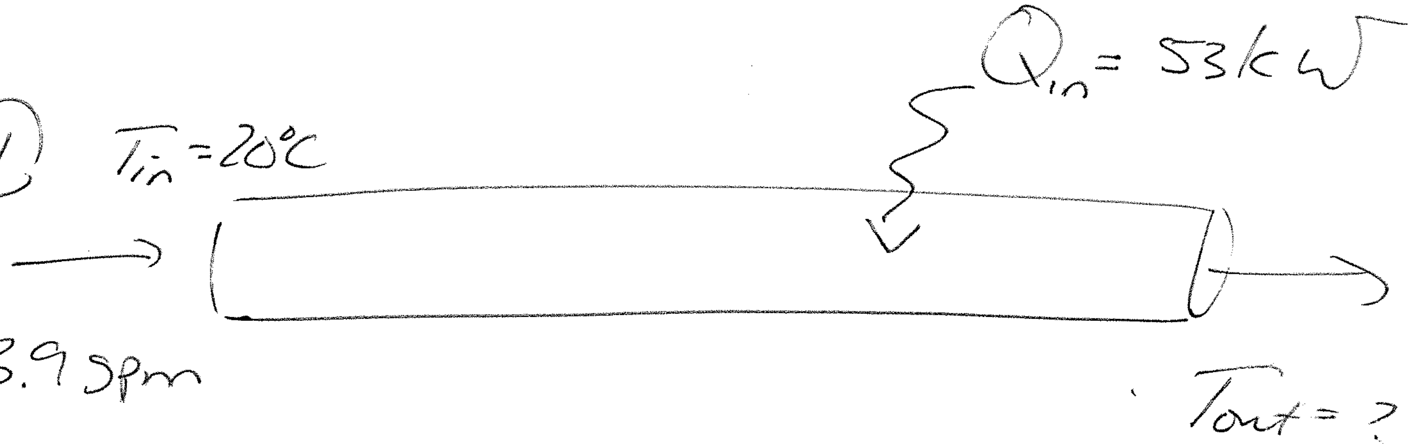
CM3120
EXAM 1
SPRING 2020

1




MACROSCOPIC ENERGY BALANCE

$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = Q_{in} + \cancel{W_{s,on}}$$

negligible          no shafts

$$Q_{in} = \Delta H = \sum_{outs} \dot{m}_i \hat{H}_i - \sum_{ins} \dot{m}_i \hat{H}_i$$

$$Q_{in} = \dot{m}\, C_p \,(T_{out} - T_{in})$$

calc $\dot{m}$      Then calc $T_{out}$.

$$\dot{m} = (3.9\ \cancel{gpm})\left(\frac{m^3/s}{15850.32\ \cancel{gpm}}\right)$$

$$\times\left(997.08\ \frac{kg}{\cancel{m^3}}\right)$$

$$\boxed{\dot{m} = 0.245333\ \frac{kg}{s}}$$

check later if we need a more accurate value

$$Q_{in} = m\hat{C}_p\,(T_{out} - T_{in})$$

$$53\ \frac{\cancel{kJ}}{s} = \left(0.24533\frac{kg}{s}\right)\left(\frac{4.182\ \cancel{kJ}}{kg\ K}\right)(T_{out} - 20\,^{\circ}C)$$

$$51.6577 = T_{out} - 20$$

$$\boxed{T_{out} = 72\,^{\circ}C}$$

$= 344.8\ K$
$= 345\ K$

## ASIDE

We could get even more accurate physical property data if we interpolate:

$$\begin{array}{ccc} \overset{(^\circ C)}{T} & \overset{kg/m^3}{\rho} & \overset{kJ/kg\,K}{\hat{C}_p} \\ 15.6 & 998.0 & 4.187 \\ 20.0 & x & y \\ 26.7 & 996.4 & 4.183 \end{array}$$

$$\frac{20-15.6}{26.7-15.6} = \frac{x-998}{996.4-998} = \frac{y-4.187}{4.183-4.187}$$

$= 0.396396$

$\Rightarrow \quad x = 997.37 = \rho \qquad y = 4.18541 = \hat{C}_p$

$\Rightarrow \dot{m} = 0.2454 \frac{kg}{s}$

very close to what we used.

