F_z = (P_1 - P_2)A$ which is the same result. //