(u-tube)

9-29-0

BSL 1st edition, pbm 7.D1 — WATER FLOWING IN A HORIZONTAL U-TUBE (turbulent). WHAT is FORCE ON WALLS?

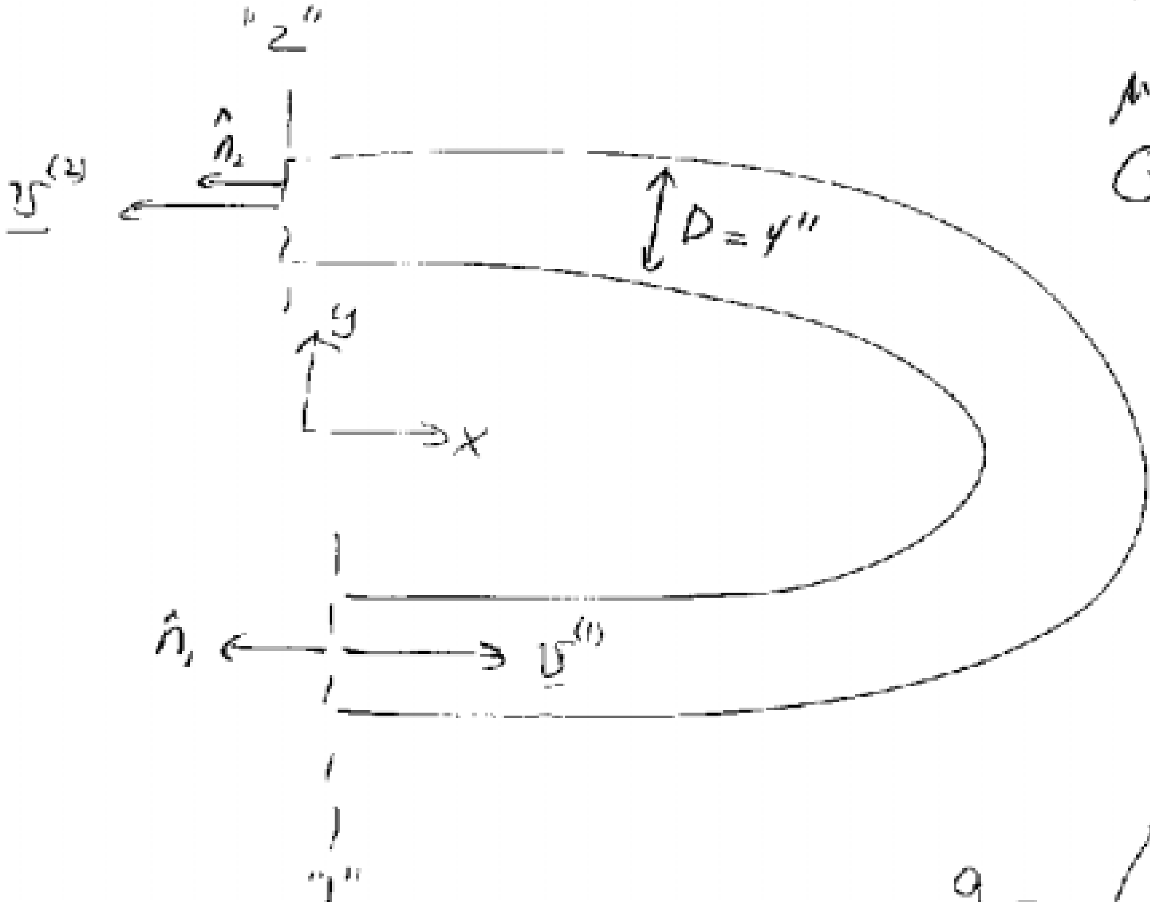
$P_2 = 19$ psia

68°F

$\rho = 62.4 \ \frac{lbm}{ft^3}$

$\mu = 1$ cP

$Q = 3 \ \frac{ft^3}{s}$



$P_1 = 21$ psia

$$\underline{g} = \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}_{xyz}$$

(U tube)

