

FACTORS FOR UNIT CONVERSIONS

Quantity	Equivalent Values
Mass	$1 \text{ kg} = 1000 \text{ g} = 0.001 \text{ metric ton} = 2.20462 \text{ lb}_m = 35.27392 \text{ oz}$ $1 \text{ lb}_m = 16 \text{ oz} = 5 \times 10^{-4} \text{ ton} = 453.593 \text{ g} = 0.453593 \text{ kg}$
Length	$1 \text{ m} = 100 \text{ cm} = 1000 \text{ mm} = 10^6 \text{ microns } (\mu\text{m}) = 10^{10} \text{ angstroms } (\text{\AA})$ $= 39.3701 \text{ in} = 3.28084 \text{ ft} = 1.09361 \text{ yd} = 0.000621371 \text{ mile}$ $1 \text{ ft} = 12 \text{ in.} = 1/3 \text{ yd} = 0.3048 \text{ m} = 30.48 \text{ cm}$
Volume	$1 \text{ m}^3 = 1000 \text{ liters} = 10^6 \text{ cm}^3 = 10^6 \text{ ml}$ $= 35.31467 \text{ ft}^3 = 219.969 \text{ imperial gallons} = 264.172 \text{ gal}$ $= 1056.69 \text{ qt}$ $1 \text{ ft}^3 = 1728 \text{ in}^3 = 7.48052 \text{ gal} = 0.028317 \text{ m}^3 = 28.3168 \text{ liters}$ $= 28,316.8 \text{ cm}^3$
Force	$1 \text{ N} = 1 \text{ kg}\cdot\text{m}/\text{s}^2 = 10^5 \text{ dynes} = 10^5 \text{ g}\cdot\text{cm}/\text{s}^2 = 0.22481 \text{ lb}_f$ $1 \text{ lb}_f = 32.174 \text{ lb}_m\cdot\text{ft}/\text{s}^2 = 4.4482 \text{ N} = 4.4482 \times 10^5 \text{ dynes}$
Pressure	$1 \text{ atm} = 1.01325 \times 10^5 \text{ N}/\text{m}^2 \text{ (Pa)} = 101.325 \text{ kPa} = 1.01325 \text{ bars}$ $= 1.01325 \times 10^6 \text{ dynes}/\text{cm}^2$ $= 760 \text{ mm Hg at } 0^\circ \text{ C (torr)} = 10.333 \text{ m H}_2\text{O at } 4^\circ \text{ C}$ $= 14.696 \text{ lb}_f/\text{in}^2 \text{ (psi)} = 33.9 \text{ ft H}_2\text{O at } 4^\circ \text{ C}$ $100 \text{ kPa} = 1 \text{ bar}$
Energy	$1 \text{ J} = 1 \text{ N}\cdot\text{m} = 10^7 \text{ ergs} = 10^7 \text{ dyne}\cdot\text{cm}$ $= 2.778 \times 10^{-7} \text{ kW}\cdot\text{h} = 0.23901 \text{ cal}$ $= 0.7376 \text{ ft}\cdot\text{lb}_f = 9.47817 \times 10^{-4} \text{ Btu}$
Power	$1 \text{ W} = 1 \text{ J}/\text{s} = 0.23885 \text{ cal}/\text{s} = 0.7376 \text{ ft}\cdot\text{lb}_f/\text{s} = 9.47817 \times 10^{-4} \text{ Btu}/\text{s} = 3.4121 \text{ Btu}/\text{h}$ $= 1.341 \times 10^{-3} \text{ hp (horsepower)}$
Viscosity	$1 \text{ Pa}\cdot\text{s} = 1 \text{ N}\cdot\text{s}/\text{m}^2 = 1 \text{ kg}/\text{m}\cdot\text{s}$ $= 10 \text{ poise} = 10 \text{ dynes}\cdot\text{s}/\text{cm}^2 = 10 \text{ g}/\text{cm}\cdot\text{s}$ $= 10^3 \text{ cp (centipoise)}$ $= 0.67197 \text{ lb}_m/\text{ft}\cdot\text{s} = 2419.088 \text{ lb}_m/\text{ft}\cdot\text{h}$
Density	$1 \text{ kg}/\text{m}^3 = 10^{-3} \text{ g}/\text{cm}^3$ $= 0.06243 \text{ lb}_m/\text{ft}^3$ $10^3 \text{ kg}/\text{m}^3 = 1 \text{ g}/\text{cm}^3 = 62.428 \text{ lb}_m/\text{ft}^3$
Volumetric Flow	$1 \text{ m}^3/\text{s} = 35.31467 \text{ ft}^3/\text{s} = 15,850.32 \text{ gal}/\text{min} \text{ (gpm)}$ $1 \text{ gpm} = 6.30902 \times 10^{-5} \text{ m}^3/\text{s} = 2.228009 \times 10^{-3} \text{ ft}^3/\text{s} = 3.7854 \text{ liter}/\text{min}$ $1 \text{ liter}/\text{min} = 0.26417 \text{ gpm}$

