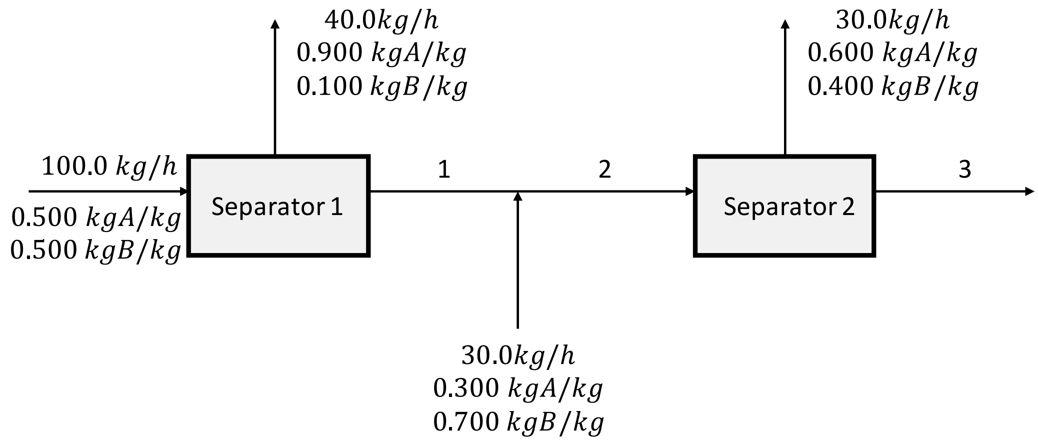
HW1

Michigan Technological University
Chemical Engineering

CM3120 Transport/Unit Ops 2—Prerequisite Material

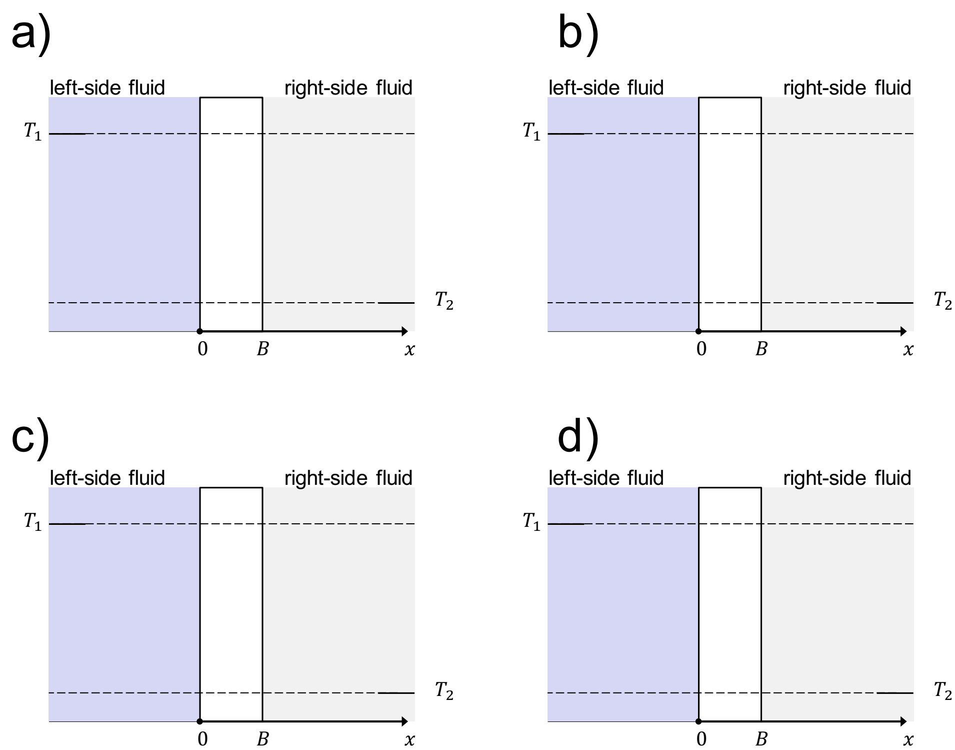
1. (FR) One thousand kilograms per hour of a mixture of benzene (B) and toluene (T) containing 50wt% benzene by mass is separated by distillation into two fractions. The mass flow rate of benzene in the overhead stream is $450\ kg\ B/h$ and that of toluene in the bottom stream is $475\ kg\ T/h$. The operation is at steady state. What are the mass flow rates and compositions of all the streams? Answer: mass fraction benzene in overhead $y_B = 0.95$, in bottoms $x_B = 0.095$.
2. (FR) For the process depicted in the flowchart below, calculate the unknown flow rates and compositions of streams 1, 2, and 3. Partial Answer: $m_1 = 60.0\ kg/h$ total; $m_3 = 60.0\ kg/h$; $x_2 = 0.255\ kg\ A/kg$



