SOLUTION

EXAM 2

CM3120

SPRING 2019 MORRISON

1. (20 points) Dimensional analysis is an important technique we introduce when we wish to understand very complex systems. For three of the following dimensionless numbers that arise in momentum or heat transfer give the following two pieces of information a) the formula for the definition of the quantity and b) a sentence indicating the significance of this dimensionless quantity.
    a. Reynolds number
    b. Nusselt number
    c. Biot number
    d. Peclet number
    e. Prandtl number

SOLUTION:

a. **Reynolds number**: $\mathrm{Re} = \frac{\rho V D}{\mu}$
Reynolds number appears in the non-dimensionalization of the microscopic momentum balance; it represents the ratio of inertial and viscous forces in a flow

b. **Nusselt number**: $\mathrm{Nu} = \frac{hD}{k}$
Nusselt number is the dimensionless heat transfer coefficient in forced convection flows.

c. **Biot number**: $\mathrm{Bi} = \frac{hD}{k}$
Reynolds number appears in the non-dimensionalization of unsteady state heat transfer solutions; it represents the ratio of external resistance to heat transfer (h) to internal resistances to heat transfer (k).

d. **Peclet number**: $\mathrm{Pe} = \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha} = \mathrm{PrRe}$
Peclet number appears in the non-dimensionalization of the microscopic energy balance; it represents the ratio of convective heat-transfer effects and conductive heat-transfer effects in a problem.

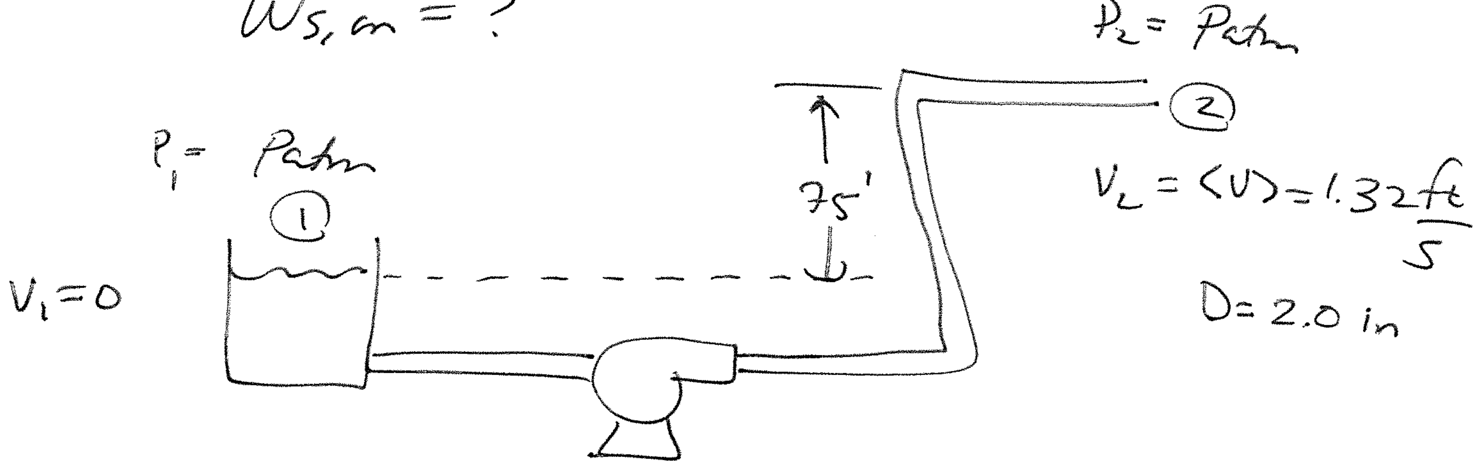
e. **Prandtl number**: $\mathrm{Pr} = \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$
Prandtl number appears as part of the Peclet number and along with Reynolds number appears in data corellations for dimensionless heat transfer coefficient, Nusselt number. The Peclet number represents the ratio of a materials ability to hold and store energy ($\hat{C}_p$) and the rate of its ability to conduct energy (k).