Temperature	$T(^{\circ}C) = \frac{5}{9}[T(^{\circ}F) - 32]$ $T(^{\circ}F) = \frac{9}{5}T(^{\circ}C) + 32 = 1.8T(^{\circ}C) + 32$
Absolute Temperature	$T(K) = T(^{\circ}C) + 273.15$ $T(^{\circ}R) = T(^{\circ}F) + 459.67$
Temperature Interval (ΔT)	$1 C^{\circ} = 1 K = 1.8 F^{\circ} = 1.8 R^{\circ}$ $1 F^{\circ} = 1 R^{\circ} = (5/9) C^{\circ} = (5/9) K$

USEFUL QUANTITIES

$$SG = \rho(20^{\circ}C) / \rho_{\text{water}}(4^{\circ}C)$$

$$\rho_{\text{water}}(4^{\circ}C) = 1000 \text{ kg/m}^3 = 62.43 \text{ lb}_m/\text{ft}^3 = 1.000 \text{ g/cm}^3$$

$$\rho_{\text{water}}(25^{\circ}C) = 997.08 \text{ kg/m}^3 = 62.25 \text{ lb}_m/\text{ft}^3 = 0.99709 \text{ g/cm}^3$$

$$g = 9.8066 \text{ m/s}^2 = 980.66 \text{ cm/s}^2 = 32.174 \text{ ft/s}^2$$

$$\begin{aligned} \mu_{\text{water}}(25^{\circ}C) &= 8.937 \times 10^{-4} \text{ Pa}\cdot\text{s} = 8.937 \times 10^{-4} \text{ kg/m}\cdot\text{s} \\ &= 0.8937 \text{ cp} = 0.8937 \times 10^{-2} \text{ g/cm}\cdot\text{s} = 6.005 \times 10^{-4} \text{ lb}_m/\text{ft}\cdot\text{s} \end{aligned}$$

Composition of air:	N ₂	78.03%
	O ₂	20.99%
	Ar	0.94%
	CO ₂	0.03%
	H ₂ , He, Ne, Kr, Xe	<u>0.01%</u>
		100.00%

$$M_{\text{air}} = 29 \text{ g/mol} = 29 \text{ kg/kmol} = 29 \text{ lb}_m/\text{lbmole}$$

$$\hat{C}_{p,\text{water}}(25^{\circ}C) = 4.182 \text{ kJ/kg}\cdot\text{K} = 0.9989 \text{ cal/g}\cdot^{\circ}C = 0.9997 \text{ Btu/lb}_m\cdot^{\circ}F$$

$$\begin{aligned} R &= 8.314 \text{ m}^3\cdot\text{Pa/mol}\cdot\text{K} = 0.08314 \text{ liter}\cdot\text{bar/mol}\cdot\text{K} = 0.08206 \text{ liter}\cdot\text{atm/mol}\cdot\text{K} \\ &= 62.36 \text{ liter}\cdot\text{mm Hg/mol}\cdot\text{K} = 0.7302 \text{ ft}^3\cdot\text{atm/lbmole}\cdot^{\circ}R \\ &= 10.73 \text{ ft}^3\cdot\text{psia/lbmole}\cdot^{\circ}R \\ &= 8.314 \text{ J/mol}\cdot\text{K} \\ &= 1.987 \text{ cal/mol}\cdot\text{K} = 1.987 \text{ Btu/lbmole}\cdot^{\circ}R \end{aligned}$$