3. (FR) Air is bubbled through a drum of liquid hexane at a rate of $0.100\ kmol/min$. The gas stream leaving the drum contains $10.0\ mol\%$ hexane vapor. Air may be considered insoluble in liquid hexane. How long will it take to vaporize $10.0\ m^3$ of the liquid? Answer: $6880\ minutes$.
4. Water enters the inside of a double-pipe heat exchanger and flows at $3.4\ gpm$ (measured at 25^oC). The inlet water temperature is 17.0^oC and the outlet water temperature is 42.3^oC. What is the heat input to the water stream? Answer: $23\ kJ/s$
5. (Geankoplis 4.5-4) A double-pipe heat exchanger is used to heat water flowing at the rate of $13.85\ kg/s$ from 54.5 to 87.8^oC. The outer fluid is a hot gas flowing at $54{,}430\ kg/h$, flowing countercurrently, and entering at 427^oC. The mean heat capacity of the gas is $1.005\ kJ/kg\ K$. The overall heat transfer coefficient of the heat exchanger is $69.1\ W/m^2K$. What is the exit-gas temperature? What is the heat transfer area of the heat exchanger? Answer: $299.5^oC, 97m^2$.
6. (Geankoplis 4.1-1). What is the heat loss per m^2 of surface area for a temporary insulating wall of a food cold storage room, where the outside wall temperature is 299.9 K and the inside wall temperature is 276.5 K. The wall is made of one inch of corkboard having a thermal conductivity of 0.0433 W/m K. Answer: $40\ W/m^2$
7. A very tall, very wide slab (thermal conductivity $= k$) of thickness B is positioned between two fluids as shown in the figures below. For each of the following five situations, sketch the steady

state temperature profile, both in the fluids and in the slab itself. Use the axes shown and draw your answers carefully. Note: $T_1 > T_2$. Answer: See instructor.

a. Boundary conditions are specified that the left wall temperature (and left fluid temperature) is T_1 and the right wall temperature (and right fluid temperature) is T_2.
b. Same as a), except k of the slab is twice as large.
c. Boundary conditions are specified that the bulk fluid temperature on the left is T_1 and the bulk fluid temperature on the right is T_2. Heat transfer coefficients h_1 (left side) and h_2 (right side) are finite.
d. Same as c), except h_1 is very large (infinite).

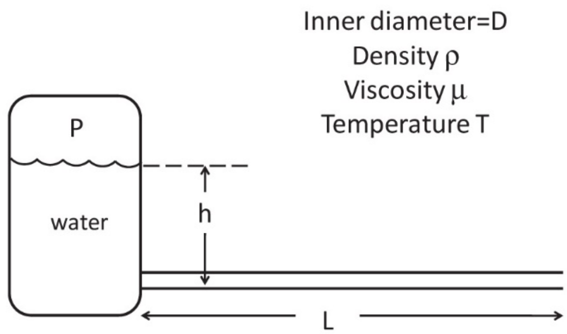


8. (IFM 2.30 & 8.33) What is a boundary layer? Give an example of a boundary layer that you have experienced. What types of forces dominate inside a momentum boundary layer? What type dominate outside the momentum boundary layer? Answer: See notes below
9. Solve this simplified momentum balance for the velocity component $v_z(r)$ for pressure-driven flow in a pipe of radius R. The boundary conditions are: at the wall ($r = R$) the velocity is zero; at the center ($r = 0$), the velocity is at a maximum.

$$\frac{\Delta p}{L} = \frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)$$

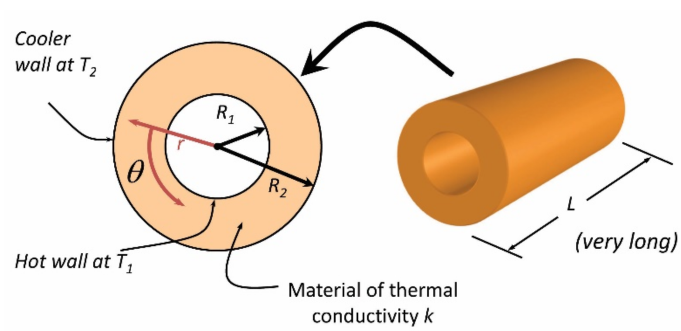
where μ is the viscosity (constant), r is the coordinate variable of the cylindrical coordinate system, and $\Delta p/L$ is the pressure drop per unit length (constant). Answer: $v_z(r) = \frac{\Delta p}{4\mu L}(r^2 - R^2)$

10. (IFM 1.36) For the piping system sketched in Figure 1.58, what is the average fluid velocity at the pipe discharge? Please write the answer in terms of the variables defined in the figure. You may neglect friction in the apparatus. The tank is not open to the atmosphere; the pipe discharges fluid to the atmosphere. P is the absolute pressure inside the vapor space over the fluid in the tank, and P is approximately constant (large tank).