2. water

25°C

$W_{s,in} = ?$

$\frac{F}{g} = 4.2 \text{ ft}$



MECHANICAL ENERGY BALANCE

$$\cancel{\frac{P_2 - P_1}{\rho}} + \frac{V_2^2 - \overset{0}{\cancel{V_1^2}}}{2\underset{=1}{\alpha}} + g(\overset{75'}{z_2} - \overset{0}{z_1}) + F = \frac{W_{s,in}}{\dot{m}}$$

both atmospheric

turbulent

$$\left(\frac{1.32 \text{ ft}}{s}\right)^2 \left(\frac{1}{2}\right) + \left(\frac{32.174 \text{ ft}}{s^2}\right)(75 \text{ ft})$$

$$+ (4.2 \text{ ft})\left(\frac{32.174 \text{ ft}}{s^2}\right) = \frac{W_{s,in}}{\dot{m}}$$

③

$$0.8712 + 2413.05 + 135.1308 = \frac{W_{s,m}}{\dot{m}}$$

$$\frac{W_{s,m}}{\dot{m}} = 2549.052 \ \frac{ft^2}{s^2}$$

?

$$\dot{m} = Q\rho = \langle v \rangle (\pi R^2) \rho$$

$$R = \frac{1}{12} ft$$

$$\rho = 62.25 \frac{lb_m}{ft^3}$$

$$\dot{m} = \left(1.32 \frac{\cancel{ft}}{s}\right)\left(\pi \left(\frac{1}{12}\right)^2 \cancel{ft^2}\right)\left(62.25 \frac{lb_m}{\cancel{ft^3}}\right)$$

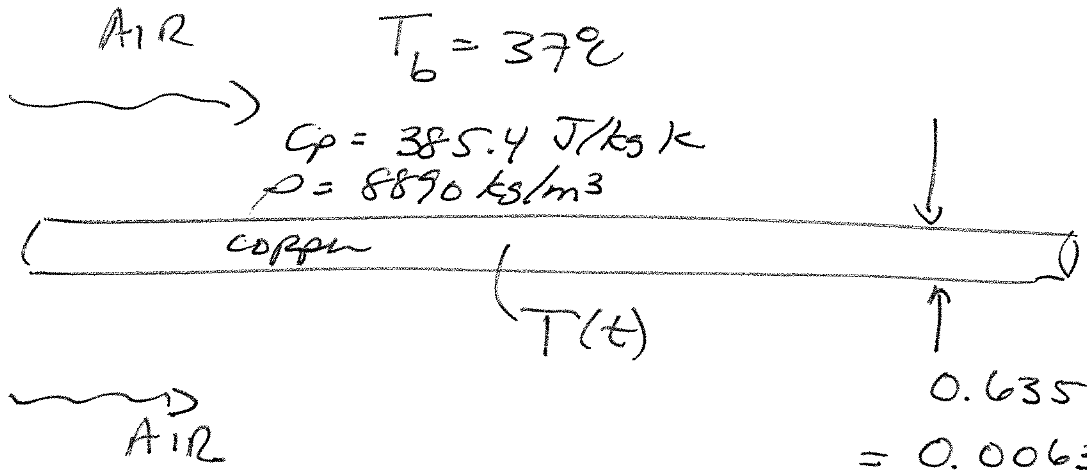
$$\boxed{\dot{m} = 1.79267 \ \frac{lb_m}{s}}$$

$$W_{s,m} = \left(2549.052 \frac{ft^{\cancel{2}}}{\cancel{s^2}}\right)\left(1.79267 \frac{\cancel{lbm}}{s}\right) \frac{\cancel{s^2}\, lb_f}{32.174 \frac{\cancel{ft}}{\cancel{lbm}}}$$

$$= 142.028 \ ft\ lb_f/s$$

$$= \boxed{140 \ \frac{ft\ lb_f}{s}}$$

3.



$0.635 \text{ cm} \; \frac{m}{100 \text{ cm}}$
$= 0.00635 \text{ m}$

$V = \pi R^2 L = \frac{\pi D^2}{4} L$
$A = \pi D L$

$t < 0, \; T = 7°C$

$t \geq 0$ in air w/ $h = 150 \; \frac{W}{m^2 K}$

How long for $T \to 36°C$?