The Equation of Continuity and the Equation of Motion in Cartesian, cylindrical, and spherical coordinates

CM3110 Fall 2011 Faith A. Morrison

Continuity Equation, Cartesian coordinates

$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$

Continuity Equation, cylindrical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r} \frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

Continuity Equation, spherical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial(\rho r^2 v_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\rho v_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial(\rho v_\phi)}{\partial \phi} = 0$$

Equation of Motion for an incompressible fluid, 3 components in Cartesian coordinates

$$\begin{aligned} \rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) &= -\frac{\partial P}{\partial x} + \left(\frac{\partial \tilde{\tau}_{xx}}{\partial x} + \frac{\partial \tilde{\tau}_{yx}}{\partial y} + \frac{\partial \tilde{\tau}_{zx}}{\partial z} \right) + \rho g_x \\ \rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) &= -\frac{\partial P}{\partial y} + \left(\frac{\partial \tilde{\tau}_{xy}}{\partial x} + \frac{\partial \tilde{\tau}_{yy}}{\partial y} + \frac{\partial \tilde{\tau}_{zy}}{\partial z} \right) + \rho g_y \\ \rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) &= -\frac{\partial P}{\partial z} + \left(\frac{\partial \tilde{\tau}_{xz}}{\partial x} + \frac{\partial \tilde{\tau}_{yz}}{\partial y} + \frac{\partial \tilde{\tau}_{zz}}{\partial z} \right) + \rho g_z \end{aligned}$$

Equation of Motion for an incompressible fluid, 3 components in cylindrical coordinates

$$\begin{aligned} \rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) &= -\frac{\partial P}{\partial r} + \left(\frac{1}{r} \frac{\partial(r \tilde{\tau}_{rr})}{\partial r} + \frac{1}{r} \frac{\partial \tilde{\tau}_{\theta r}}{\partial \theta} - \frac{\tilde{\tau}_{\theta\theta}}{r} + \frac{\partial \tilde{\tau}_{zr}}{\partial z} \right) + \rho g_r \\ \rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) &= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \left(\frac{1}{r^2} \frac{\partial(r^2 \tilde{\tau}_{r\theta})}{\partial r} + \frac{1}{r} \frac{\partial \tilde{\tau}_{\theta\theta}}{\partial \theta} + \frac{\partial \tilde{\tau}_{z\theta}}{\partial z} + \frac{\tilde{\tau}_{\theta r} - \tilde{\tau}_{r\theta}}{r} \right) + \rho g_\theta \\ \rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) &= -\frac{\partial P}{\partial z} + \left(\frac{1}{r} \frac{\partial(r \tilde{\tau}_{rz})}{\partial r} + \frac{1}{r} \frac{\partial \tilde{\tau}_{\theta z}}{\partial \theta} + \frac{\partial \tilde{\tau}_{zz}}{\partial z} \right) + \rho g_z \end{aligned}$$

Equation of Motion for an incompressible fluid, 3 components in spherical coordinates

