HW3

Michigan Technological University
Chemical Engineering

Module	2021 Problems	Topic
3	1	For steady EMCD calc mol, mass frac, flux
3	2	Show Fick's law in mass units becomes the molar version
3	3	Flux identities: show relation to mass/molar average velocity
3	4	Calculate molar flux for EMCD given partial pressures
3	5	**Micro SAMB: evaporation of a hemispherical drop**
3	6	**Micro SAMB: water dripping from condensation on cold pipe**
3	7	**Micro SAMB: film model around sphere; diffusion varies with T**
3	8	Plot the concentration profile from Example 2 from lecture
3	9	Given a scenario, identify the driving force for mass transfer
3	10	**Micro SAMB: Reaction at catalyst surface; calc conc distribution**
3	11	For stagnant B, calc individual velocities in diffusion
3	12	For stead EMCD, calculate individual velocities
3	13	Extend example 1 from class; part c quasi steady state
3	14	**Micro SAMB: Evaporating droplet; what is the flux**
3	15	**Micro SAMB: Water evaporating in tank; dilute case**

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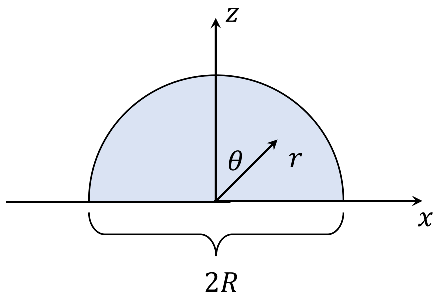
CM3120 Transport/Unit Ops 2

1. A gas mixture of helium (species A) and nitrogen (species B) is contained in a pipe at constant pressure $1.00\ atm$ and temperature $298\ K$ (ideal gas). The bulk molar velocity $\underline{v}^*$ is zero (no bulk flow); helium diffuses in the z-direction and nitrogen diffuses in the $(-z)$-direction. At one end of the pipe (point 1) the partial pressure of helium is $0.60\ atm$ at the other end, $20.0 cm$ away (point 2), the partial pressure is $0.20\ atm$.
    a. What are the mole fractions of the two species at the two points?
    b. What are the mass fractions of the two species at the two points?
    c. What is the average molecular weight of the mixture at the two points?
    d. If the diffusion coefficient of the mixture at steady state is $\mathcal{D}_{AB} = 0.687 \times 10^{-4} m^2/s$ what is the flux of helium between the two points at steady state? What is the flux of nitrogen?
    e. What is the diffusion velocity for each species at each location? (Diffusion velocity for species A $\equiv (\underline{v}_A - \underline{v}^*)$)

    Answer: a) $x_A = 0.60, x_A = 0.20, b)\ \omega_A = 0.1765$, $\omega_A = 0.0345$, c) $13.61 \frac{g}{mol}, 23.21 \frac{g}{mol}$, d) flux=$5.6 \times 10^{-3} mol\ A/\ s\ m^2$, e) for point 1 $(\underline{v}_A - \underline{v}^*) = 2.3 \times 10^{-4} m/s$, for point 2 $(\underline{v}_A - \underline{v}^*) = 6.9 \times 10^{-4} m/s$
2. **(stretch)** The fundamental form of Fick's law is
$$\underline{j}_A = \rho_A(\underline{v}_A - \underline{v}) = -\rho\mathcal{D}_{AB}\nabla\omega_A$$
This expression is written in mass units. Show that in molar units Fick's law becomes:
$$\underline{J}_A^* = c_A(\underline{v}_A - \underline{v}^*) = -c\mathcal{D}_{AB}\nabla x_A$$
3. Show that the following relationships for the various versions of the species mass/molar fluxes hold (do not assume Fick's law to show the equivalence):
    a. $\underline{N}_A + \underline{N}_B = c\underline{v}^*$
    b. $\underline{n}_A + \underline{n}_B = \rho\underline{v}$
    c. $\underline{j}_A + \underline{j}_B = 0$
    d. $\underline{J}_A^* + \underline{J}_B^* = 0$

    In a sentence or two, what are the differences among the various fluxes in the question above? Why have we chosen to use such a variety of nomenclature?
4. Geankoplis 6.1-1, revised. A gas mixture of methane (species A) and helium (species B) is contained in a tube at constant pressure $101.32\ kPa$ and temperature $298\ K$. At one point the partial pressure of methane is $60.79\ kPa$ and at a point $0.0200\ m$ away the partial pressure is $20.26\ kPa$. The two species are undergoing equimolar counter diffusion. What are the combined molar fluxes N_{Ax} and N_{Bx} for each component? Answer: $5.52 \times 10^{-5} kmol/s\ m^2$
5. A hemispherical drop of liquid water lies on a flat surface (see figure below). The water evaporates from the surface through the still air adjacent to the droplet. The temperature and pressure are constant, and the diffusion is slow, so the size of the droplet is almost unchanged during the evaporation. Model this problem in such a way that we can determine the concentration distribution near the surface of the droplet. How can we

calculate the evaporation rate? Answer for concentration distribution in the film: $\left(\frac{1-x_A}{1-x_{A\infty}}\right) = \left(\frac{1-x_{AR}}{1-x_{A\infty}}\right)^{r/R}$; see also notes.