$\Rightarrow T = 71.6\,^\circ C$

still $72^\circ C$ ✓

OK to use approx. values ✓

2. a. $f(Re)$ for turbulent flow:

simplified correlation:

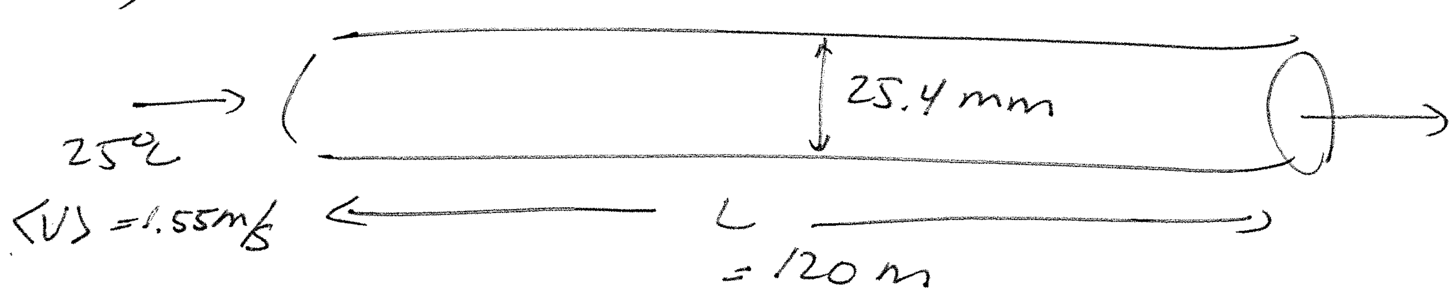
$$f = \frac{1.02}{4}\left(\log Re\right)^{-2.5}$$

or Prandtl correlation:

$$\frac{1}{\sqrt{f}} = -4.0\log\left(\frac{4.67}{Re\sqrt{f}}\right) + 2.28$$

or Moody diagram (but you need the chart)

b)




$\Delta p = ?$

Soln:

1. Calc Re
2. ⇒ f (from correlation)
3. $f = \dfrac{\Delta P\, D}{2L\rho\langle V\rangle^2}$

$$Re = \frac{\rho V D}{\mu}$$

$$= \frac{\left(997.08\ \frac{kg}{m^3}\right)\left(1.55\ \frac{m}{s}\right)\left(25.4\times10^{-3}\ m\right)}{\left(8.937\times10^{-4}\ \frac{kg}{m\,s}\right)}$$

$$\boxed{Re = 43924}$$ (turbulent)

44,000

Simplified correlation:

$$f = \left(\frac{1.02}{4}\right)\left(\log(43924)\right)^{-2.5}$$

$$f = 5.4905\times10^{-3}$$

$$\boxed{f = 0.0055}$$

$$\Delta p = \frac{f(2) L \rho \langle v \rangle^2}{D}$$

$$= \frac{(5.49 \times 10^{-3})(2)(120\,\cancel{m})(997.08\,\frac{\cancel{kg}}{\cancel{m^3}})(1.55\,\frac{\cancel{m}}{\cancel{s}})^2}{(25.4 \times 10^{-3}\,\cancel{m})}$$

$$\times \; \frac{Pa\,\cancel{m^2}}{\cancel{N}} \; \frac{\cancel{N}\,\cancel{s^2}}{\cancel{kg}\,\cancel{m}}$$

$$= 124275\ Pa \; \frac{kPa}{10^3\ Pa}$$

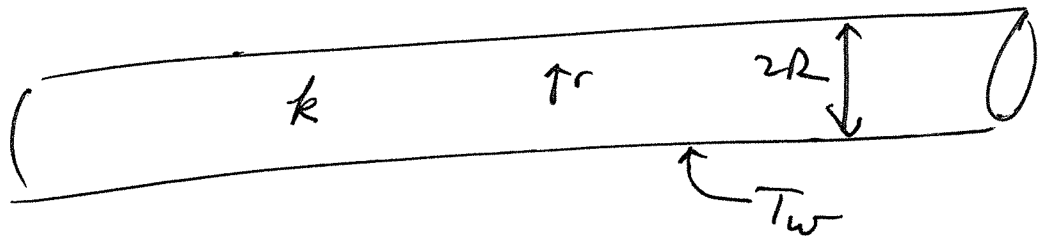
$$= 124\ kPa$$

$$\boxed{120\ kPa}$$ no more than 2 sig figs on correlation. //

$$124.275\ kPa \; \frac{14.696\ psi}{101.325\ kPa} = 18.02\ psi = 1.23\ ATM$$

$$= 18\ psi = 1.2\ ATM$$

3.)