$$\begin{aligned} &\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \right) \\ &= -\frac{\partial P}{\partial r} + \left(\frac{1}{r^2} \frac{\partial(r^2 \tilde{\tau}_{rr})}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\tilde{\tau}_{\theta r} \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \tilde{\tau}_{\phi r}}{\partial \phi} - \frac{\tilde{\tau}_{\theta\theta} + \tilde{\tau}_{\phi\phi}}{r} \right) + \rho g_r \\ &\rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2 \cot \theta}{r} \right) \\ &= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \left(\frac{1}{r^3} \frac{\partial(r^3 \tilde{\tau}_{r\theta})}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\tilde{\tau}_{\theta\theta} \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \tilde{\tau}_{\phi\theta}}{\partial \phi} + \frac{\tilde{\tau}_{\theta r} - \tilde{\tau}_{r\theta}}{r} - \frac{\tilde{\tau}_{\phi\phi} \cot \theta}{r} \right) + \rho g_\theta \\ &\rho \left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} + \frac{v_\theta v_\phi \cot \theta}{r} \right) \\ &= -\frac{1}{r \sin \theta} \frac{\partial P}{\partial \phi} + \left(\frac{1}{r^3} \frac{\partial(r^3 \tilde{\tau}_{r\phi})}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\tilde{\tau}_{\theta\phi} \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \tilde{\tau}_{\phi\phi}}{\partial \phi} + \frac{\tilde{\tau}_{\phi r} - \tilde{\tau}_{r\phi}}{r} + \frac{\tilde{\tau}_{\theta\theta} \cot \theta}{r} \right) + \rho g_\phi \end{aligned} \quad 3$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation) 3 components in Cartesian coordinates

$$\begin{aligned}\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) &= -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) + \rho g_x \\ \rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) &= -\frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right) + \rho g_y \\ \rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) &= -\frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z\end{aligned}$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in cylindrical coordinates

$$\begin{aligned}\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) &= -\frac{\partial P}{\partial r} + \mu \left(\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (r v_r)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right) + \rho g_r \\ \rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) &= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \mu \left(\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (r v_\theta)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right) + \rho g_\theta \\ \rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) &= -\frac{\partial P}{\partial z} + \mu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z\end{aligned}$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in spherical coordinates

$$\begin{aligned}\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \right) &= -\frac{\partial P}{\partial r} + \mu \left(\frac{\partial}{\partial r} \left(\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v_r}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_r}{\partial \phi^2} \right. \\ &\quad \left. - \frac{2}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) - \frac{2}{r^2 \sin \theta} \frac{\partial v_\phi}{\partial \phi} \right) + \rho g_r \\ \rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2 \cot \theta}{r} \right) &= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \mu \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial v_\theta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_\theta}{\partial \phi^2} \right. \\ &\quad \left. + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} - \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial v_\phi}{\partial \phi} \right) + \rho g_\theta \\ \rho \left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} + \frac{v_\phi v_\theta \cot \theta}{r} \right) &= -\frac{1}{r \sin \theta} \frac{\partial P}{\partial \phi} + \mu \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial v_\phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (v_\phi \sin \theta) \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_\phi}{\partial \phi^2} \right. \\ &\quad \left. + \frac{2}{r^2 \sin \theta} \frac{\partial v_r}{\partial \phi} + \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial v_\theta}{\partial \phi} \right) + \rho g_\phi\end{aligned}$$

Note: the r -component of the Navier-Stokes equation in spherical coordinates may be simplified by adding $0 = \frac{2}{r} \nabla \cdot \underline{v}$ to the component shown above. This term is zero due to the continuity equation (mass conservation). See Bird et. al.

References:

1. R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena*, 2nd edition, Wiley: NY, 2002.
2. R. B. Bird, R. C. Armstrong, and O. Hassager, *Dynamics of Polymeric Fluids: Volume 1 Fluid Mechanics*, Wiley: NY, 1987.

The **Equation of Energy** in Cartesian, cylindrical, and spherical coordinates for Newtonian fluids of constant density, with source term S . Source could be electrical energy due to current flow, chemical energy, etc. Two cases are presented: the general case where thermal conductivity may be a function of temperature (vector flux $\underline{\tilde{q}} = \underline{q}/A$ appears in the equations); and the more usual case, where thermal conductivity is constant.