Answer: $v_2 = \sqrt{2(gh + (P - P_{atm})/\rho)}$, (turbulent flow assumed).

11. What is the steady state temperature profile in a cylindrical metal shell (a pipe) if the inner wall has a temperature of T_1 and the outer wall is at a lower temperature T_2?



Answer: $\frac{T_2 - T(r)}{T_2 - T_1} = \frac{\ln\frac{R_2}{r}}{\ln\frac{R_2}{R_1}}$ or, equivalently, $\frac{T_1 - T(r)}{T_2 - T_1} = \frac{\ln\frac{R_1}{r}}{\ln\frac{R_2}{R_1}}$

12. In the situation in problem 11, if we did not know the temperature of the inner wall, but instead we know the bulk fluid temperature of hot fluid moving in turbulent flow, what would be the appropriate boundary conditions for the wall temperature at R_1 and R_2? Answer: See notes below.

13. The solution for the temperature profile $T(r)$ in radial heat flow in an annulus with Newton's law of cooling boundary conditions is (see problem 11 for geometry and problem 12 for the boundary conditions; T_{b1} and T_{b2} are the constant bulk fluid temperatures on the inside and outside respectively):

$$T(r) - T_{b2} = \frac{(T_{b1} - T_{b2})\left(\ln\left(\frac{R_2}{r}\right) + \frac{k}{h_2 R_2}\right)}{\frac{k}{h_2 R_2} + \ln\left(\frac{R_2}{R_1}\right) + \frac{k}{h_1 R_1}}$$

What is the energy flux in the radial direction for this temperature profile? Answer: See notes below.

14. Water (temperature $= 25.0^0 C$) flows through a horizontal pipe (nominal 2-in Schedule 40 steel pipe, inner radius $0.02625\ m$; outer radius 0.03016, length $= 2500\ m$) in turbulent flow with Reynolds number of Re=1.2×10^4. Calculate the following:
    a. Average velocity in m/s
    b. Pressure drop in psi
    Answers: $0.20\ m/s$; $4.4\ psi$
15. We plan to heat a fluid (material properties given below) by sending it through the inside pipe of a double pipe heat exchanger (inner pipe dimensions: nominal 2-in Schedule 40 steel pipe, inner radius $0.02625\ m$; outer radius 0.03016, length $= 1.6\ m$) with condensing steam flowing on the outside; due to the condensing steam, the inside surface temperature of the inner pipe is maintained constant at $95^o C$ along the entire length of the pipe. The fluid enters at $13^o C$ at a mass flow rate of 3.2 kg/s. We do not know what the exit temperature will be.

    The fluid's material properties, <u>which do not vary significantly with temperature</u>, are:
    density = $1022\ kg/m^3$
    heat capacity = $4.3\ kJ/kgK$
    thermal conductivity = $0.605\ W/mK$
    viscosity = $8.3 \times 10^{-4}\ Pa\ s$)
    a. What is the value of heat transfer coefficient h_{lm} that characterizes the heat transfer in this apparatus?
    b. **(Stretch)** What is the expected exit temperature of the fluid?

    Answers: $h_{lm} = 5300\ W/m^2 K, T_{exit} = 21^o C$
16. (FR Ex8.4-2) One hundred moles per hour of liquid n-hexane at $25^o C$ and $7.0\ bar$ is vaporized and heated to $300^o C$ at constant pressure. How much heat $\dot{Q}$ must be supplied? Answer: $2.4\ kW$
17. (IFM 6.21, p487) If the velocity vector $\underline{v}$ in m/s for flow through a pipe of radius 0.012 m is given by the following expression, what is the volumetric flow rate Q of fluid through the pipe? Answer: $0.0027 m^3/s$

$$\underline{v}[m/s] = \begin{pmatrix} 0 \\ 0 \\ 12.0\left(1 - \left(\frac{r}{0.012}\right)^2\right) \end{pmatrix}_{r\theta z}$$

18. (IFM 6.23, p488) For a pressure-driven flow in a slit, the total stress tensor $\underline{\underline{\tilde{\Pi}}}$ is given here, where $P = Cx$ is the pressure, μ is viscosity, B is the gap half-height, and A is the velocity at the centerline. The fluid is an incompressible Newtonian fluid. What is the x-component of the force due to fluid on the bottom plate? Answer: $F_x = 2\mu ALW/B$

$$v_x(z) = A\left(1 - \frac{z^2}{B^2}\right)$$

$$\underline{\underline{\tilde{\Pi}}} = \begin{pmatrix} -P & 0 & -\left(\frac{2\mu A}{B^2}\right)z \\ 0 & -P & 0 \\ -\left(\frac{2\mu A}{B^2}\right)z & 0 & -P \end{pmatrix}_{xyz}$$

