CM3110
Transport I
Part I: Fluid Mechanics

More Complicated Flows II: External Flow

(or applying fluid-mechanics problem-solving to a new category of flows)

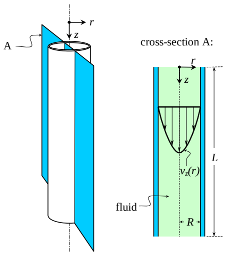
Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

More complicated flows II: From *Nice* to ***Powerful***

Nice:

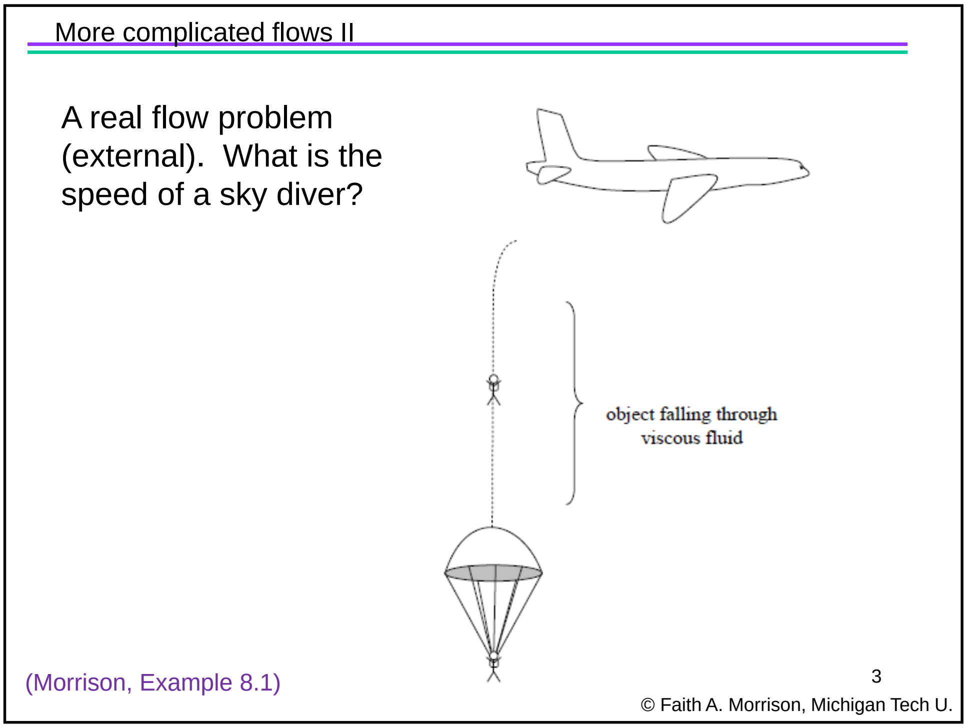
Learning to solve one particular problem (or a group of related problems)



Powerful:

Solving <u>never-before-solved</u> problems.

?

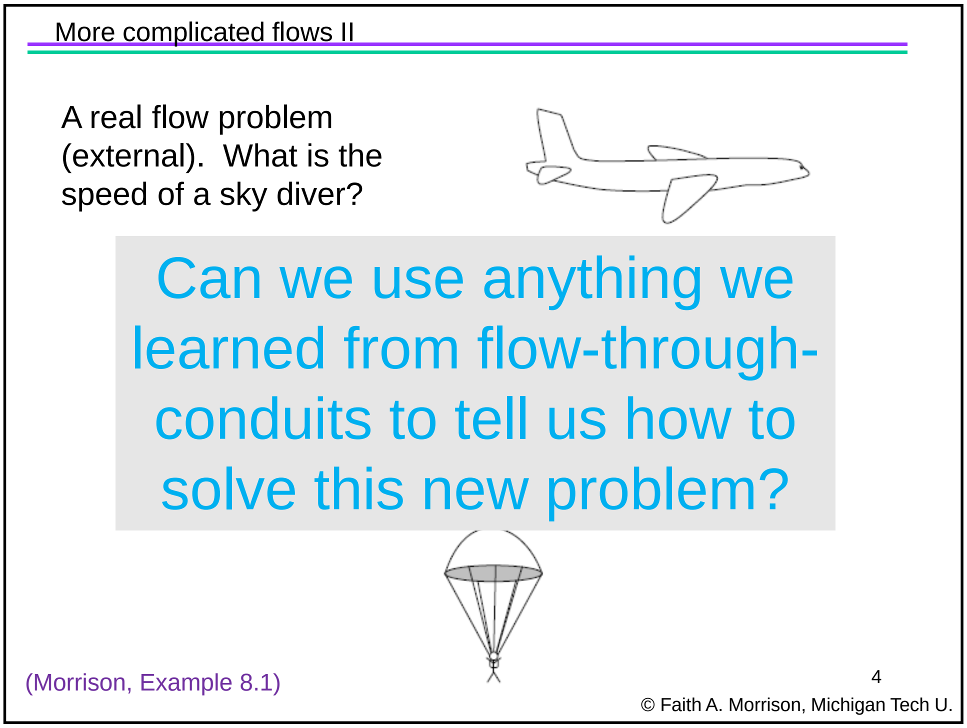


More complicated flows II

A real flow problem (external). What is the speed of a sky diver?

object falling through viscous fluid

(Morrison, Example 8.1)



More complicated flows II

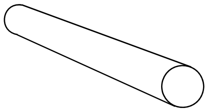
A real flow problem (external). What is the speed of a sky diver?

Can we use anything we learned from flow-through-conduits to tell us how to solve this new problem?

(Morrison, Example 8.1)

More complicated flows II

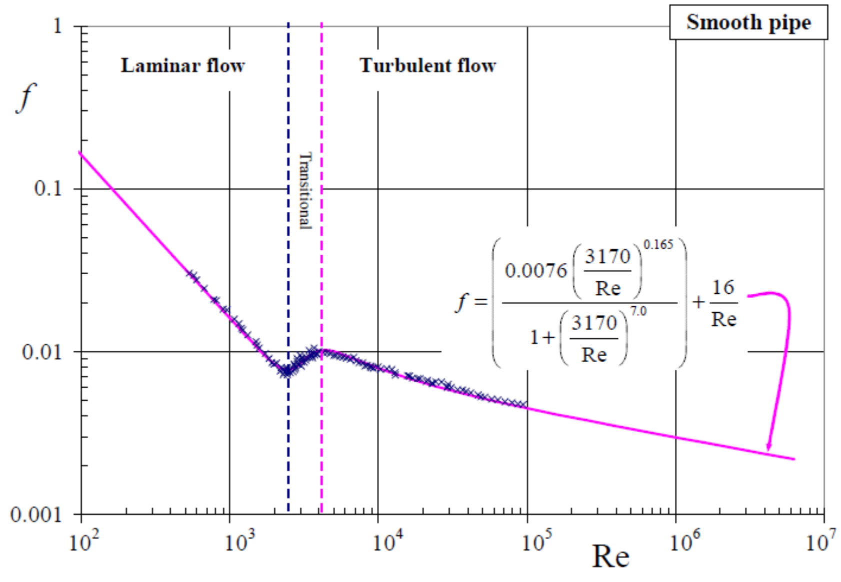
Flow through Conduits



So far we have talked about internal flows

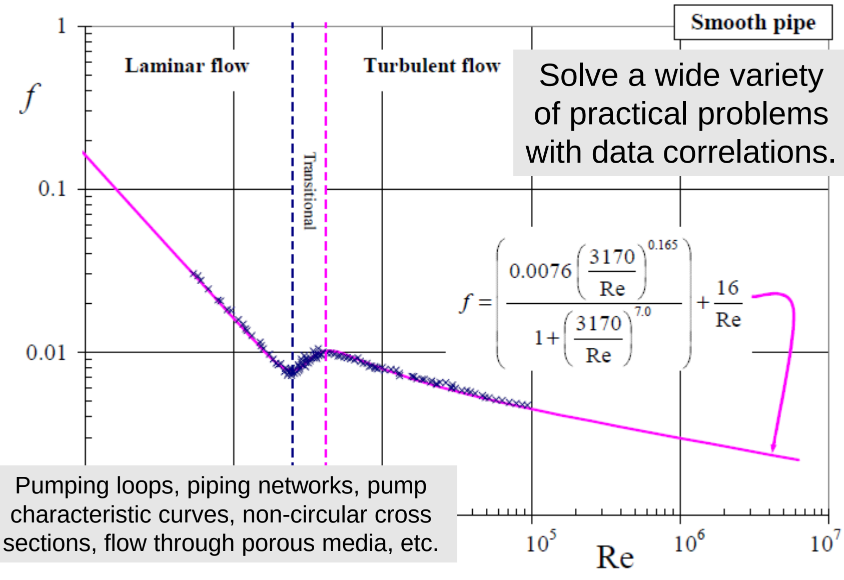
- ideal flows (Poiseuille flow in a tube)
- real flows (turbulent flow in a tube)

Strategy for handling real flows:	Dimensional analysis and data correlations, $f(\mathrm{Re})$
How did we arrive at correlations?	non-Dimensionalize ideal flow; use to guide expts on similar non-ideal flows; take data; develop empirical correlations from data
What do we do with the correlations?	use in MEB; calculate pressure-drop flow-rate relations



$$f = \left(\frac{0.0076\left(\frac{3170}{\mathrm{Re}}\right)^{0.165}}{1+\left(\frac{3170}{\mathrm{Re}}\right)^{7.0}} \right) + \frac{16}{\mathrm{Re}}$$

F. A. Morrison, *An Introduction to Fluid Mechanics*, Cambridge University Press 2013.



Solve a wide variety of practical problems with data correlations.

$$f=\left(\frac{0.0076\left(\frac{3170}{\mathrm{Re}}\right)^{0.165}}{1+\left(\frac{3170}{\mathrm{Re}}\right)^{7.0}}\right)+\frac{16}{\mathrm{Re}}$$

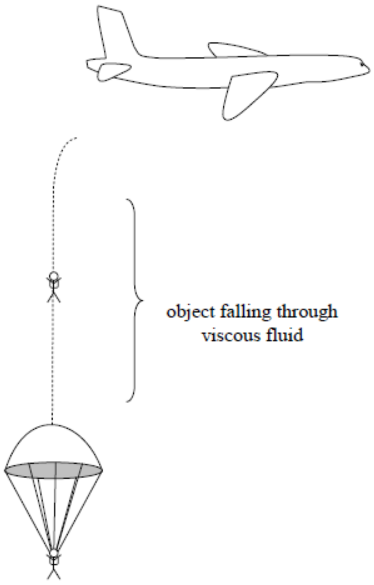
Pumping loops, piping networks, pump characteristic curves, non-circular cross sections, flow through porous media, etc.

