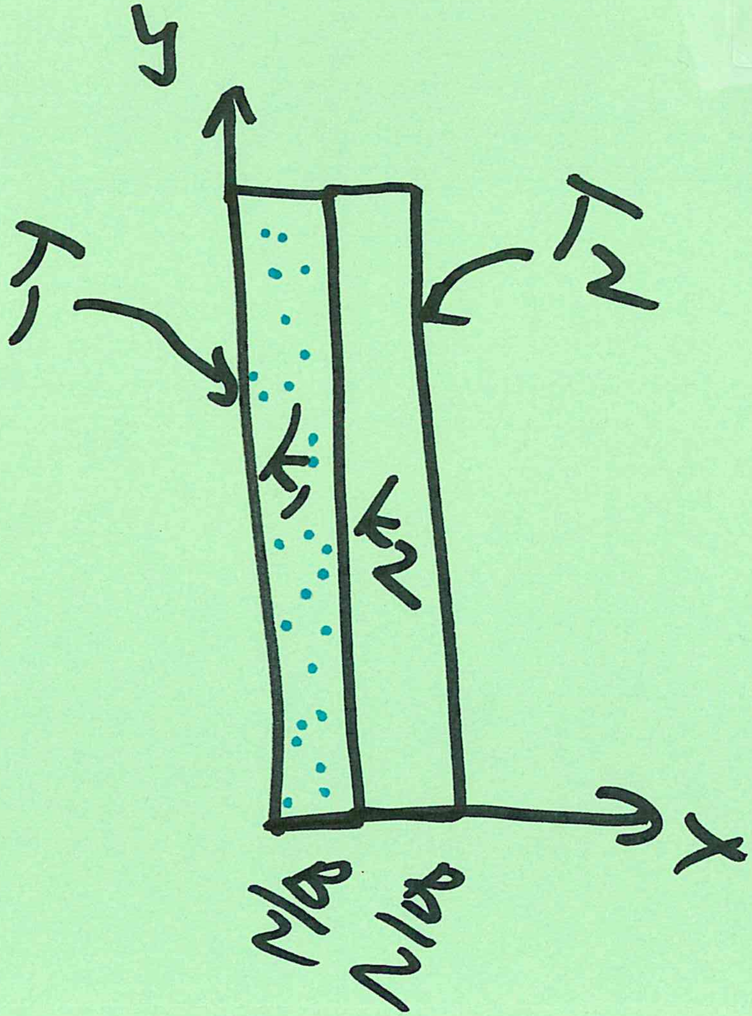
LEC 14-15

10-29-2019

1/3

1D Steady heat conduction in composite slab



What is the temp profile?

Soln: micro E-bal separately in each material

k_1 material

k_2 material

micro E bal in k_1 material:

(same as before w/
$k \to k_1$,
different BCs)

SOLN:

$T = c_1 x + c_2$ BC: $x = 0$ $T = T_1$

$\frac{q_x}{A} = -k_1 \frac{dT}{dx} = -k_1 c_1$ $x = \frac{B}{2}$ $T = T_3$

unknown

$x = 0$: $\boxed{T_1 = c_2}$

$x = \frac{B}{2}$: $T_3 = c_1 \frac{B}{2} + T_1$

$\boxed{c_1 = \frac{2}{B}(T_3 - T_1)}$

k_1-material

$\Rightarrow \boxed{T = \frac{2}{B}(T_3 - T_1)x + T_1}$

Micro E bal in $\boxed{k_2}$ material:

(Same as before w/
$k \to k_2$, different BC's)

SOLN

$$T = C_3 x + C_4$$

$$\frac{q_x}{A} = -k_2 \frac{dT}{dx} = -k_2 C_3$$

BC: $x = \frac{B}{2}$ $T = T_3$

$x = B$ $T = T_2$

$$\left.\begin{aligned} x = \tfrac{B}{2}: \quad & T_3 = C_3 \tfrac{B}{2} + C_4 \\ x = B: \quad & T_2 = C_3 B + C_4 \end{aligned}\right\}$$

solve
(2 eqns,
2 unknowns)

Subtract:

$$(T_3 - T_2) = c_3\left(\frac{B}{2} - B\right)$$

$$= c_3\left(-\frac{B}{2}\right)$$

$$\boxed{c_3 = (T_3 - T_2)\left(-\frac{2}{B}\right)}$$

substitute:

$$c_4 = T_2 - c_3 B = T_2 + \frac{2}{\cancel{B}}(T_3 - T_2)\cancel{B}$$

$$c_4 = T_2 + 2T_3 - 2T_2$$

$$\boxed{c_4 = 2T_3 - T_2}$$

Substitute back:

$$T = C_3 x + C_4$$

k-2 mat'l:

$$\boxed{T = (T_3 - T_2)\left(-\frac{2}{B}\right)x + 2T_3 - T_2}$$

But what is T_3?

<u>FLUXES</u> must be the <u>same</u>

$$\Rightarrow \quad \frac{q_x}{A} = -k_2 \frac{dT}{dx}\bigg|_{\substack{\text{in} \\ k_2 \text{ mat'l}}} = -k_1 \frac{dT}{dx}\bigg|_{\substack{\text{in} \\ k_1 \text{ mat'l}}}$$

Evaluating:

$$\cancel{-}k_2\left(\frac{\cancel{2}}{\cancel{B}}\right)(T_2 - T_3) = \cancel{-}k_1\left(\frac{\cancel{2}}{\cancel{B}}\right)(T_3 - T_1)$$

$$k_2 T_2 - k_2 T_3 = k_1 T_3 - k_1 T_1$$

$$(k_1 + k_2)\, T_3 = k_1 T_1 + k_2 T_2$$

$$\boxed{T_3 = \left(\frac{k_1 T_1 + k_2 T_2}{k_1 + k_2}\right)}$$

What is the flux?

$$\frac{q_x}{A} = -k_1 \frac{2}{B}(T_3 - T_1) = -k_1\left(\frac{2}{B}\right) T_3 + k_1\left(\frac{2}{B}\right) T_1$$

$$= k_1\left(\frac{2}{B}\right) T_1 - k_1 \frac{2}{B}\left(\frac{k_1 T_1 + k_2 T_2}{k_1 + k_2}\right)$$

$$\frac{q_x}{A} = k_1\left(\frac{2}{B}\right)\left[\frac{T_1 \cancel{k_1} + T_1 k_2 - \cancel{k_1 T_1} - k_2 T_2}{k_1 + k_2}\right]$$

$$\boxed{\frac{q_x}{A} = k_1 k_2 \left(\frac{2}{B}\right) \frac{(T_1 - T_2)}{(k_1 + k_2)}}$$