19. **(Stretch)** (IFM Ex7.5) The water utility (water at 25^oC) in a home supplies water at $60psig$. The water flows into 80 ft of 1-inch Schedule 40 pipe (ID= $0.0266m$) connected to 20 ft of half-inch Schedule 40 pipe (ID= $0.0158m$). What is the water flow rate Q? Answer: $31\ gpm$
20. (IFM Ex7.16) What is the pressure drop per unit length for room-temperature toluene flowing at $1.0\ ml.min$ through a packed column with the following characteristics: void fraction= 0.39; packing specific surface area= $72.0\ cm^{-1}$, diameter= $1.0cm$. Answer: $170\ Pa/m$
21. For the situation described below, heat transfer is taking place that can be described by Newton's law of cooling, $q = hA\Delta T$. How would you determine a good value for the heat transfer coefficient h in this case? What quantities would need to be <u>measured</u> so that you could complete your calculation? What quantities would you need to <u>look up</u>? Briefly explain your reasoning. Answer: See notes below
    a. *A steel pipe (length = $15m$) with an extremely hot liquid passing through it loses heat to the environment as it crosses a room in a chemical plant. What is the heat loss per meter of pipe?*
22. (IFM 3.21, p221, modified). The velocity vector for steady pressure-driven flow through a slit, in Cartesian coordinates, is given below. Sketch the velocity field. Use the appropriate Cartesian coordinate system, which may be deduced from the velocity field.

$$\underline{v} = \begin{pmatrix} 0 \\ v_y(x) \\ 0 \end{pmatrix}_{xyz}$$

$$v_y(x) = A\left(1 - \frac{(x-H)^2}{H^2}\right)$$

23. **(Stretch)** (IFM 6.25). What is the torque on a rod turning in an infinite bath of fluid? The radius of the rod is R, the length is L, and the rod turns at angular velocity Ω in a fluid of viscosity μ. You may leave your answer in terms of the unknown velocity distribution.

Notes

4. What are the assumptions that lead to the final version of the macroscopic energy balance that you use?
5. Take a look a t the significant figures on this problem. Do you have any comments?
6. What does the energy balance tell you in this problem?
7. What is the meaning of heat transfer coefficient h? What is the difference between heat transfer coefficients h and U?
8. CM3110 discusses momentum boundary layers. We now will have thermal and mass-transfer boundary layers. Why do you think this physics exists in all three transport fields?
9. This is Poiseuille flow (laminar pipe flow).
10. What is the pressure at the bottom of the tank? Answer: $P + \rho g h$.
11. Can you solve for $T(r)$ if the wall temperatures are not known? Solve for the case when all that is known is the bulk fluid temperatures of the inside and outside fluids. The associated heat transfer coefficients are h_1 and h_2 in this case. Answer: See problem 13.
12. At $r = R_1, \frac{q_r}{A} = -k\frac{dT}{dr} = h(T_{b1} - T(R_1))$. This is the Newton's law of cooling boundary condition.
13. $\text{Flux} = \frac{k(T_{b1}-T_{b2})}{\frac{k}{h_2R_2}+\ln\left(\frac{R_2}{R_1}\right)+\frac{k}{h_1R_1}}\left(\frac{1}{r}\right)$. Can you derive the temperature profile in this problem?
14. Use correlations associated with the Moody plot. See the exam handout sheet.
15. Use correlations associated with the flow (Seider-Tate). See the exam handout sheet.
16. Enthalpy is a state function. Look up $\Delta\hat{H}_{vap}$ and devise a path from the current temperature to the temperature of the vaporization and then for the vapor to final temperature.
17. Two dimensional integral. See exam handout sheet.
18. Two dimensional integral. See exam handout sheet.
19. Prandtl equation is a handy data correlation for this, but you could use others.
20. Ergun equation is the appropriate data correlation for flow through packed beds.
21. Both natural convection and radiation may be important. For natural convection, use GrPr correlations. See exam handout sheet for data correlations. For radiation, use the expression for h_{rad}.
22. Note the boundary locations ($x = 0, x = H, x = 2H$; this gives you a hint of what coordinate system was in use. Draw a vector centered at a variety of values of x, for example $x = 0, 0.2H, 0.4H, 0.6H, 0.8H, H, 1.2H, 1.4H, 1.6H, 1.8H, 2H$. Evaluate v_y at each point and sketch. It's a parabola.
23. Two dimensional integral. See exam handout sheet for the expression. Answer: $\underline{\mathcal{T}} = 2\pi R^2 L\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]\Big|_{r=R}\hat{e}_z$