F. A. Morrison, *An Introduction to Fluid Mechanics*, Cambridge University Press 2013.

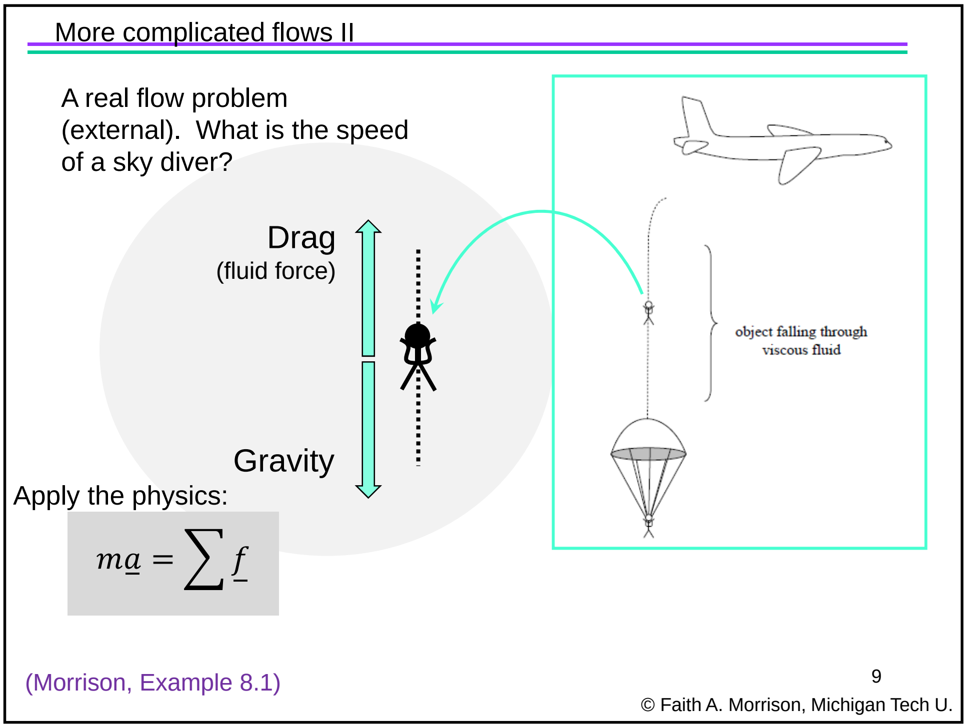
More complicated flows II

A real flow problem (external). What is the speed of a sky diver?

We can apply the *conduit* process to this new problem.



(Morrison, Example 8.1)



More complicated flows II

A real flow problem (external). What is the speed of a sky diver?

Drag
(fluid force)

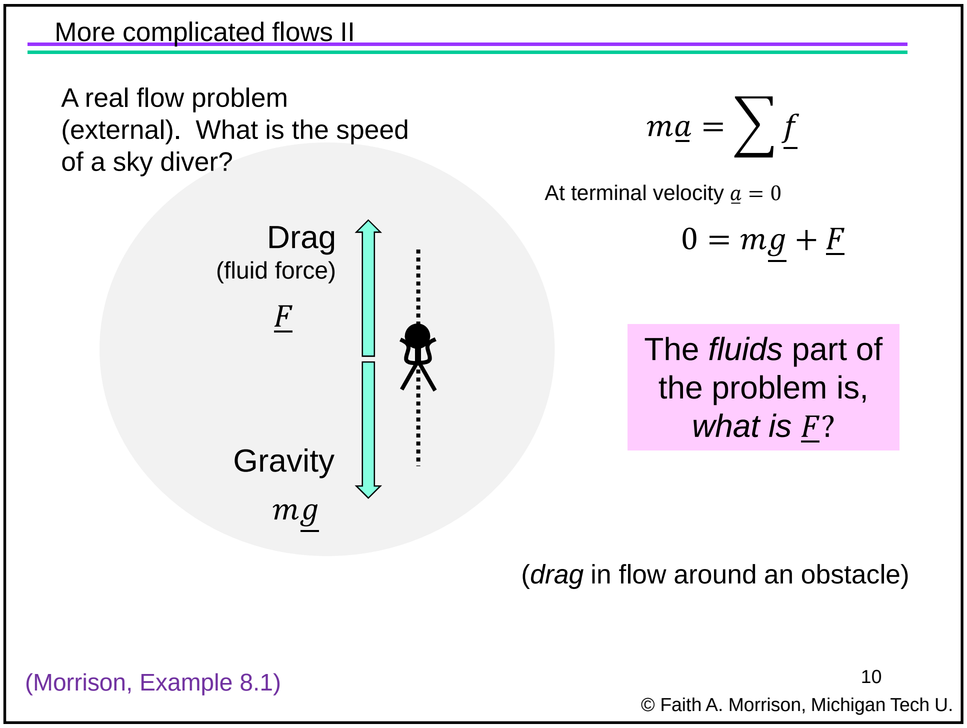
Gravity

object falling through viscous fluid

Apply the physics:

$$m\underline{a} = \sum \underline{f}$$

(Morrison, Example 8.1)



More complicated flows II

A real flow problem (external). What is the speed of a sky diver?

Drag
(fluid force)
$\underline{F}$

Gravity
$m\underline{g}$

$$m\underline{a} = \sum \underline{f}$$

At terminal velocity $\underline{a} = 0$

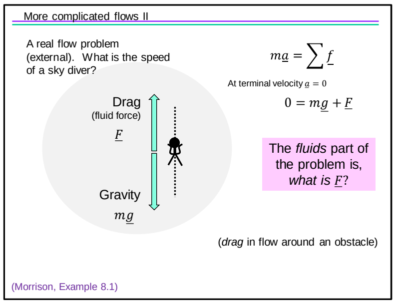
$$0 = m\underline{g} + \underline{F}$$

The *fluids* part of the problem is, *what is* $\underline{F}$?

(*drag* in flow around an obstacle)

(Morrison, Example 8.1)

Can we address this **new** problem (new to us) . . .



More complicated flows II

A real flow problem (external). What is the speed of a sky diver?

$$m\underline{a} = \sum \underline{f}$$

At terminal velocity $\underline{a} = 0$

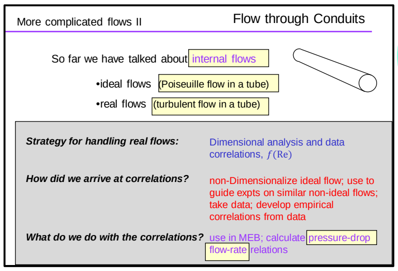
$$0 = m\underline{g} + \underline{F}$$

The *fluids* part of the problem is, *what is* $\underline{F}$?

(*drag* in flow around an obstacle)

(Morrison, Example 8.1)

By using the same approach as used for pipe flow?



More complicated flows II — Flow through Conduits

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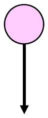
More complicated flows II

Flow around Obstacles



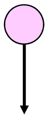
Now, we will talk about external flows

- ideal flows (flow around a sphere)
- real flows (turbulent flow around a sphere, sky diver, other obstacles)

Strategy for handling real flows:	Dimensional analysis and data correlations
How did we arrive at correlations?	non-Dimensionalize ideal flow; use to guide expts on similar non-ideal flows; take data; develop empirical correlations from data
What do we do with the correlations?	calculate drag – free-stream velocity relations

More complicated flows II

Flow around Obstacles



Now, we will talk about external flows

- ideal flows (flow around a sphere)
- rea er obstacles)

Let’s try

Strategy f d data

How did we arrive at correlations? non-Dimensionalize ideal flow; use to guide expts on similar non-ideal flows; take data; develop empirical correlations from data

What do we do with the correlations? calculate drag – free-stream velocity relations

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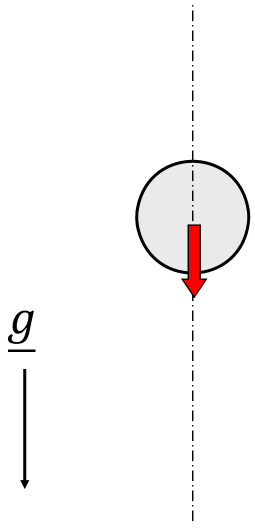
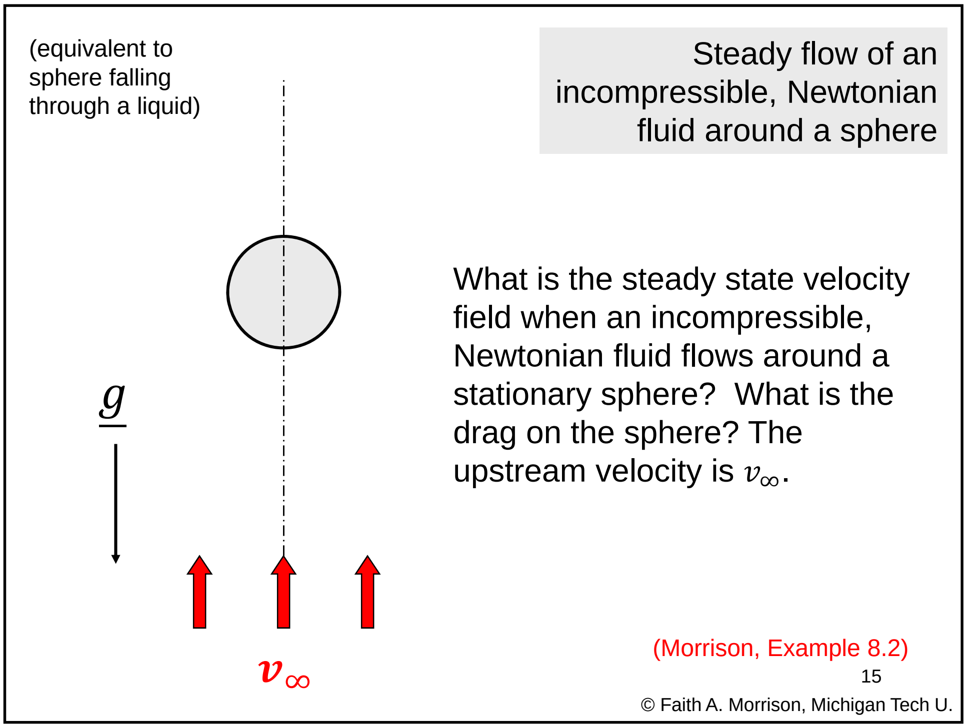


