Show:

$$\underline{J}_A^* = -c \mathcal{D}_{AB} \nabla x_A$$

Soln

The fundamental form of Fick's law is based on mass:

$$\underline{j}_A = \rho_A (\underline{v}_A - \underline{v}) = -\rho \mathcal{D}_{AB} \nabla w_A$$

$$\rho = Mc \qquad \nabla w_A = \frac{M_A M_B \nabla x_A}{M^2}$$

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$$-\cancel{M}c\,\mathcal{D}_{AB}\,\frac{M_A M_B\,\nabla x_A}{M^{\cancel{2}}} = \rho_A(\underline{v}_A - \underline{v})$$

$$-c\,\mathcal{D}_{AB}\,\nabla x_A = \frac{M}{M_A M_B}\,\rho_A(\underline{v}_A - \underbrace{\underline{v}}_{w_A\underline{v}_A + w_B\underline{v}_B})$$

$$= \frac{M\rho_A}{M_A M_B}\,(\underbrace{\underline{v}_A - w_A\underline{v}_A}_{\underline{v}_A(1-w_A) = \underline{v}_A w_B} - w_B\underline{v}_B)$$

$$-c\mathcal{D}_{AB}\nabla x_A = \left(w_B \underline{v}_A - w_B \underline{v}_B\right)\frac{\cancel{M}\rho_A}{M_A \cancel{M_B}}$$

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$$w_B = \frac{x_B \cancel{M_B}}{\cancel{M}} \qquad w_B = \frac{x_B \cancel{M_B}}{\cancel{M}} \qquad \frac{\rho_A}{M_A} = c_A$$

$$-c\mathcal{D}_{AB}\nabla x_A = \left[\underbrace{(1-x_A)\underline{v}_A - x_B \underline{v}_B}_{\underbrace{\underline{v}_A - x_A v_A - x_B v_B}_{\underline{v}_A - \underline{v}^*}}\right] c_A$$

$$-c\mathcal{D}_{AB}\nabla x_A = c_A\left(\underline{v}_A - \underline{v}^*\right) = \underline{J}_A^* \quad //$$