MACRO UNSTEADY E-BAL

- no potential, kinetic energy effects
- no flow $\Rightarrow \Delta H = 0$
- no shafts $\Rightarrow W_{s,m} = 0$

$$\frac{dU}{dt} = Q_{in} = hA(T_b - T) = \rho V \hat{C}_p \frac{dT}{dt}$$

We know the soln:

$$\ln\left(\frac{T_\infty - T}{T_\infty - T_0}\right) = -\frac{hA}{(\rho\hat{C}_p V)}t$$

$$\frac{hA}{\rho C_p V} = \frac{h\,\cancel{\pi}\cancel{D}\cancel{L}}{\rho C_p \frac{\cancel{\pi} D^{\cancel{2}}}{4}\cancel{L}} = \frac{4h}{\rho C_p D}$$

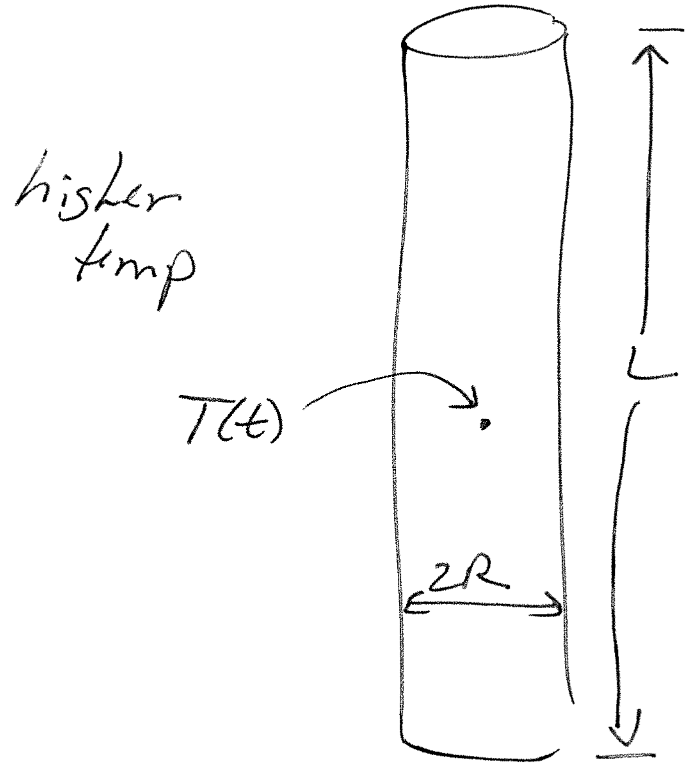
$$= \frac{(4)\left(150\ \frac{\cancel{W}}{\cancel{m^2}\cancel{K}}\right)\frac{\cancel{J}}{\cancel{W}s}}{\left(8890\ \frac{\cancel{kg}}{\cancel{m^3}}\right)\left(385.4\ \frac{\cancel{J}}{\cancel{kg}\cancel{K}}\right)(0.00635\cancel{m})}$$

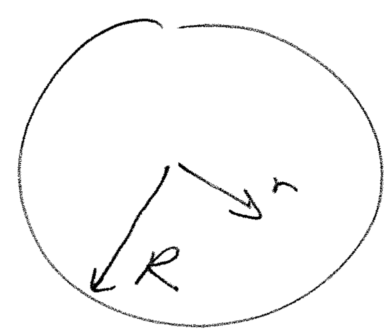
$$= 0.0275781\ s^{-1}$$

$$\ln\left(\frac{\overbrace{37-36}^{1}}{\underbrace{37-7}_{30}}\right) = -\frac{0.0275781}{s}t$$

$$t = 123.3\ s = \boxed{120\ s} = \boxed{2\ mn}$$

4)





$t \leq 0 \quad T = T_{bath}$

$t > 0 \quad T = T(t)$ in fluid w h

$T_{bulk} = T_{new}$

MICROSCOPIC ENERGY BALANCE:

(see sheet)

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \cancel{v_r \frac{\partial T}{\partial r}} + \cancel{\frac{v_\theta}{r} \frac{\partial T}{\partial \theta}} + \cancel{v_z \frac{\partial T}{\partial z}} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \cancel{\frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} \right) + \cancel{S}$$

long (no end effects)

no elec. current; no reaction

Symmetry with respect to θ

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{\sin\theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S$$