What is the steady state velocity field around a sphere dropping through an incompressible Newtonian fluid

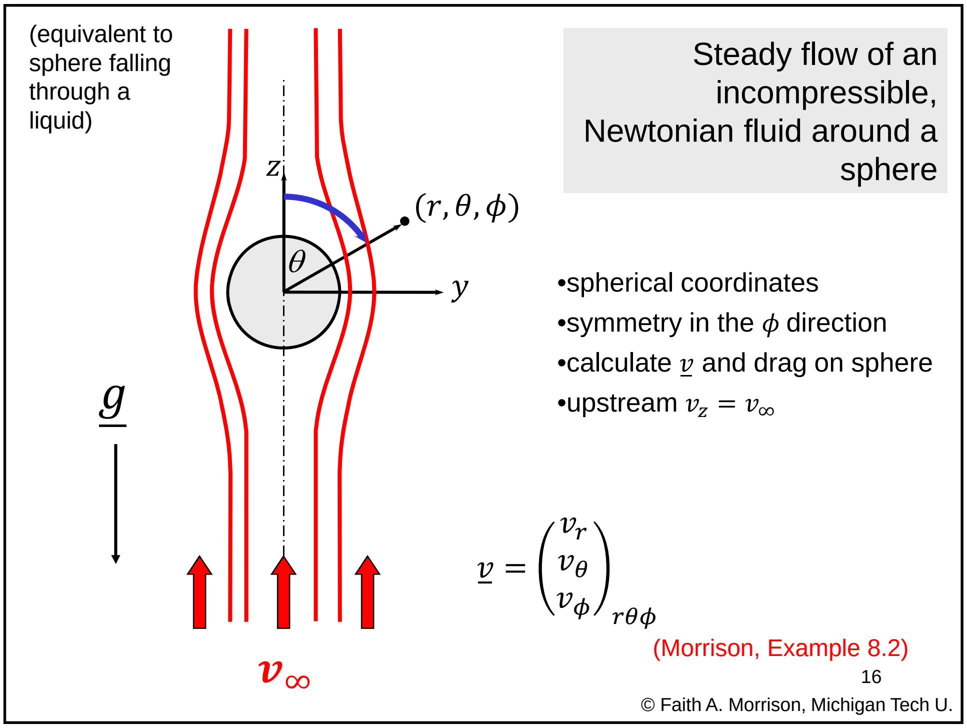


(equivalent to sphere falling through a liquid)

Steady flow of an incompressible, Newtonian fluid around a sphere

What is the steady state velocity field when an incompressible, Newtonian fluid flows around a stationary sphere? What is the drag on the sphere? The upstream velocity is v_∞.

(Morrison, Example 8.2)



(equivalent to sphere falling through a liquid)

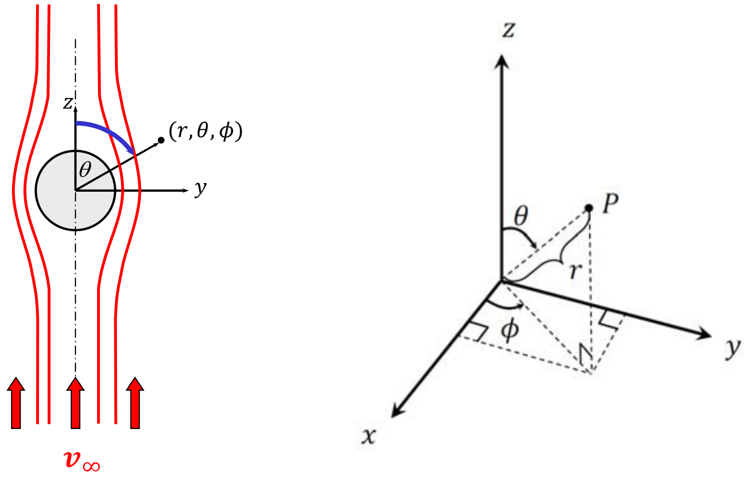
Steady flow of an incompressible, Newtonian fluid around a sphere

- •spherical coordinates
- •symmetry in the ϕ direction
- •calculate $\underline{v}$ and drag on sphere
- •upstream $v_z = v_\infty$

$$\underline{v} = \begin{pmatrix} v_r \\ v_\theta \\ v_\phi \end{pmatrix}_{r\theta\phi}$$

(Morrison, Example 8.2)

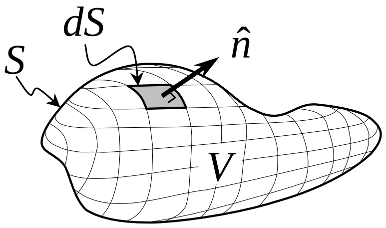
Slow flow around a sphere (Stokes Flow)



Slow flow around a sphere (Stokes Flow)

Microscopic Balances

(Microscopic balances on an arbitrary control volume)



Continuity Equation (mass)

$$\boxed{\frac{\partial\rho}{\partial t}+\underline{v}\cdot\nabla\rho=-\rho(\nabla\cdot\underline{v})}$$

Equation of Motion (momentum)

$$\boxed{\rho\left(\frac{\partial\underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla P+\mu\nabla^2\underline{v}+\rho\underline{g}}$$

Newtonian fluid

Navier-Stokes Equation

Slow flow around a sphere (Stokes Flow)

Continuity (spherical coordinates)

The Equation of Continuity and the Equation of Motion in Cartesian, cylindrical, and spherical coordinates

CM3110 Fall 2011 Faith A. Morrison

Continuity Equation, Cartesian coordinates

$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$

Continuity Equation, cylindrical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho v_\theta)}{\partial \theta} + \frac{\partial (\rho v_z)}{\partial z} = 0$$

Continuity Equation, spherical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial (\rho r^2 v_r)}{\partial r} + \frac{1}{r \sin\theta}\frac{\partial (\rho v_\theta \sin\theta)}{\partial \theta} + \frac{1}{r \sin\theta}\frac{\partial (\rho v_\phi)}{\partial \phi} = 0$$

www.chem.mtu.edu/~fmorriso/cm310/Navier.pdf

Slow flow around a sphere (Stokes Flow)

Navier-Stokes (spherical coordinates)

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in spherical coordinates

$$\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \right)$$
$$= -\frac{\partial P}{\partial r} + \mu \left(\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right)\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial v_r}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2} \right.$$
$$\left. - \frac{2}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(v_\theta \sin\theta\right) - \frac{2}{r^2 \sin\theta}\frac{\partial v_\phi}{\partial \phi} \right) + \rho g_r$$

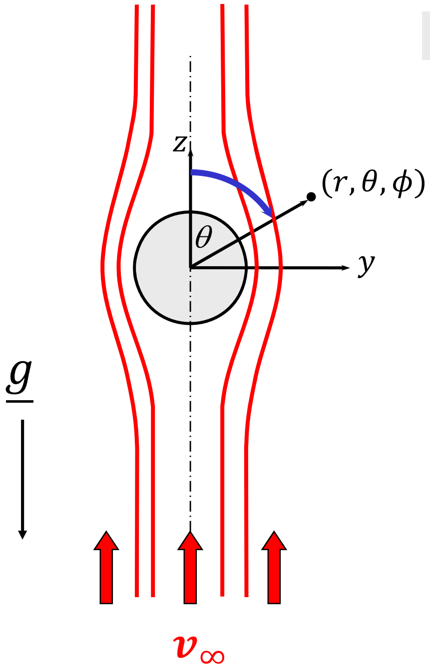
$$\rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2 \cot\theta}{r} \right)$$
$$= -\frac{1}{r}\frac{\partial P}{\partial \theta} + \mu \left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(v_\theta \sin\theta\right)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2} \right.$$
$$\left. + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r^2 \sin\theta}\frac{\partial v_\phi}{\partial \phi} \right) + \rho g_\theta$$

$$\rho \left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} + \frac{v_\phi v_\theta \cot\theta}{r} \right)$$
$$= -\frac{1}{r\sin\theta}\frac{\partial P}{\partial \phi} + \mu \left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(v_\phi \sin\theta\right)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2} \right.$$
$$\left. + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cot\theta}{r^2 \sin\theta}\frac{\partial v_\theta}{\partial \phi} \right) + \rho g_\phi$$

Note: the r-component of the Navier-Stokes equation in spherical coordinates may be simplified by adding $0 = \frac{2}{r}\nabla \cdot \underline{v}$ to the component shown above. This term is zero due to the continuity equation (mass conservation). See Bird et. al.

www.chem.mtu.edu/~fmorriso/cm310/Navier.pdf

Slow flow around a sphere (Stokes Flow)



Gravity

$$\underline{g} = -g\hat{e}_z$$

$$\hat{e}_z = \cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta$$

(See inside back cover; do some algebra)

$$\boxed{\underline{g} = -\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta = \begin{pmatrix} -\cos\theta \\ \sin\theta \\ 0 \end{pmatrix}_{r\theta\phi}}$$

(Morrison, Example 8.2)

Slow flow around a sphere (Stokes Flow)

Because the flow is not unidirectional, we have to consider the left-hand-side of the Navier-Stokes

Steady flow of an incompressible, Newtonian fluid around a sphere

Equation of Motion:

$$\boxed{\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla P + \mu\nabla^2\underline{v} + \rho\underline{g}}$$

(do we *have* to?)

