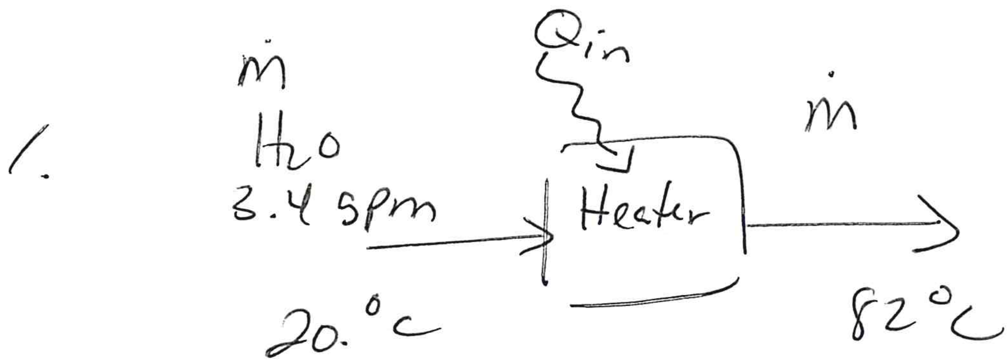
EXAM 1
SOLUTION
CM3120
22 Jan 2019

1.



MASS flow $= 3.4\ \cancel{gpm}\ \dfrac{m^3/s}{15850.32\ \cancel{gpm}}\ \dfrac{997.08\ kg}{\cancel{m^3}}$

$$\boxed{\dot{m} = 0.21388\ \frac{kg}{s}}$$

ALT. UNITS

$$\times \frac{60\,s}{min} = 12.8328\ \frac{kg}{min}$$

MACRO ENERGY BALANCE

$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = Q_{in} + \cancel{W_{s,on}}$$

negligible potential + kinetic

no shafts

$$Q_{in} = \Delta H = \sum_{outs} \dot{m}_i \hat{H}_i - \sum_{ins} \dot{m}_i \hat{H}_i$$

$$= \dot{m}\left(\hat{H}_{out} - \hat{H}_{in}\right)$$

$$= \dot{m}\, C_p \left(T_{out} - T_{in}\right)$$

for a liquid
at constant P
only T changes

$$Q_{in} = \left(0.21388 \frac{\cancel{kg}}{s}\right)\left(\frac{4.182\ kJ}{\cancel{kg}\ \cancel{K}}\right)(82-20)\cancel{K}$$

Kelvin intervals

$$Q_{in} = 55.456 \underbrace{\frac{kJ}{s}}_{kWatt}$$

$$= \boxed{55 kW}$$

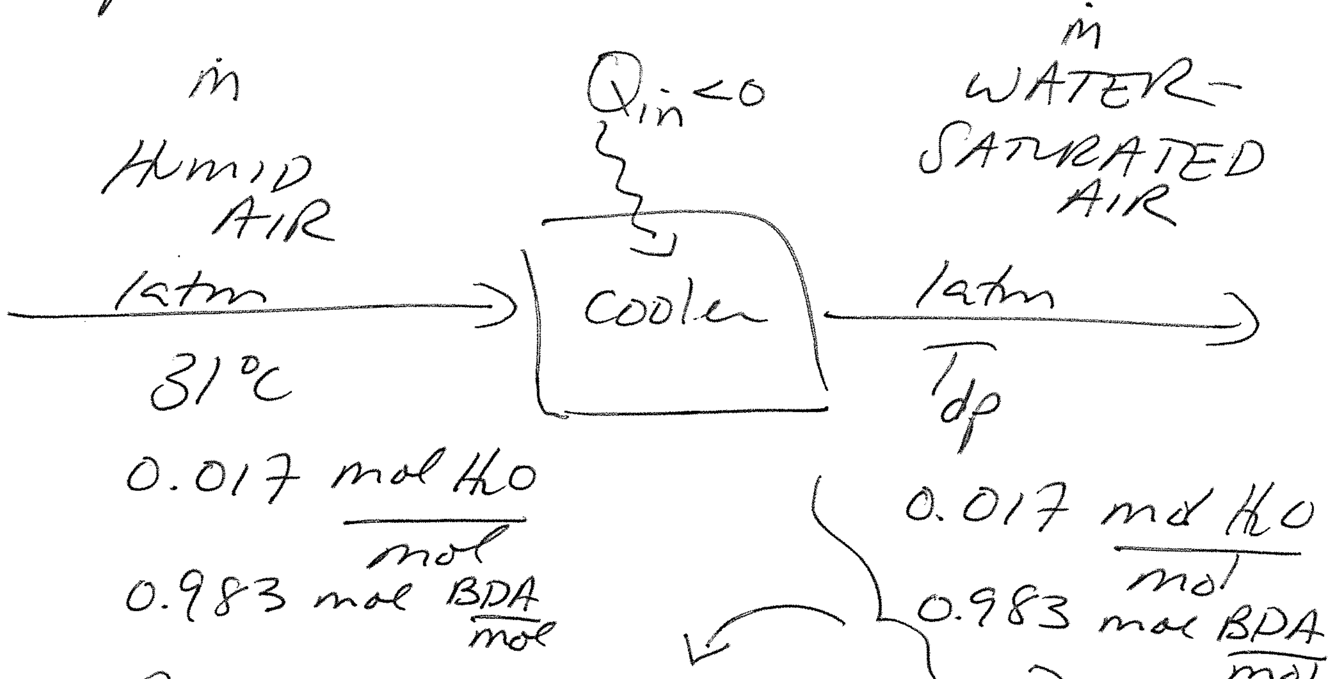
ALT. UNITS:

$$\left(55.456 \frac{kJ}{s}\right)$$

$= 3153.7$ BTU/min

$= 3327$ kJ/min

2. To determine dew point we cool a stream at constant pressure until the condensing species *just* starts to condense.



Raoult's Law (saturation)

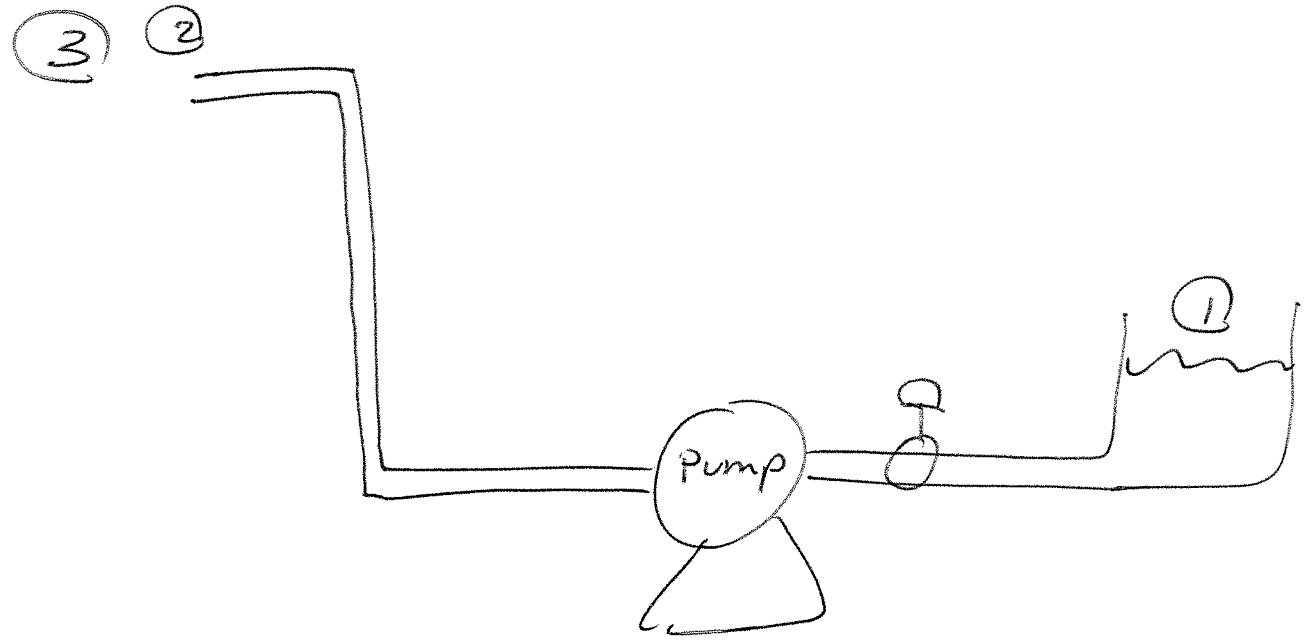
$$y_A P = P_A^*$$

$y_A = 0.017$

$P = 760 \text{ mmHg}$

$\Rightarrow P_A^* = 12.92 \text{ mmHg}$

on vapor pressure table look for T where this is P^* That's dew point



We'll solve with MEB:

steady, single input, single output, no rxn, no phase change, little heat transfered

$$\frac{\cancel{P_2 - P_1}}{\rho} + \frac{V_2^2 - \cancel{V_1^2}}{2\alpha} + g(h_2 - h_1) + \cancel{F} = \frac{W_{s,on}}{\dot{m}}$$

large diameter

both atmospheric

$\alpha = 1$, turbulent

neglect for now

$\boxed{\text{need } V_2}$

$$\langle V \rangle = \frac{\text{volumetric flow rate}}{\text{cross sectional area}}$$