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Microscopic energy balance, in terms of flux; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = -\nabla \cdot \underline{\tilde{q}} + S$$

Microscopic energy balance, in terms of flux; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = - \left(\frac{\partial \tilde{q}_x}{\partial x} + \frac{\partial \tilde{q}_y}{\partial y} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = - \left(\frac{1}{r} \frac{\partial (r \tilde{q}_r)}{\partial r} + \frac{1}{r} \frac{\partial \tilde{q}_\theta}{\partial \theta} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) = - \left(\frac{1}{r^2} \frac{\partial (r^2 \tilde{q}_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (\tilde{q}_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \tilde{q}_\phi}{\partial \phi} \right) + S$$

Fourier's law of heat conduction, Gibbs notation: $\underline{\tilde{q}} = -k \nabla T$

Fourier's law of heat conduction, Cartesian coordinates:

$$\begin{pmatrix} \tilde{q}_x \\ \tilde{q}_y \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial x} \\ -k \frac{\partial T}{\partial y} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$$

Fourier's law of heat conduction, cylindrical coordinates:

$$\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial r} \\ -\frac{k}{r} \frac{\partial T}{\partial \theta} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{r\theta z}$$

Fourier's law of heat conduction, spherical coordinates:

$$\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_\phi \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial r} \\ -\frac{k}{r} \frac{\partial T}{\partial \theta} \\ -\frac{k}{r \sin \theta} \frac{\partial T}{\partial \phi} \end{pmatrix}_{r\theta\phi}$$

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\begin{aligned} \rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) \\ = k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S \end{aligned}$$

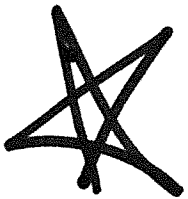
Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013. On the web at www.chem.mtu.edu/~fmorriso/IFM_WebAppendixCD2013.pdf

A.2-11 Heat-Transfer Properties of Liquid Water, SI Units

T (°C)	T (K)	ρ (kg/m ³)	c_p (kJ/kg·K)	$\mu \times 10^3$ (Pa·s, or kg/m·s)	k (W/m·K)	N_{Pr}	$\beta \times 10^4$ (1/K)	$(g\beta\rho^2/\mu^2)$ $\times 10^{-8}$ (1/K·m ³)
0	273.2	999.6	4.229	1.786	0.5694	13.3	-0.630	
15.6	288.8	998.0	4.187	1.131	0.5884	8.07	1.44	10.93
26.7	299.9	996.4	4.183	0.860	0.6109	5.89	2.34	30.70
37.8	311.0	994.7	4.183	0.682	0.6283	4.51	3.24	68.0
65.6	338.8	981.9	4.187	0.432	0.6629	2.72	5.04	256.2
93.3	366.5	962.7	4.229	0.3066	0.6802	1.91	6.66	642
121.1	394.3	943.5	4.271	0.2381	0.6836	1.49	8.46	1300
148.9	422.1	917.9	4.312	0.1935	0.6836	1.22	10.08	2231
204.4	477.6	858.6	4.522	0.1384	0.6611	0.950	14.04	5308
260.0	533.2	784.9	4.982	0.1042	0.6040	0.859	19.8	11 030
315.6	588.8	679.2	6.322	0.0862	0.5071	1.07	31.5	19 260

A.2-11 Heat-Transfer Properties of Liquid Water, English Units

T (°F)	ρ ($\frac{lb_m}{ft^3}$)	c_p ($\frac{btu}{lb_m \cdot ^\circ F}$)	$\mu \times 10^3$ ($\frac{lb_m}{ft \cdot s}$)	k ($\frac{btu}{h \cdot ft \cdot ^\circ F}$)	N_{Pr}	$\beta \times 10^4$ (1/°R)	$(g\beta\rho^2/\mu^2)$ $\times 10^{-6}$ (1/°R·ft ³)
32	62.4	1.01	1.20	0.329	13.3	-0.350	
60	62.3	1.00	0.760	0.340	8.07	0.800	17.2
80	62.2	0.999	0.578	0.353	5.89	1.30	48.3
100	62.1	0.999	0.458	0.363	4.51	1.80	107
150	61.3	1.00	0.290	0.383	2.72	2.80	403
200	60.1	1.01	0.206	0.393	1.91	3.70	1010
250	58.9	1.02	0.160	0.395	1.49	4.70	2045
300	57.3	1.03	0.130	0.395	1.22	5.60	3510
400	53.6	1.08	0.0930	0.382	0.950	7.80	8350
500	49.0	1.19	0.0700	0.349	0.859	11.0	17 350
600	42.4	1.51	0.0579	0.293	1.07	17.5	30 300