Slow flow around a sphere (Stokes Flow)

$$\underline{v} = \begin{pmatrix} v_r \\ v_\theta \\ v_\phi \end{pmatrix}_{r\theta\phi} \qquad \underline{g} = \begin{pmatrix} -\cos\theta \\ \sin\theta \\ 0 \end{pmatrix}_{r\theta\phi} \qquad p = p(r,\theta)$$

Steady flow of an incompressible, Newtonian fluid around a sphere

Creeping Flow

Eqn of Continuity:

$$\left(\frac{1}{r^2}\frac{\partial\left(r^2 v_r\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial v_\theta \sin\theta}{\partial\theta}\right) = 0$$

Eqn of Motion:

$$\rho\left(\underbrace{\cancel{\frac{\partial \underline{v}}{\partial t}}}_{\text{steady state}} + \underbrace{\cancel{\underline{v}\cdot\nabla\underline{v}}}_{\text{neglect inertia}}\right) = \underbrace{-\nabla P + \mu\nabla^2\underline{v} + \rho\underline{g}}_{\text{SOLVE}}$$

BC1: no slip at sphere surface

BC2: velocity goes to v_∞ far from sphere

Slow flow around a sphere (Stokes Flow)

SOLUTION: *Creeping Flow around a sphere*

(we neglected inertia, i.e. LHS of Navier-Stokes)

$$\underline{v} = \begin{pmatrix} v_\infty\left[1 - \frac{3}{2}\frac{R}{r} + \frac{1}{2}\left(\frac{R}{r}\right)^3\right]\cos\theta \\ -v_\infty\left[1 - \frac{3}{4}\frac{R}{r} - \frac{1}{4}\left(\frac{R}{r}\right)^3\right]\sin\theta \\ 0 \end{pmatrix}_{r\theta\phi}$$

$$P = P_0 - \rho g r\cos\theta - \frac{3}{2}\frac{\mu v_\infty}{R}\left(\frac{R}{r}\right)^2\cos\theta$$

all the stresses can be calculated from $\underline{v}$

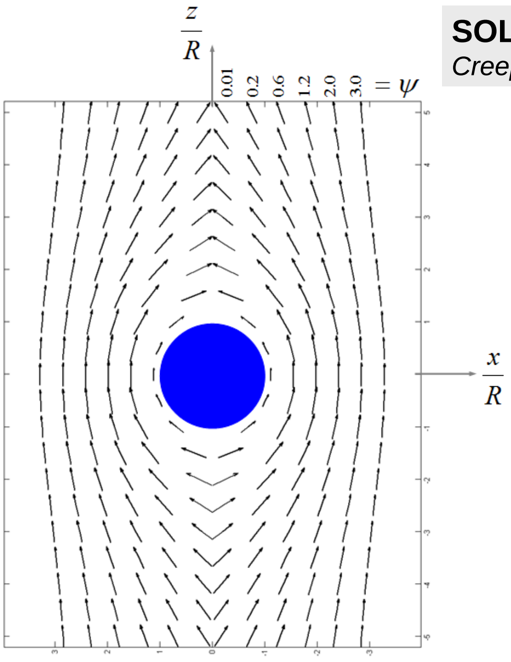
$$\underline{\underline{\tilde{\tau}}} = \mu\left(\nabla\underline{v} + (\nabla\underline{v})^T\right)$$

$$\underline{\underline{\tilde{\Pi}}} = -p\underline{\underline{I}} + \underline{\underline{\tilde{\tau}}}$$

(see the usual handout for stress components)

Morrison, Example 8.2; complete solution steps in Denn, Process Fluid Mechanics (Prentice Hall, 1980)

Slow flow around a sphere (Stokes Flow)



SOLUTION: Velocity Field for *Creeping Flow around a sphere*

- No slip at the walls of the sphere
- Far from the sphere the flow does not feel the presence of the sphere

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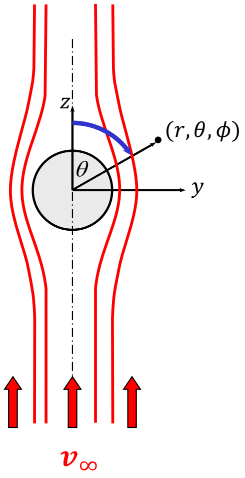
Slow flow around a sphere (Stokes Flow)

To get the force on the sphere (drag), we ask,

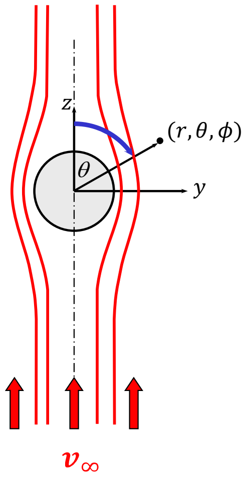


What is the total *z*-direction force on the sphere?

$$\underline{F} = \iint_S \left(\hat{n}\cdot\underline{\underline{\tilde{\Pi}}}\right)\Big|_{surface}\, dS$$

Slow flow around a sphere (Stokes Flow)

To get the force on the sphere, we ask,



Let's try

What is the total *z*-direction force on the sphere?

$$\underline{F} = \iint_S \left(\hat{n}\cdot\underline{\underline{\tilde{\Pi}}}\right)\Big|_{surface} dS$$

Slow flow around a sphere (Stokes Flow)

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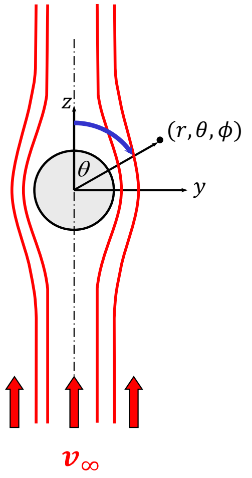

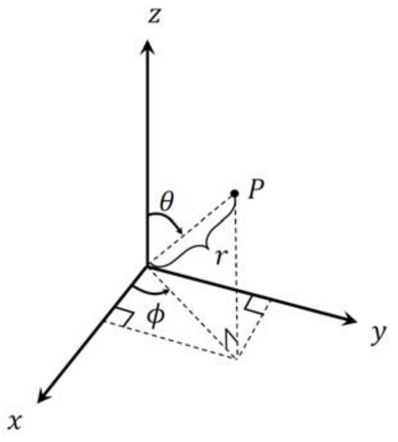

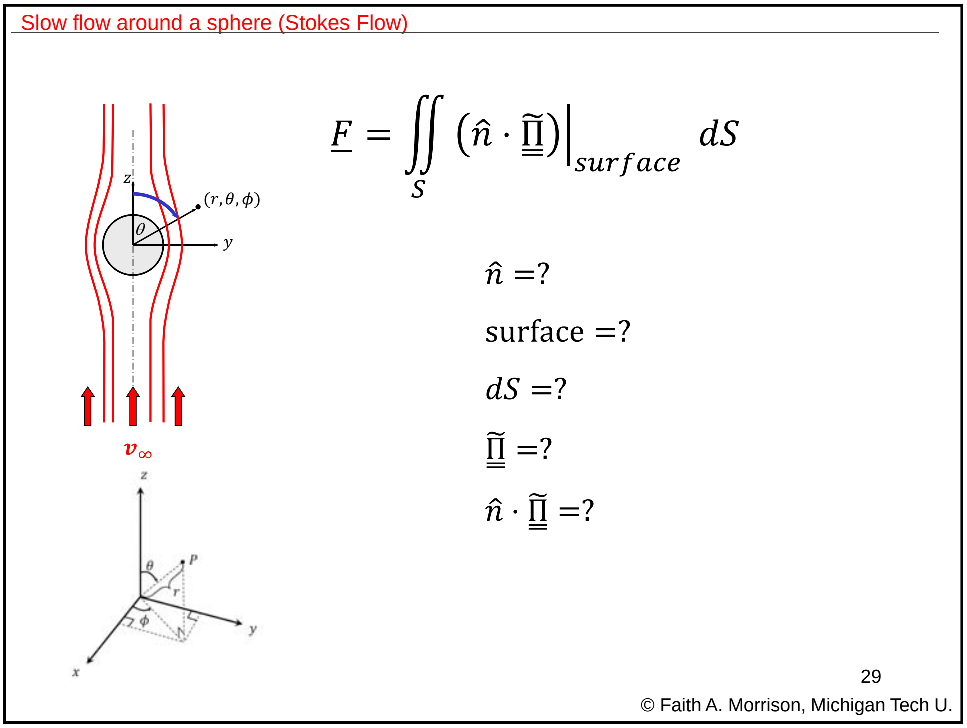

Slide flow around a sphere (Stokes Flow)

$$\underline{F} = \iint_S \left(\hat{n}\cdot\underline{\underline{\tilde{\Pi}}}\right)\Big|_{surface}\, dS$$

$\hat{n}$ =?

surface =?

dS =?

$\underline{\underline{\tilde{\Pi}}}$ =?

$\hat{n}\cdot\underline{\underline{\tilde{\Pi}}}$ =?

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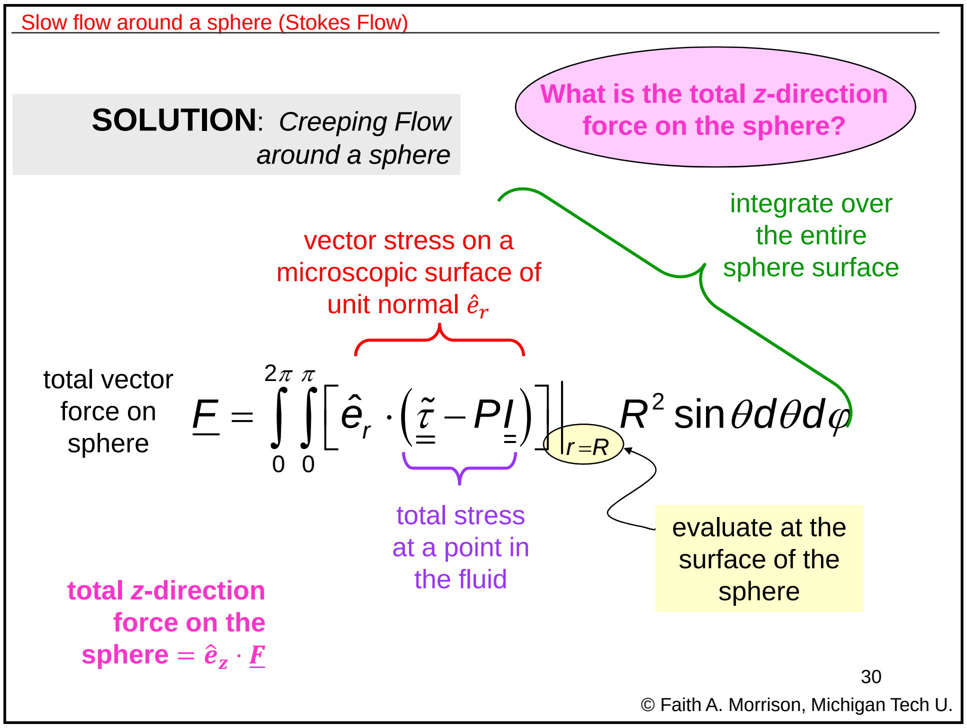

Slow flow around a sphere (Stokes Flow)