$\underline{v} = 0$

$$\Rightarrow \rho \hat{C}_p \frac{\partial T}{\partial t} = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right)$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013. On the web at www.chem.mtu.edu/~fmorriso/IFM_WebAppendixCD2013.pdf

E-BAL

$$\rho \hat{C}_p \frac{\partial T}{\partial t} = k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right)$$

initial condition:

IC: $\boxed{t = 0 \quad T = T_{bath} \quad \text{for all } r}$

boundary conditions:

BC1 $\boxed{r = 0 \quad \frac{dT}{dr} = 0 \quad \text{for all } t > 0}$

$r = R$ Newton's law of cooling:

$$-k\underbrace{\frac{dT}{dr}}_{>0} = \underbrace{\frac{q_r}{A}}_{<0} = h(\overset{\text{cool}}{T} - \overset{\text{hot}}{T_{new}}) \quad <0$$

Since $T_{new} > T_{bath}$

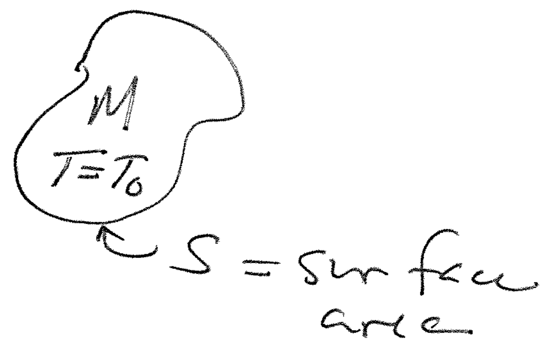
$\frac{q_r}{A} < 0$ (heat goes in negative "r" direction)

2nd BC:

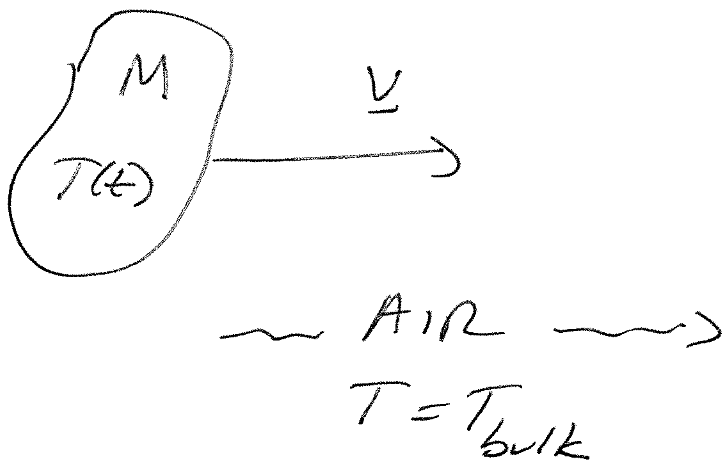
$$\boxed{r = R \qquad -k\frac{dT}{dr} = h\left(T - T_{rews}\right) \qquad \text{for all } t > 0}$$

5) brass

$t<0$



HOT



COOLING in AIR

MACRO UNSTEADY ENERGY BALANCE

system = the part itself

Assume: – no flow (it's solid) $\Rightarrow \Delta H = 0$
$\Delta E_K = \Delta E_p = 0$

– kinetic + potential energy of the part are negligible

– no shafts $\Rightarrow$ no shaft work

$$\underbrace{\frac{dU_{sys}}{dt}}_{M C_v \frac{dT}{dt}} = Q_{in} = \sum_i \underbrace{q_{in,i}}$$

C_v ← need.

no conduction
no current
no reaction
yes convection
yes radiation

$$M C_p \frac{dT}{dt} = h S (T_{bulk} - T) + \Sigma \sigma S (T_{bulk}^4 - T^4)$$

h: ? make as big as reasonable ⇒ 250 W/m²K

this is the high end for forced convection in gases (see Table)

Σ: ? = 0.22 (see Table, Engineering Toolbox)

brass:

$$C_p = 381.2 \frac{J}{kg\,K}$$

$$h = 250 \; W/m^2 K$$

$$\sigma = 5.676 \times 10^{-8} \frac{W}{m^2 K^4}$$