Note: $\mu \times 10^3 = 1.786 \text{ Pa s}$
 $\Rightarrow \mu = 1.786 \times 10^{-3} \text{ Pa s}$

A.3-3 Physical Properties of Air at 101.325 kPa (1 Atm Abs), SI Units

T (°C)	T (K)	ρ (kg/m ³)	c_p (kJ/kg·K)	$\mu \times 10^5$ (Pa·s, or kg/m·s)	k (W/m·K)	N_{Pr}	$\beta \times 10^3$ (1/K)	$g\beta\rho^2/\mu^2$ (1/K·m ³)
-17.8	255.4	1.379	1.0048	1.62	0.02250	0.720	3.92	2.79×10^8
0	273.2	1.293	1.0048	1.72	0.02423	0.715	3.65	2.04×10^8
10.0	283.2	1.246	1.0048	1.78	0.02492	0.713	3.53	1.72×10^8
37.8	311.0	1.137	1.0048	1.90	0.02700	0.705	3.22	1.12×10^8
65.6	338.8	1.043	1.0090	2.03	0.02925	0.702	2.95	0.775×10^8
93.3	366.5	0.964	1.0090	2.15	0.03115	0.694	2.74	0.534×10^8
121.1	394.3	0.895	1.0132	2.27	0.03323	0.692	2.54	0.386×10^8
148.9	422.1	0.838	1.0174	2.37	0.03531	0.689	2.38	0.289×10^8
176.7	449.9	0.785	1.0216	2.50	0.03721	0.687	2.21	0.214×10^8
204.4	477.6	0.740	1.0258	2.60	0.03894	0.686	2.09	0.168×10^8
232.2	505.4	0.700	1.0300	2.71	0.04084	0.684	1.98	0.130×10^8
260.0	533.2	0.662	1.0341	2.80	0.04258	0.680	1.87	0.104×10^8

A.3-3 Physical Properties of Air at 101.325 kPa (1 Atm Abs), English Units

T (°F)	ρ ($\frac{lb_m}{ft^3}$)	c_p ($\frac{btu}{lb_m \cdot ^\circ F}$)	μ (centipoise)	k ($\frac{btu}{h \cdot ft \cdot ^\circ F}$)	N_{Pr}	$\beta \times 10^3$ (1/°R)	$g\beta\rho^2/\mu^2$ (1/°R·ft ³)
0	0.0861	0.240	0.0162	0.0130	0.720	2.18	4.39×10^6
32	0.0807	0.240	0.0172	0.0140	0.715	2.03	3.21×10^6
50	0.0778	0.240	0.0178	0.0144	0.713	1.96	2.70×10^6
100	0.0710	0.240	0.0190	0.0156	0.705	1.79	1.76×10^6
150	0.0651	0.241	0.0203	0.0169	0.702	1.64	1.22×10^6
200	0.0602	0.241	0.0215	0.0180	0.694	1.52	0.840×10^6
250	0.0559	0.242	0.0227	0.0192	0.692	1.41	0.607×10^6
300	0.0523	0.243	0.0237	0.0204	0.689	1.32	0.454×10^6
350	0.0490	0.244	0.0250	0.0215	0.687	1.23	0.336×10^6
400	0.0462	0.245	0.0260	0.0225	0.686	1.16	0.264×10^6
450	0.0437	0.246	0.0271	0.0236	0.674	1.10	0.204×10^6
500	0.0413	0.247	0.0280	0.0246	0.680	1.04	0.163×10^6

Source: National Bureau of Standards, Circular 461C, 1947; 564, 1955; NBS-NACA, Tables of Thermal Properties of Gases, 1949; F. G. Keyes, Trans. A.S.M.E., 73, 590, 597 (1951); 74, 1303 (1952); D. D. Wagman, Selected Values of Chemical Thermodynamic Properties, Washington, D.C.: National Bureau of Standards, 1953.

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