SOLUTION: *Creeping Flow around a sphere*

What is the total *z*-direction force on the sphere?

integrate over the entire sphere surface

vector stress on a microscopic surface of unit normal $\hat{e}_r$

total vector force on sphere

$$\underline{F} = \int_0^{2\pi}\int_0^{\pi}\left[\overbrace{\hat{e}_r\cdot\underbrace{\left(\underline{\underline{\tilde{\tau}}}-P\underline{\underline{I}}\right)}_{\text{total stress at a point in the fluid}}}^{\text{vector stress on a microscopic surface of unit normal } \hat{e}_r}\right]\Bigg|_{r=R} R^2\sin\theta\, d\theta\, d\varphi$$

evaluate at the surface of the sphere

total *z*-direction force on the sphere $= \hat{e}_z\cdot\underline{F}$

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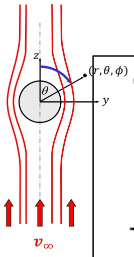
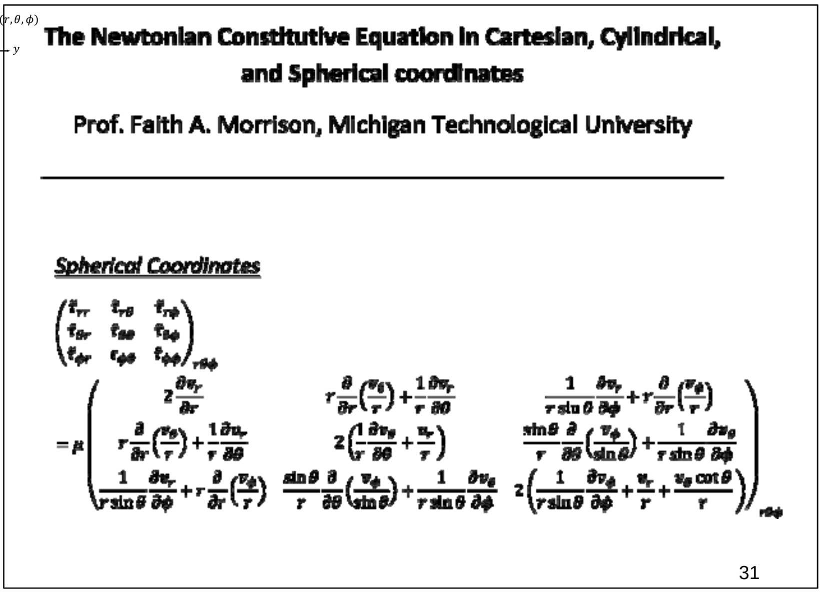
Slow flow around a sphere (Stokes Flow)

$$\underline{F} = \iint_S \left(\hat{n}\cdot\underline{\underline{\tilde{\Pi}}}\right)\Big|_{surface}\, dS$$





The Newtonian Constitutive Equation in Cartesian, Cylindrical, and Spherical coordinates

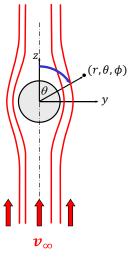
Prof. Faith A. Morrison, Michigan Technological University

Spherical Coordinates

$$\begin{pmatrix} \tilde{\tau}_{rr} & \tilde{\tau}_{r\theta} & \tilde{\tau}_{r\phi} \\ \tilde{\tau}_{\theta r} & \tilde{\tau}_{\theta\theta} & \tilde{\tau}_{\theta\phi} \\ \tilde{\tau}_{\phi r} & \tilde{\tau}_{\phi\theta} & \tilde{\tau}_{\phi\phi} \end{pmatrix}_{r\theta\phi}$$

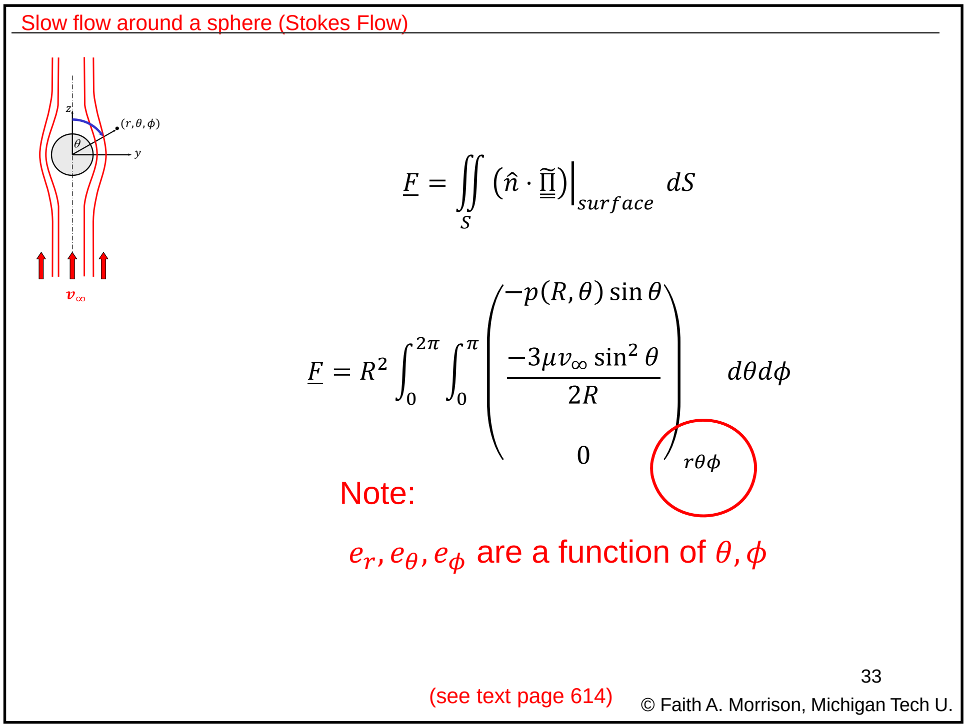
$$= \mu \begin{pmatrix} 2\frac{\partial v_r}{\partial r} & r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial \theta} & \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi}+r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) \\ r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial \theta} & 2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r}{r}\right) & \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} \\ \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi}+r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) & \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} & 2\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}+\frac{v_r}{r}+\frac{v_\theta\cot\theta}{r}\right) \end{pmatrix}_{r\theta\phi}$$

Slow flow around a sphere (Stokes Flow)



$$\underline{F} = \iint_S \left(\hat{n}\cdot\underline{\underline{\tilde{\Pi}}}\right)\Big|_{surface}\, dS$$

$$\underline{F} = R^2 \int_0^{2\pi}\int_0^{\pi} \begin{pmatrix} -p(R,\theta)\sin\theta \\ \frac{-3\mu v_\infty \sin^2\theta}{2R} \\ 0 \end{pmatrix}_{r\theta\phi} d\theta d\phi$$



Slide flow around a sphere (Stokes Flow)

$$\underline{F} = \iint_S \left(\hat{n} \cdot \tilde{\underline{\underline{\Pi}}}\right)\Big|_{surface} dS$$

$$\underline{F} = R^2 \int_0^{2\pi} \int_0^{\pi} \begin{pmatrix} -p(R,\theta)\sin\theta \\ \dfrac{-3\mu v_\infty \sin^2\theta}{2R} \\ 0 \end{pmatrix}_{\boxed{r\theta\phi}} d\theta d\phi$$

Note:

e_r, e_θ, e_ϕ are a function of θ, ϕ

(see text page 614)

Slow flow around a sphere (Stokes Flow)

Force on a sphere (creeping flow limit)

$$\hat{e}_z \cdot \underline{F} = F_z = \overbrace{\underbrace{\frac{4}{3}\pi R^3 \rho g}_{\text{buoyant force}} + \overbrace{2\pi\mu R v_\infty}^{\text{form drag}}}^{\text{comes from pressure}} + \overbrace{\underbrace{4\pi\mu R v_\infty}_{\text{friction drag}}}^{\text{comes from shear stresses}}$$

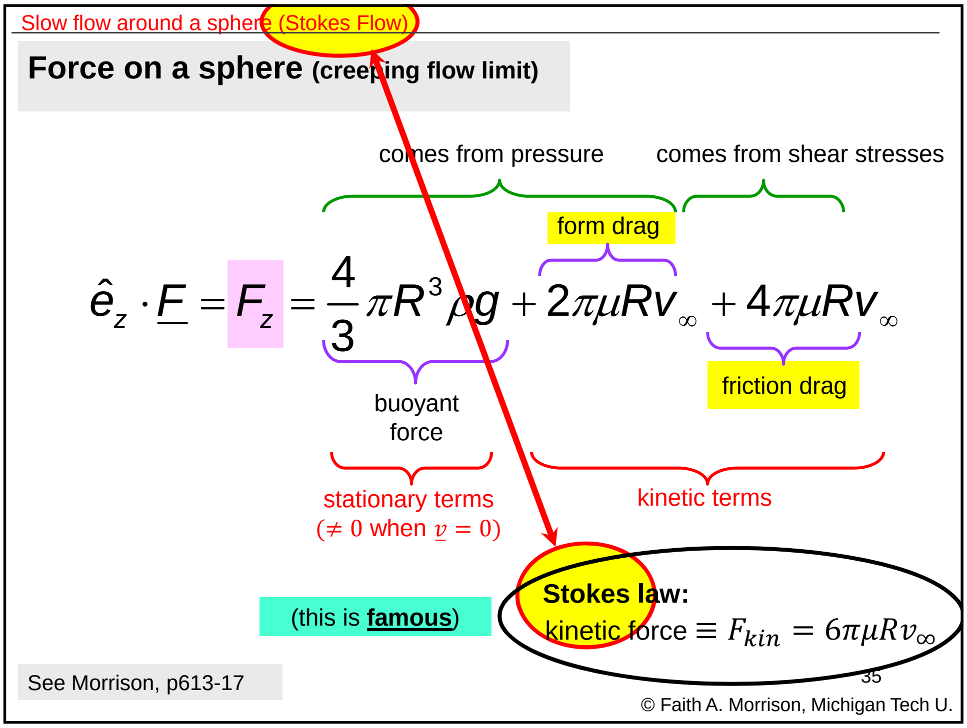
stationary terms ($\neq 0$ when $\underline{v} = 0$)

kinetic terms

(this is **famous**)

Stokes law:
kinetic force $\equiv F_{kin} = 6\pi\mu R v_\infty$

See Morrison, p613-17



Slow flow around a sphere (Stokes Flow)