9-29-03

MACRO
MASS
BAL:

$$m_1 = m_2$$

$$\rho \langle v^{(1)} \rangle A = \rho \langle v^{(2)} \rangle A$$

$$\boxed{\langle v^{(1)} \rangle = \langle v^{(2)} \rangle = \langle v \rangle}$$

MACRO
MOMENTUM
BAL:

$$-\left\{ \hat{v}_1 \left(\frac{m_1 \cos\theta_1 \langle v^{(1)} \rangle}{\beta_1} \right) + \hat{v}_2 \left(\frac{m_2 \cos\theta_2 \langle v^{(2)} \rangle}{\beta_2} \right) + \sum \underline{F} = \right.$$

FORCES
ON

$$\hat{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz} \qquad \hat{v}_2 = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}_{xyz} \qquad n_1 = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}_{xyz} \quad n_2 = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

$$\cos\theta_1 = \hat{n}_1 \cdot \hat{v}_1 = -1$$

$$\cos\theta_2 = \hat{n}_2 \cdot \hat{v}_2 = 1$$

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$\beta_1 = \beta_2 = 1$ (turbulent flow)

Pressure on at "1" $= P_1 A\, v^{(1)} = \begin{pmatrix} P_1 A \\ 0 \\ 0 \end{pmatrix}_{xyz}$

Pressure on at "2" $= -P_2 A\, v^{(2)} = \begin{pmatrix} +P_2 A \\ 0 \\ 0 \end{pmatrix}_{xyz}$

gravity on $= M_{tot} \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}_{xyz}$

Force on fluid due to walls $= \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix}_{xyz}$

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ASSEMBLE:

$$-\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz} \rho \langle v \rangle^2 A \, (-1) + \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}_{xyz} \rho \langle v \rangle^2 A \, (1) \right.$$

$$\left. + \begin{pmatrix} P_1 A \\ 0 \\ 0 \end{pmatrix}_{xyz} + \begin{pmatrix} P_2 A \\ 0 \\ 0 \end{pmatrix}_{xyz} + \begin{pmatrix} 0 \\ 0 \\ M_{tot}\, g \end{pmatrix}_{xyz} + \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix}_{xyz} \right)$$

y-component: $\boxed{F_y = 0}$

z-component: $\boxed{F_z = -M_{tot}\, g}$

x-component: $0 = \rho \langle v \rangle^2 A + \rho \langle v \rangle^2 A + (P_1 + P_2) A + F_x$

$$0 = 2 \rho \langle v \rangle^2 A + (P_1 + P_2) \frac{\pi D^2}{4} + F_x$$

$$\underline{F} = \begin{pmatrix} -2 \rho \langle v \rangle^2 \frac{\pi D^2}{4} - \frac{P_1 + P_2}{4} \pi D^2 \\ 0 \\ -M_{tot}\, g \end{pmatrix}_{xyz}$$

the negative of this is the x-force on the walls.

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$$F_{wall_x} = -F_x = 2\rho \langle v \rangle^2 \frac{\pi D^2}{4} + \frac{P_1 + P_2}{4} \pi D^2$$

$$= \frac{\pi D^2}{4} \left[2\rho \langle v \rangle^2 + P_1 + P_2 \right]$$

$$\langle v \rangle = \frac{4Q}{\pi D^2} = \left(\frac{3 ft^3}{s} \right) \left(\frac{4}{\pi} \right) \left(\frac{1}{4/12 ft} \right)^2$$

$$= 34.37747 \frac{ft}{s}$$

$$\rho \langle v \rangle^2 = \left(\frac{62.4 \cancel{lb_m}}{\cancel{ft^3}} \right) \left(34.37747 \frac{\cancel{ft}}{\cancel{s}} \right)^2 \frac{\cancel{s^2}\, lb_f}{32.174 \cancel{ft}\, \cancel{lb_m}} \left(\frac{ft}{12 in} \right)$$

$$= 15.91713 \quad psi$$

$$F_{wall_x} = \frac{\pi\, 16\, in^2}{4} \left[2(15.91713) + 21 + 19 \right] \frac{lb_f}{in^2}$$

$$\boxed{F_{wall_x} = 903\ lb_f}$$