6. A cold-water pipeline runs through a hot, humid space in a processing plant. Water condenses onto the pipe and drips onto the ground. Model this problem in such a way that we can estimate the rate of water dripping from the pipe. Answer:

$$\ln\left(\frac{1-x_A}{1-x_{A1}}\right) = \left(\frac{\ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)}{\ln\left(\frac{R_2}{R_f}\right)}\right)\ln\left(\frac{r}{R_f}\right)$$

7. **Stretch:** A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. We can model the diffusion process as shown in the figure below. The temperature in the film is not constant but varies as $T(r)/T(\mathcal{R}_1) = (r/\mathcal{R}_1)^n$; note that this means that both the diffusivity D_{AB} and the concentration $c = P/RT$ are a function of position through their temperature dependences. What is the water mole fraction in the film as a function of radial position? You may assume ideal gas properties for air; you may assume that the diffusivity varies with temperature as follows: BSL2 p550

$$D_{AB}(T)/D_{AB,1} = \left(T/T(\mathcal{R}_1)\right)^{3/2}$$

Answer: $\left(\frac{1-x_A}{1-x_{A1}}\right) = \left(\frac{1-x_{A2}}{1-x_{A1}}\right)^{\left(1/\mathcal{R}_1^{1+\frac{n}{2}} - 1/r^{1+\frac{n}{2}}\right)/\left(1/\mathcal{R}_1^{1+\frac{n}{2}} - 1/\mathcal{R}_2^{1+\frac{n}{2}}\right)}$

8. In lecture we solved the problem below. Using Excel, plot the concentration distribution. Consider a variety of reasonable values of the concentrations at the boundaries.

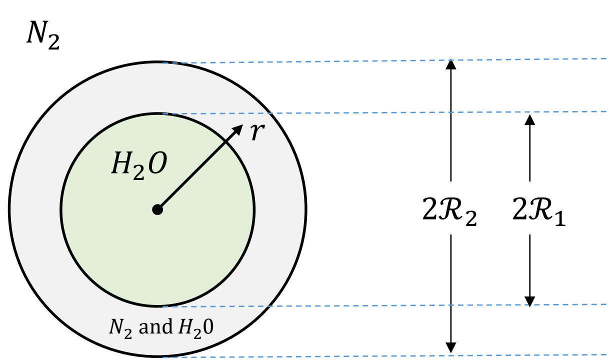
   A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). What is the water mole fraction in the air as a function of distance from the droplet? Answer for $x_A(r)$:

$$\frac{1-x_A}{1-x_{A1}} = \left(\frac{1-x_{A2}}{1-x_{A1}}\right)^{\frac{1/\mathcal{R}_1 - 1/r}{1/\mathcal{R}_1 - 1/\mathcal{R}_2}}$$

9. In your own words, answer the following:

a. What is “ordinary diffusion”? How is it different from mass transfer that is not “ordinary diffusion”?
b. For all the homework 3 problems employing the microscopic species A mass balance to determine a concentration profile, identify the driving force for mass transfer.

10. An irreversible, instantaneous chemical reaction ($2A \rightarrow B$) takes place at a catalyst surface, as discussed in class (Example 3). The reaction is “diffusion-limited” because the rate of completion of the reaction is determined by the rate of diffusion through the “film” near the catalyst surface. Calculate the steady state composition distribution in the film (x_A (z)). Answer: $(1 - x_{A0}/2)^{(1-z/\delta)} = (1 - x_A/2)$.
11. In a particular region in space, species A (gas) is diffusing through stagnant species B (also a gas) at steady state. The situation may be considered to be one-dimensional (1D) diffusion. The steady state flux of species A (with respect to stationary coordinates) is $5.0 \times 10^{-5}\ kmol\ A\ /m^2 s$. At one point in the diffusion space, the concentration of A is $0.0050\ kmol/m^3$, and the concentration of B is $0.036\ kmol/m^3$. What is your estimate of the individual velocities of species A and B along the direction of mass transfer? What is the average molar velocity? Answer: $\underline{v}^* = 0.0012\ m/s$.
12. In a particular region in space species A (gas) and species B (also a gas) form a binary mixture in which steady equimolar counter diffusion is occurring (see p499 of WRF, posted at https://pages.mtu.edu/~fmorriso/cm3120/WRF2015_pp499-500.pdf). The situation may be considered to be one-dimensional (1D) diffusion. The steady state flux of species A is $5.0 \times 10^{-5}\ kmol\ A\ /m^2 s$ (with respect to stationary coordinates). At one point in the diffusion space, the concentration of A is $0.0050\ kmol/m^3$, and the concentration of B is $0.036\ kmol/m^3$. What is your estimate of the average velocities of molecules of species A and B along the direction of mass transfer? In this region of space, what is the average molar velocity? If A is water and B is nitrogen, what is the mass-average velocity $\underline{v}$? Answer: $v_z = -4.6 \times 10^{-4} m/s$; $v_z^* \approx 0$. Comment on the difference.