steady
electric current $S_0 \ \frac{W}{m^3}$
$\underline{v} = 0$
$\theta$ symmetry
long

$$0 = \frac{k}{r} \frac{d}{dr}\left( r \frac{dT}{dr} \right) + S_0$$

<u>SOLVE</u>

$$\left( -\frac{S_0}{k} \right) r = \frac{d}{dr}\Big( \underbrace{r \frac{dT}{dr}}_{\equiv \Phi} \Big)$$

$$\frac{d\Phi}{dr} = \left( \frac{-S_0}{k} \right) r$$

$$r\frac{dT}{dr} = \Phi = \left( -\frac{S_0}{k} \right) \frac{r^2}{2} + c_1$$

$$\frac{dT}{dr} = \left(-\frac{S_0}{2k}\right) r + \frac{c_1}{r}$$

note: since r may be zero for $\frac{dT}{dr}$ to be finite, $c_1 = 0$
This is one possible BC.

integrate:

(a) $$\boxed{T = \left(-\frac{S_0}{2k}\right)\frac{r^2}{2} + c_1 \ln r + c_2}$$

(b) BC:

$r = 0$ $\quad \frac{dT}{dr} = \text{finite} \Rightarrow c_1 = 0$

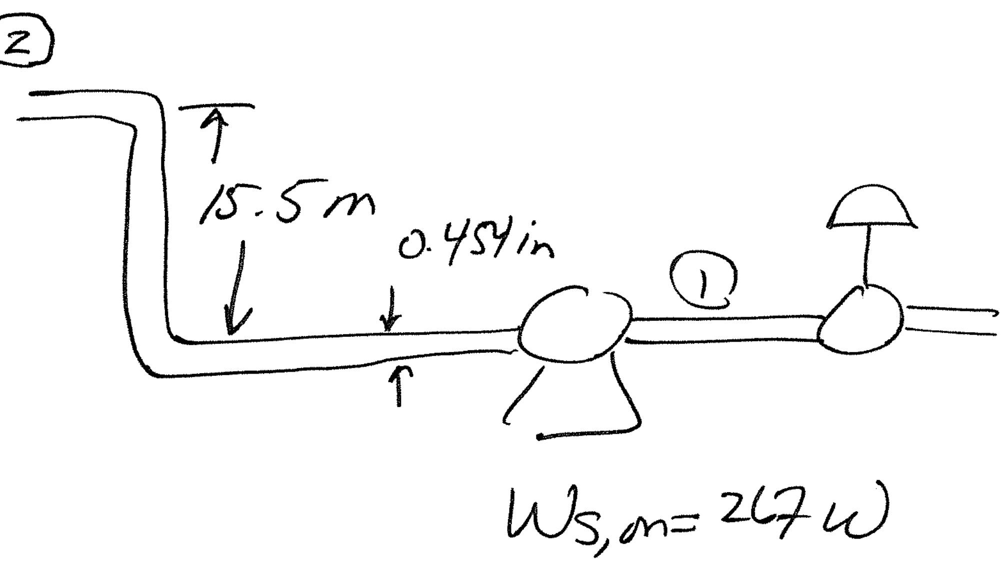
or

$r = 0$ $\quad T = \text{finite}$ ($\ln 0$ undefined)

$r = R$ $\quad T = T_w$

4.)

20 gpm
25°C




What is $P_1$?

MECH ENERGY BAL:

$$\underbrace{\frac{P_2 - P_1}{\rho}}_{P_2 = \text{atm}} + \cancel{\frac{V_2^2 - V_1^2}{2\alpha}}_{V_2 = V_1} + g\overbrace{(z_2 - z_1)}^{15.5\,m} + \cancel{F_{21}}_{\text{neglect}} = \frac{W_{s,on}}{\dot{m}}$$