Force on a sphere (creeping flow limit)

comes from pressure

comes from shear stresses

$$\hat{e}_z \cdot \underline{F} = F_z = \underbrace{\frac{4}{3}\pi R^3 \rho g}_{\text{buoyant force}} + \overbrace{2\pi\mu R v_\infty}^{\text{form drag}} + \underbrace{4\pi\mu R v_\infty}_{\text{friction drag}}$$

stationary terms ($\neq 0$ when $\underline{v} = 0$)

kinetic terms

(this is **famous**)

Stokes law:

kinetic force $\equiv F_{kin} = 6\pi\mu R v_\infty$

See Morrison, p613-17

35

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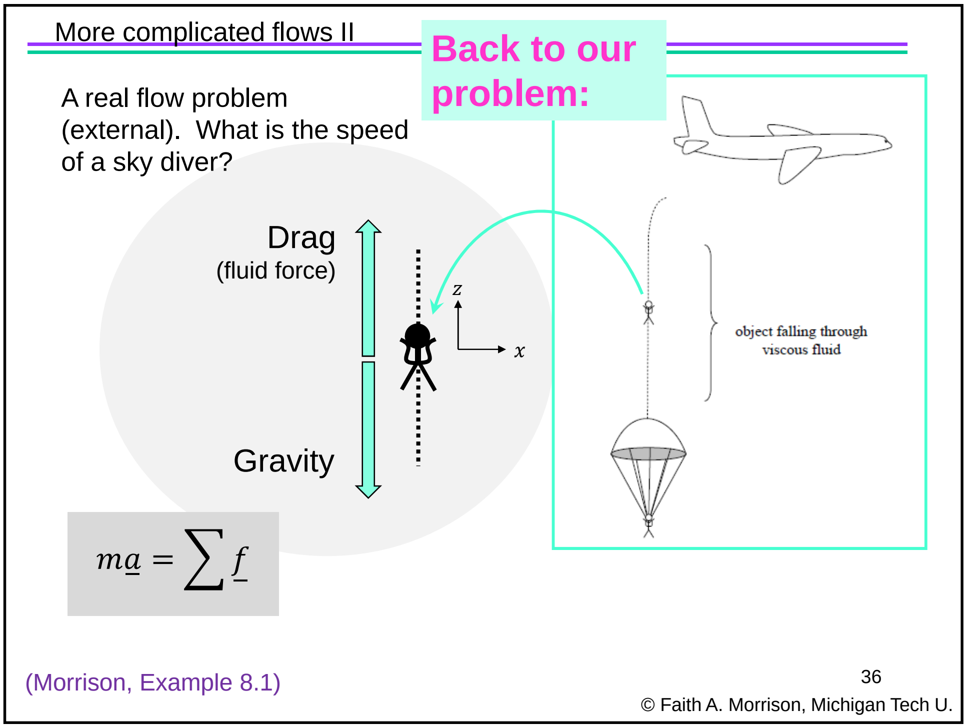

More complicated flows II

Back to our problem:

A real flow problem (external). What is the speed of a sky diver?

$$m\underline{a} = \sum \underline{f}$$

(Morrison, Example 8.1)

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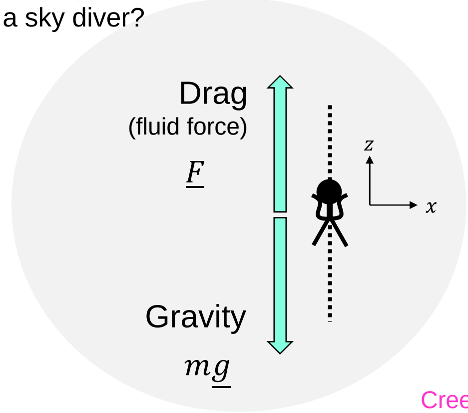
More complicated flows II

A real flow problem (external). What is the speed of a sky diver?



$$m\underline{a} = \sum \underline{f}$$

$$0 = m\underline{g} + \boxed{\underline{F}}$$

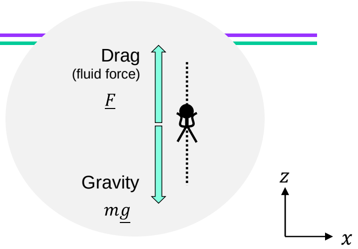
We can get this from the creeping flow solution

Creeping flow drag:

$$\boxed{\hat{e}_z \cdot \underline{F} = F_z = \frac{4}{3}\pi R^3 \rho g + 6\pi\mu R v_\infty}$$

(Morrison, Example 8.1)

More complicated flows II



A real flow problem (external). What is the speed of a sky diver?

$$m\underline{a} = \sum \underline{f}$$

$$0 = m\underline{g} + \underline{F}$$

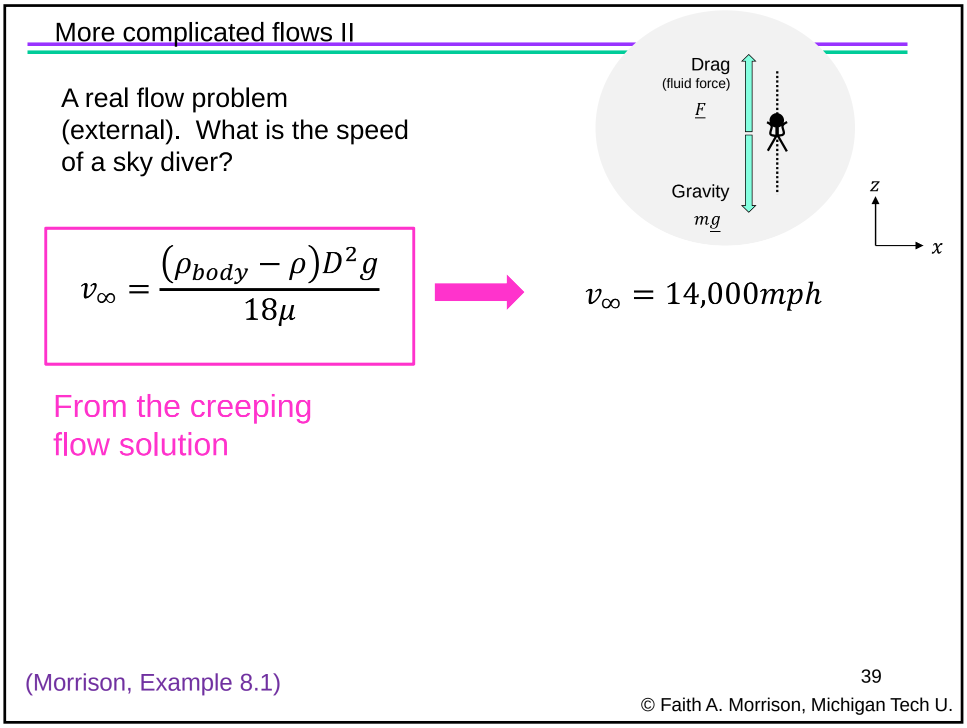
$$\begin{pmatrix} 0 \\ 0 \\ -\dfrac{4\pi R^3 \rho_{body} g}{3} \end{pmatrix}_{xyz} + \begin{pmatrix} 0 \\ 0 \\ \dfrac{4\pi R^3 \rho g}{3} + 6\pi R\mu v_\infty \end{pmatrix}_{xyz} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

$$\boxed{v_\infty = \frac{(\rho_{body} - \rho)D^2 g}{18\mu}}$$

From the creeping flow solution

(see inside front cover)

(Morrison, Example 8.1)



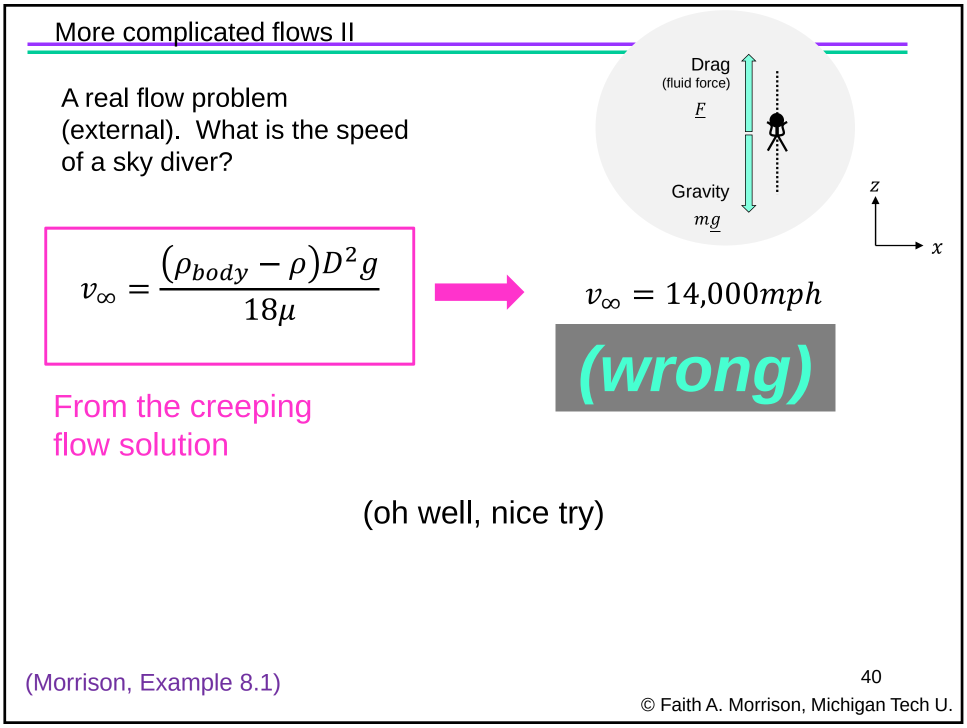
More complicated flows II

A real flow problem (external). What is the speed of a sky diver?

$$\boxed{v_\infty = \frac{(\rho_{body} - \rho)D^2 g}{18\mu}} \Rightarrow v_\infty = 14{,}000mph$$

From the creeping flow solution

(Morrison, Example 8.1)



More complicated flows II

A real flow problem (external). What is the speed of a sky diver?

$$\boxed{v_\infty = \frac{(\rho_{body} - \rho)D^2 g}{18\mu}} \Rightarrow v_\infty = 14{,}000mph$$

(wrong)

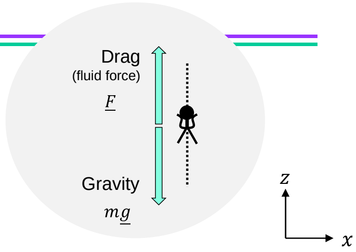
From the creeping flow solution

(oh well, nice try)

(Morrison, Example 8.1)

More complicated flows II

A real flow problem (external). What is the speed of a sky diver?



$$\boxed{v_{\infty} = \frac{\left(\rho_{body} - \rho\right)D^{2}g}{18\mu}} \Longrightarrow v_{\infty} = 14{,}000mph$$

(wrong)

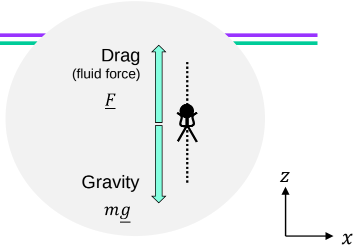
From the creeping flow solution

But, wait! . . .