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$$\langle v \rangle = \left(20\ \cancel{gpm}\right)\left(\frac{\frac{m^3}{s}}{15850.32\ \cancel{gpm}}\right)\left(\frac{\left(\frac{39.3701\ \cancel{in}}{\cancel{m}}\right)^2}{\pi\left[\left(\frac{0.454}{2}\right)\cancel{in}\right]^2}\right)$$

$$\boxed{\langle v \rangle = 12.08157\ \frac{m}{s}}$$

$$\frac{W_{s,on}}{\dot{m}} = \left(12.08157\frac{m}{s}\right)^2\frac{1}{2}$$

$$+\left(9.8066\frac{m}{s^2}\right)(\underbrace{15.5-1.3}_{14.2\ m})m$$

$$= 72.982\frac{m^2}{s^2} + 139.2537$$

$$= 212.236\ \frac{\cancel{m^2}}{\cancel{s^2}}\ \frac{\cancel{N}\ \cancel{s^2}}{kg\ \cancel{m}}\ \frac{J}{\cancel{N}\ \cancel{m}}$$

$$= 212\ \frac{J}{kg}$$

$$\dot{m} = (20\ \cancel{gpm})\ \frac{\frac{\cancel{m^3}}{s}\left(\frac{997.08\ kg}{\cancel{m^3}}\right)}{15850.32\ \cancel{gpm}}$$

$$\boxed{\dot{m} = 1.25812\ \frac{kg}{s}}$$

$$W_{s,on} = \left(212\ \frac{J}{\cancel{kg}}\right)\left(1.25812\ \frac{\cancel{kg}}{s}\right)$$

$$= 267\ \frac{J}{s}$$

$$= \boxed{270\ W}$$

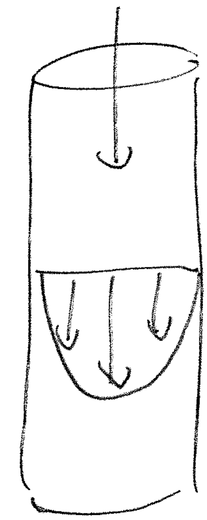
or

$$= 300\ W$$

(1 sig fig)

This would increase if we included friction //

4.



$$v_z(r) = \frac{\Delta P}{4\mu L}\left(R^2 - r^2\right)$$

$$Q = \int_0^{2\pi}\int_0^{R} v_z \; r\,dr\,d\theta$$

$$\int_0^{2\pi} d\theta = \theta\Big|_0^{2\pi} = 2\pi$$

$$Q = \cancel{2}\pi \int_0^{R} \frac{\Delta P}{{}_2\cancel{4}\mu L}\left(R^2 - r^2\right) r\,dr$$

$$Q\,\frac{2\mu L}{\pi \Delta P} = \int_0^{R}\left(rR^2 - r^3\right) dr$$

$$\frac{2\mu L}{\pi \Delta P} Q = \int_0^R \left(r R^2 - r^3 \right) dr$$

$$= \left(R^2 \frac{r^2}{2} - \frac{r^4}{4} \right) \Big|_0^R$$

$$= \frac{R^4}{2} - \frac{R^4}{4} = \frac{R^4}{4}$$

$$Q = \frac{R^4}{4} \frac{\pi \Delta P}{2 \mu L}$$

$$\boxed{Q = \frac{\Delta P \pi R^4}{8 \mu L}}$$

Hagen
Poiseuille
Eqn.