1. calc $\dot{m}$
2. calc $P_1$

$$\dot{m} = (20\ \cancel{gpm})\left(\frac{\cancel{m^3}/s}{15850.32\ \cancel{gpm}}\right)\left(\frac{997.08\,kg}{\cancel{m^3}}\right)$$

$$\boxed{\dot{m} = 1.25812\ \frac{kg}{s}}$$

$$\frac{P_{atm} - P_1}{\rho} + g\,\Delta z = \frac{W_{s,on}}{\dot{m}}$$

$$g\,\Delta z - \frac{W_{s,on}}{\dot{m}} = \frac{P_1 - P_{atm}}{\rho}$$

$$P_1 - P_{atm} = \rho g \Delta z - \rho \frac{W_{s,on}}{\dot{m}}$$

$$= \left(997.08\ \frac{kg}{m^3}\right)\left(9.8066\ \frac{m}{s^2}\right)(15.5\,m)\ \frac{N\,s^2}{kg\,m}\ \frac{Pa}{N/m^2}$$

$$-\left(997.08\ \frac{\cancel{kg}}{m^3}\right)\left(267\ \frac{\cancel{J}}{s}\ \frac{N\,m}{\cancel{J}}\right)\left(\frac{s}{1.25812\ \cancel{kg}}\right)$$

$$\times \frac{Pa}{N/m^2}$$

$$P_1 - P_{atm} = 151{,}558 - 211602$$

$$= -60043 \text{ Pa}$$

$$P_1 = 1.01325 \times 10^5 - 60043$$

$$= 41{,}282 \text{ Pa}$$

$$\boxed{P_1 = 41 \text{ kPa}}$$

(one or two sig fig)

5.)

12

## The **Equation of Energy** for systems with **constant $k$**

**Microscopic energy balance**, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left( \frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

**Microscopic energy balance**, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left( \frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

no current
no rxn

**Microscopic energy balance**, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left( \cancel{\frac{\partial T}{\partial t}} + \cancel{v_r \frac{\partial T}{\partial r}} + \cancel{\frac{v_\theta}{r} \frac{\partial T}{\partial \theta}} + \cancel{v_z \frac{\partial T}{\partial z}} \right) = k \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \cancel{\frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} \right) + \cancel{S_e}$$

steady

$v = 0$

$\theta$ symmetry

Long

**Microscopic energy balance**, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left( \frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left( \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left( \sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S_e$$

$$0 = k \frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right)$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

SOLVE

$$0 = \frac{d}{dr}\Big(\underbrace{r\frac{dT}{dr}}_{\equiv \Phi}\Big)$$

$$\frac{d\Phi}{dr} = 0$$

integrate:

$$\Phi = c_1 = r\frac{dT}{dr}$$

$$\frac{dT}{dr} = \frac{c_1}{r}$$

integrate: $\boxed{T = c_1 \ln r + C_2}$

BC: $r = R_1 \quad T = T_1$

$r = R_2 \quad T = T_2$

$$\begin{cases} T_1 = c_1 \ln R_1 + C_2 \\ T_2 = c_1 \ln R_2 + C_2 \end{cases}$$

SOLVE for $c_1$, $C_2$

subtract:

$$T_1 - T_2 = c_1 \big( \underbrace{\ln R_1 - \ln R_2}_{\ln \frac{R_1}{R_2}} \big)$$

$$\boxed{c_1 = \frac{T_1 - T_2}{\ln \frac{R_1}{R_2}}}$$

substitute:

$$T_1 = \left( \frac{T_1 - T_2}{\ln \frac{R_1}{R_2}} \right) \ln R_1 + c_2$$

$$\boxed{c_2 = T_1 - \ln R_1 \left( \frac{T_1 - T_2}{\ln \frac{R_1}{R_2}} \right)}$$

substitute back :

$$T = c_1 \ln r + c_2$$

$$T = \left(\frac{T_1 - T_2}{\ln \frac{R_1}{R_2}}\right) \ln r + \overbrace{T_1 - \ln R_1 \left(\frac{T_1 - T_2}{\ln \frac{R_1}{R_2}}\right)}^{c_2}$$

$$T - T_1 = \left(\frac{T_1 - T_2}{\ln \frac{R_1}{R_2}}\right) \underbrace{(\ln r - \ln R_1)}_{\ln \frac{r}{R_1}}$$

$$\boxed{\frac{(T - T_1)}{(T_1 - T_2)} = \frac{\ln \frac{r}{R_1}}{\ln \frac{R_1}{R_2}}}$$