(Morrison, Example 8.1)

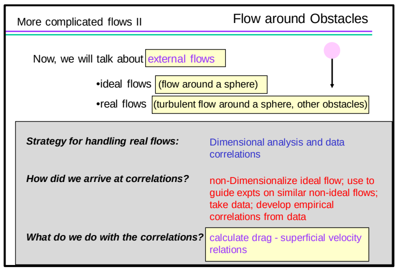
More complicated flows II

A real flow problem (external). What is the speed of a sky diver?



$$m\underline{a} = \sum \underline{f}$$

$$0 = m\underline{g} + \boxed{\underline{F}}$$



More complicated flows II Flow around Obstacles

Now, we will talk about external flows

- ideal flows (flow around a sphere)
- real flows (turbulent flow around a sphere, other obstacles)

Strategy for handling real flows:	Dimensional analysis and data correlations
How did we arrive at correlations?	non-Dimensionalize ideal flow; use to guide expts on similar non-ideal flows; take data; develop empirical correlations from data
What do we do with the correlations?	calculate drag - superficial velocity relations

How about if we do **dimensional analysis,** measure **data correlations** for **non-creeping** flow, and then use the correlations to determine $\underline{F}$?

(Morrison, Example 8.1)

Fast flow around a sphere (dimensional analysis)

Steady flow of an incompressible, Newtonian fluid around a sphere

***Turbulent** Flow*

- Nondimensionalize eqns of change:

$$\left(\frac{\partial \underline{v}^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* \underline{v}^*\right) = -\nabla^* P^* + \frac{1}{\mathrm{Re}} \nabla^{*2} \underline{v}^* + \frac{1}{Fr} \underline{g}^*$$

- Nondimensionalize eqn for $F_{z,kinetic}$:

define dimensionless kinetic force

$$f = C_D = \frac{F_{z,kinetic}}{\frac{\pi D^2}{4}\left(\frac{1}{2}\rho v_\infty^2\right)}$$

drag coefficient

- conclude *f=f(Re) or* $C_D = C_D(Re)$

- take data, plot, develop correlations

Fast flow around a sphere (dimensional analysis)

Steady flow of an incompressible, Newtonian fluid around a sphere

***Creeping** Flow*

What does this look like in **Creeping flow?**

(we have the solution)

Creeping flow:

$$f = C_D = \frac{\overbrace{6\pi\mu R v_\infty}^{F_{z,sphere}\ =\text{Stokes law}}}{\left(\frac{\pi D^2}{4}\right)\left(\frac{1}{2}\rho v_\infty^2\right)} = \frac{24}{Re}$$

From the creeping flow solution

Fast flow around a sphere (dimensional analysis)

How do we apply to
Turbulent flow?

(we need to take data)

Steady flow of an incompressible, Newtonian fluid around a sphere

***Turbulent** Flow*

Turbulent flow: Calculate C_D from terminal velocity of a falling sphere (see Figure 8.13)

$$f = C_D = \frac{\overbrace{F_{drag}}^{F_{drag} = \frac{4\pi R^3 \rho_{body} g}{3} - \frac{4\pi R^3 \rho g}{3}}}{\left(\frac{\pi D^2}{4}\right)\left(\frac{1}{2}\rho v_\infty^2\right)}$$

At terminal speed the net weight is exactly balanced by the viscous retarding force.

$$\left. f = C_D = \frac{(\rho_{sphere} - \rho)}{\rho}\frac{4}{3}\frac{Dg}{v_\infty^2} \right\}$$

all measurable quantities

(see Example 8.4)

Fast flow around a sphere (dimensional analysis)

- take data, plot, develop correlations

Steady flow of an incompressible, Newtonian fluid around a sphere

***Creeping** Flow*

Steady flow of an incompressible, Newtonian fluid around a sphere

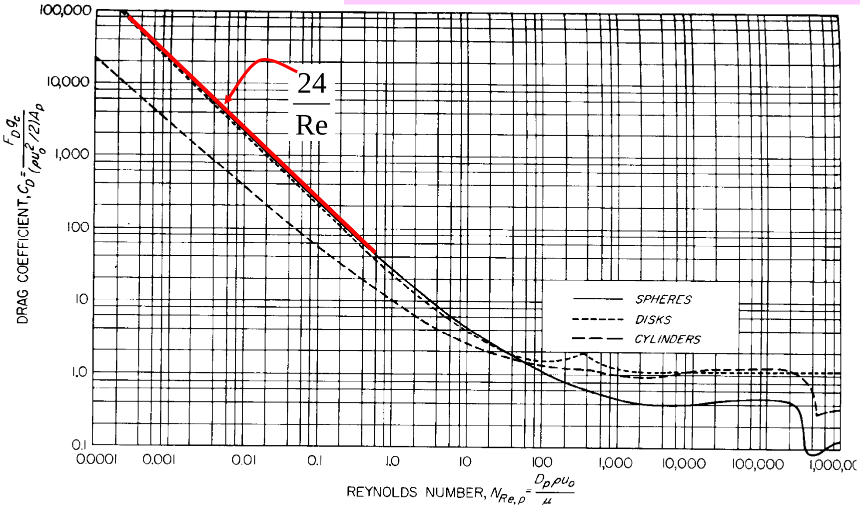
***Turbulent** Flow*

Data of $C_D(\mathrm{Re})$? ➡

Fast flow around a sphere (dimensional analysis)

graphical correlation

Steady flow of an incompressible, Newtonian fluid around a sphere

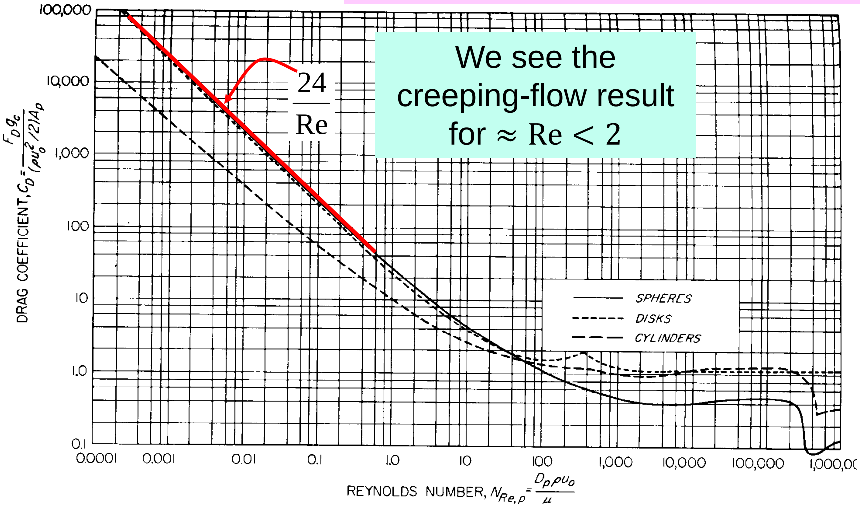


McCabe et al., *Unit Ops of Chem Eng*, 5th edition, p147

Fast flow around a sphere (dimensional analysis)

graphical correlation

Steady flow of an incompressible, Newtonian fluid around a sphere



We see the creeping-flow result for $\approx \text{Re} < 2$

McCabe et al., *Unit Ops of Chem Eng*, 5th edition, p147

Fast flow around a sphere (dimensional analysis)

correlation **equations**

Steady flow of an incompressible, Newtonian fluid around a sphere

creeping	$f = \frac{24}{Re}$	$Re < 0.10$
vortices	$f = 18.5 Re^{-0.60}$	$2 \leq Re \leq 500$
wake flow	$f = 0.44$	$500 \leq Re \leq 200{,}000$

BSL, p194

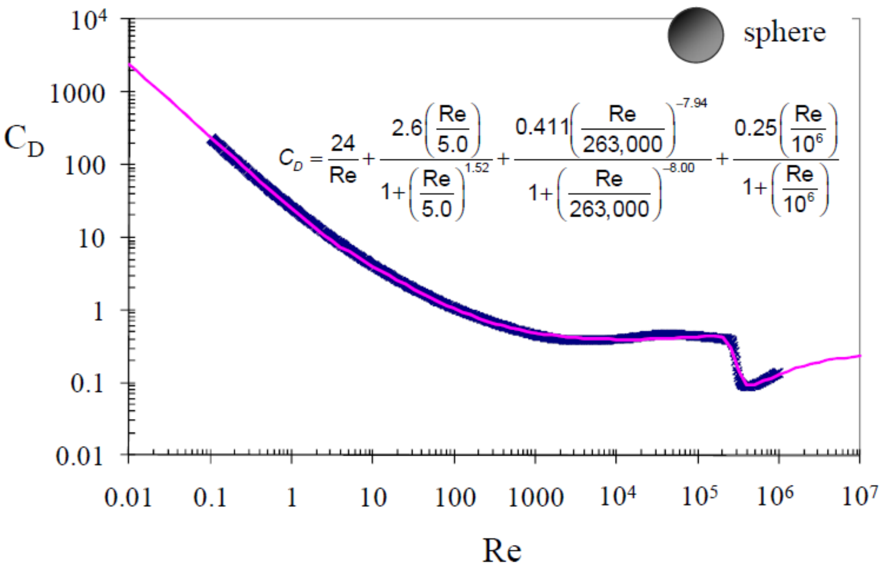
- use correlations in engineering practice
  - particle settling
  - entrained droplets in distillation columns
  - particle separators
  - drop coalescence

(See Denn, Bird et al., Perry's)