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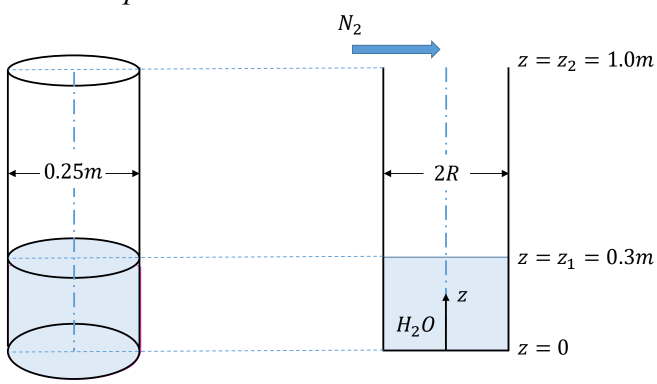
13. (Problem discussed during diffusion lecture 1).

    Water (40^oC, $1.0\ atm$) slowly and steadily evaporates into nitrogen (40^oC, $1.0\ atm$) from the bottom of a cylindrical tank ($0.25m$ diameter) as shown in the figure below. A stream of dry nitrogen flows slowly past the open tank. The mole fraction of water in the gas at the top opening of the tank is 0.02. *The geometry is as shown in the figure. What is water mole fraction as a function of vertical position? You may assume ideal gas properties. What is the rate of water evaporation?*



    a. Carry out the algebra that was skipped in lecture.
    b. Plot the water mole fraction (x_A) as a function of vertical position (z) for the situation
    c. What is the rate of water evaporation for the original situation 40^oC? Answer: $7.8 \times 10^{-5}\frac{g}{s} = 0.3\ g/hr$.
    d. (problem continues HW4)

14. A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air. What is the flux of water at the surface of the droplet? Answer:

$$\frac{1 - x_A}{1 - x_{A1}} = \left(\frac{1 - x_{A2}}{1 - x_{A1}}\right)^{\frac{1/\mathcal{R}_1 - 1/r}{1/\mathcal{R}_1 - 1/\mathcal{R}_2}}$$

15. The problem described in problem 3.13 above was solved in class for the general case. Assuming the gas is dilute in water, solve the problem for the mole fraction of water as a function of position. Answer: $x_A = x_{A1} + (x_{A1} - x_{A2})\left(\frac{z - z_1}{z_1 - z_2}\right)$

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References:

1. Bird, R. B., W. E. Stewart, and E. N. Lightfoot, Transport Phenomena, 2nd edition, Wiley, NY, 2002. Widely known in the ChemE world as "BSL." Also the 1960 edition is fine too, and available used ("the red book").
2. Christie J. Geankoplis, Transport Processes and Unit Operations, 4th Edition, Prentice Hall, New York (2003). Two hardbound copies are on reserve in the library.
3. Frank P. Incropera, David P. DeWitt, Theodore L. Bergman, Adrienne S. Lavinem, Fundamentals of Heat and Mass Transfer, 6th edition, Wiley, New York (2006).
4. Welty, James R., Gregory L. Rorrer, and David C. Foster, Fundamentals of Momentum, Heat, and Mass Transfer, 6th edition, Wiley, New York, 2015.

Notes:

1. Start with a sketch. The diffusion is 1D. Raoult's law can help you find the composition. Use a basis to switch from moles to mass. Pay attention to units.
2. Bird, Stewart and Lightfoot 2nd edition (BSL2) page 534 has some great identities to use when performing these transformations. One such identity is $\nabla\omega_A = (M_A M_B \nabla x_A)/M^2$
3. See the text, or BSL (reference at the bottom of my sheet), or my sheets. WRF 24.2
4. Equimolar counter diffusion means $N_A = -N_B$.
5. Hemisphere diffusion.
   a. The air is stagnant. Include boundary conditions at the surface and far away from the sphere.
   b. The molar flux $\underline{N}_A$ can tell us the evaporation rate at steady state. WRF 25.3
6. Sketch the cold pipe problem and see where diffusion comes into the picture. The molar flux $\underline{N}_A$ can tell us the condensation rate at steady state. Near the water film, the air will be saturated with water (Raoult's law). What temperature would you use? The location of the film edge is at R_f and far away is R_2. WRF 25.4
7. Note that for an ideal gas $c = \frac{n}{V} = \frac{P}{RT}$ and thus c is a function of temperature. $\mathcal{D}_{AB}$ is not constant. B is stagnant.
8. Look at the plot in the second lecture of module 3.
9. See module 3 lecture I.
10. Consider a film model. Work out the relationship between N_A and N_B from the reaction stoichiometry.
11. Be sure to sketch the problem. Note that B is stagnant. Stretch: If A is water and B is nitrogen, what is $\underline{v}$? Comment on the difference. Answer: $8.2 \times 10^{-4} m/s$ WRF 24.4
12. Equimolar counter diffusion implies that $N_A = -N_B$.
13. Calculate the flux and from that the water transfer rate. See Geankoplis' Table 6.2-1 for diffusion coefficients.
14. Set up a film model.
15. See lecture IIa, module 3.