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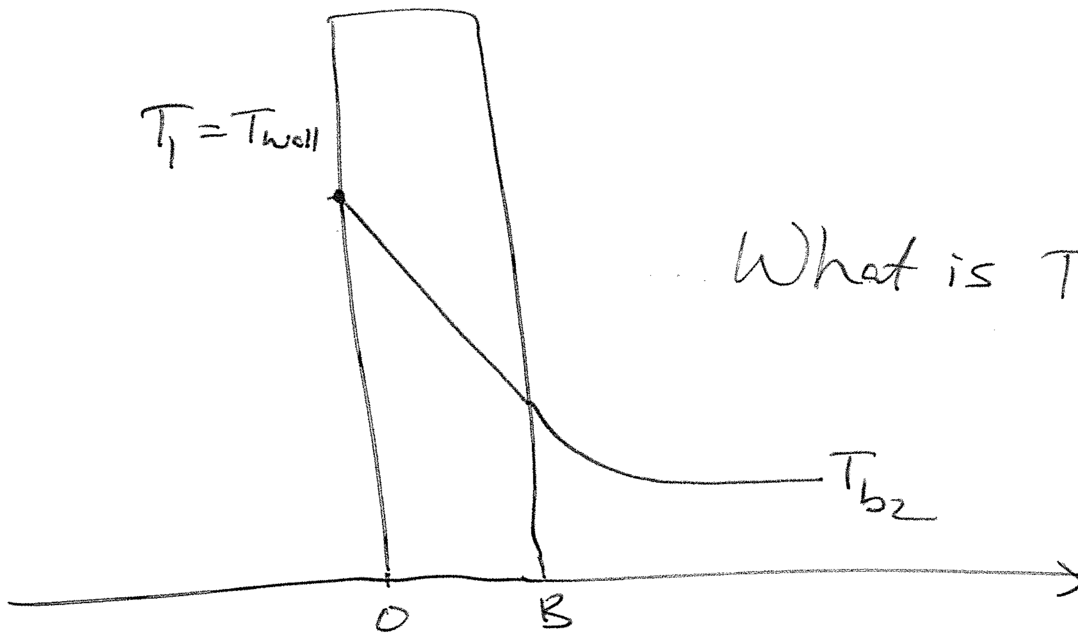

What is $T(x)$?

Microscopic energy balance

<u>assume</u>

steady | $\underline{v} = 0$
long, wide | no reaction
 | no electric current

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\cancel{\frac{\partial T}{\partial t}} + \cancel{v_x \frac{\partial T}{\partial x}} + \cancel{v_y \frac{\partial T}{\partial y}} + \cancel{v_z \frac{\partial T}{\partial z}} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} \right) + \cancel{S}$$

steady $v = 0$ wide long no reaction, no electric current

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013. On the web at www.chem.mtu.edu/~fmorriso/IFM_WebAppendixCD2013.pdf

$$0 = \cancel{k}\,\frac{d^2T}{dx^2}$$

$$\frac{d}{dx}\underbrace{\left(\frac{dT}{dx}\right)}_{\equiv \Phi} = 0$$

$$\frac{d\Phi}{dx} = 0$$

$$\boxed{\frac{dT}{dx} = \Phi = C_1}$$

$$\boxed{T = C_1 x + C_2}$$

BC: $X = 0$ $\quad T = T_1 \Rightarrow \boxed{C_2 = T_1}$

$X = B$ $\quad \dfrac{q_x}{A} = h\Big(\underbrace{T(B)}_{T(B) = C_1 B + T_1} - T_{b2}\Big)$

Fourier's law: $\dfrac{q_x}{A} = -k\,\dfrac{dT}{dx}$

$$-k \left.\frac{dT}{dx}\right|_{x=B} = h\left[\left(c_1 B + T_1\right) - T_{b2}\right]$$

$$\underbrace{}_{c_1}$$

$$-k c_1 = h c_1 B + h T_1 - h T_{b2}$$

$$c_1 (hB + k) = (T_{b2} - T_1) h$$

$$\boxed{c_1 = \frac{(T_{b2} - T_1) h}{hB + k}}$$

$$\boxed{T = \frac{(T_{b2} - T_1) h}{(hB + k)} x + T_1}$$

check units: ✓

check BC 1: $x = 0 \quad T = T_1$ ✓

$$\frac{(T - T_1)}{(T_b - T_1)} = \frac{xh}{hB + k} = \left(\frac{\frac{x}{k}}{\frac{B}{k} + \frac{1}{h}}\right)$$

check BC 2:

$$-k\frac{\partial T}{\partial x} = \frac{\cancel{(T_{b2}-T_1)}\,\cancel{h}(-k)}{hB+k}$$

$$= \cancel{h}\left[h\left(\frac{T_{b2}-T_1}{hB+k}\right)B + T_1 - T_{b2}\right]$$

$$= \cancel{(T_{b2}-T_1)}\left[\frac{(hB)}{hB+k} - 1\right]$$

$$\frac{-k}{hB+k} = \frac{\cancel{hB} - \cancel{hB} - k}{hB+k} \quad \checkmark$$