Dr. Morrison developed a single, combined correlation

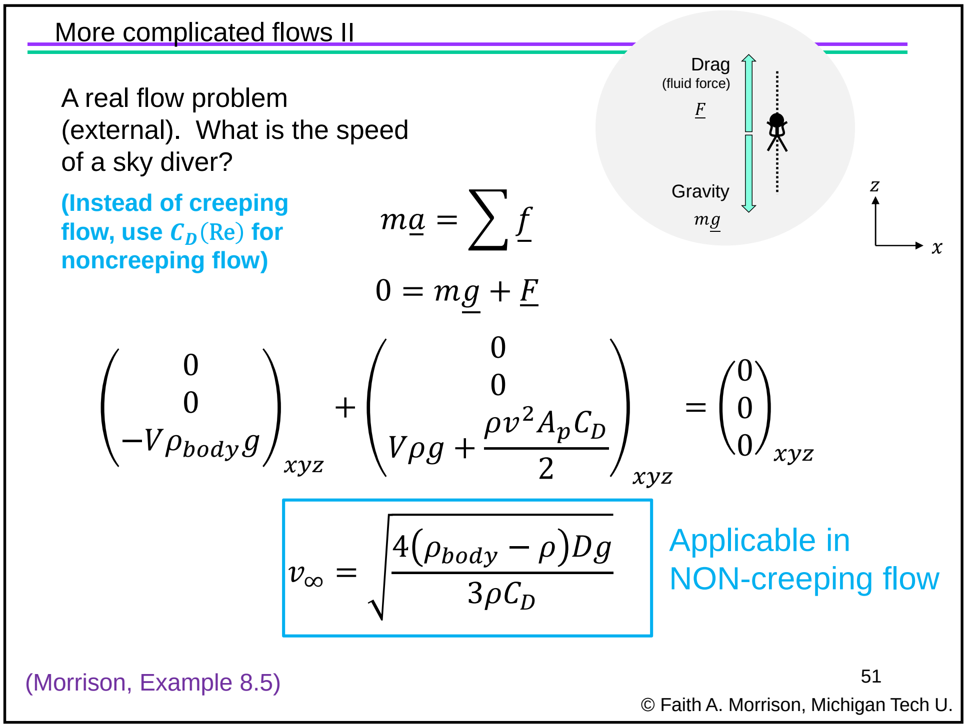
Denn, *Process Fluid Mechanics*, 1980
Bird, Stewart, Lightfoot, *Transport Phenomena*, 1960 and 2006

Fast flow around a sphere (dimensional analysis)



$$C_D = \frac{24}{Re} + \frac{2.6\left(\frac{Re}{5.0}\right)}{1+\left(\frac{Re}{5.0}\right)^{1.52}} + \frac{0.411\left(\frac{Re}{263{,}000}\right)^{-7.94}}{1+\left(\frac{Re}{263{,}000}\right)^{-8.00}} + \frac{0.25\left(\frac{Re}{10^6}\right)}{1+\left(\frac{Re}{10^6}\right)}$$

F. A. Morrison *An Introduction to Fluid Mechanics*, Cambridge 2013, Figure 8.13, p625



More complicated flows II

A real flow problem (external). What is the speed of a sky diver?

(Instead of creeping flow, use $C_D(\mathrm{Re})$ for noncreeping flow)

$$m\underline{a} = \sum \underline{f}$$

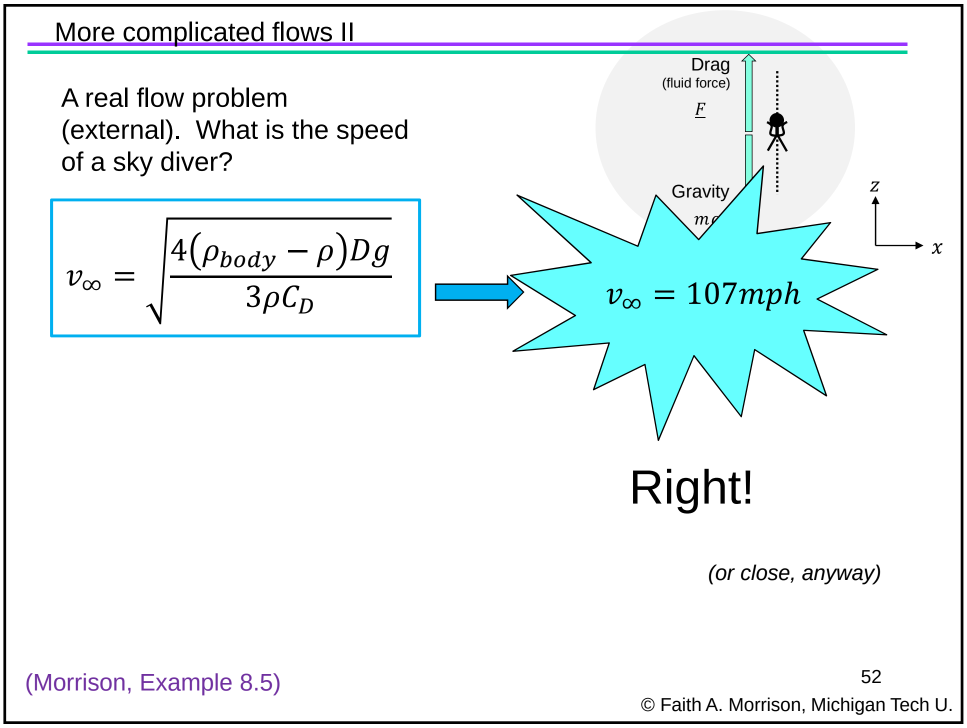
$$0 = m\underline{g} + \underline{F}$$

$$\begin{pmatrix} 0 \\ 0 \\ -V\rho_{body}g \end{pmatrix}_{xyz} + \begin{pmatrix} 0 \\ 0 \\ V\rho g + \dfrac{\rho v^2 A_p C_D}{2} \end{pmatrix}_{xyz} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

$$\boxed{v_\infty = \sqrt{\frac{4(\rho_{body} - \rho)Dg}{3\rho C_D}}}$$

Applicable in NON-creeping flow

(Morrison, Example 8.5)



More complicated flows II

A real flow problem (external). What is the speed of a sky diver?

$$\boxed{v_\infty = \sqrt{\frac{4(\rho_{body} - \rho)Dg}{3\rho C_D}}}$$

$$v_\infty = 107mph$$

Right!

(or close, anyway)

(Morrison, Example 8.5)

More complicated flows II

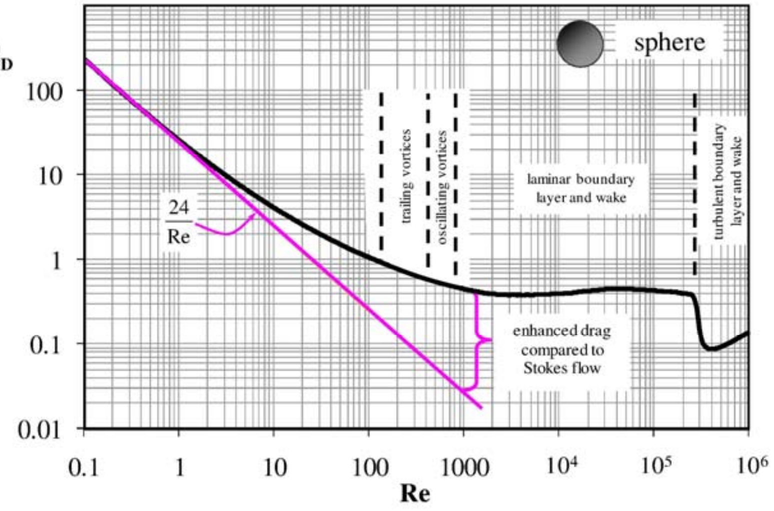
Powerful:

Solving <u>never-before-solved</u> problems.

Left to explore:

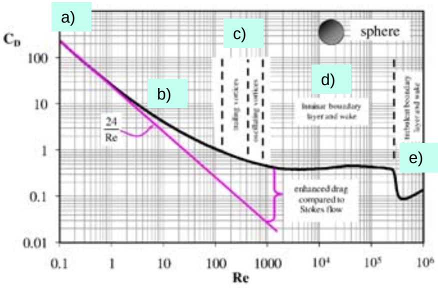
- What is non-creeping flow like?
  (boundary layers)
- Viscosity dominates in creeping flow, what about the flow where inertia dominates?
  (potential flow)
- What about mixed flows (viscous+inertial)?
  (boundary layers)
- What about really complex flows (curly)?
  (vorticity; irrotational+circulation)

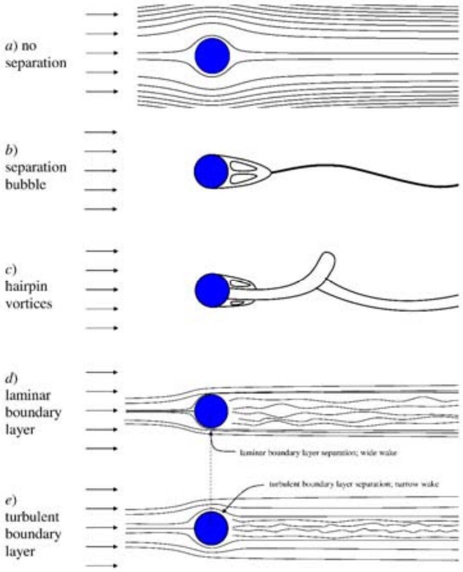
Fast flow around a sphere (dimensional analysis)



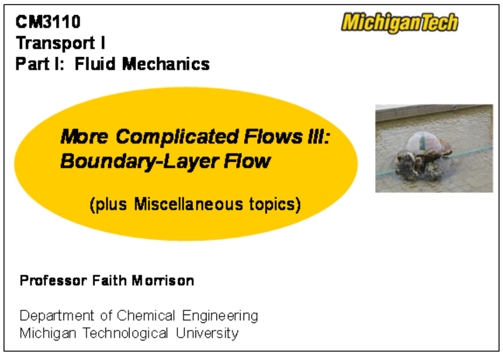
F. A. Morrison *An Introduction to Fluid Mechanics*, Cambridge 2013, Figure 8.23, p650

Fast flow around a sphere (dimensional analysis)





One more essential topic in Fluid Mechanics: **Boundary Layers**


CM3110
Transport I
Part I: Fluid Mechanics

More Complicated Flows III: Boundary-Layer Flow

(plus Miscellaneous topics)

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University