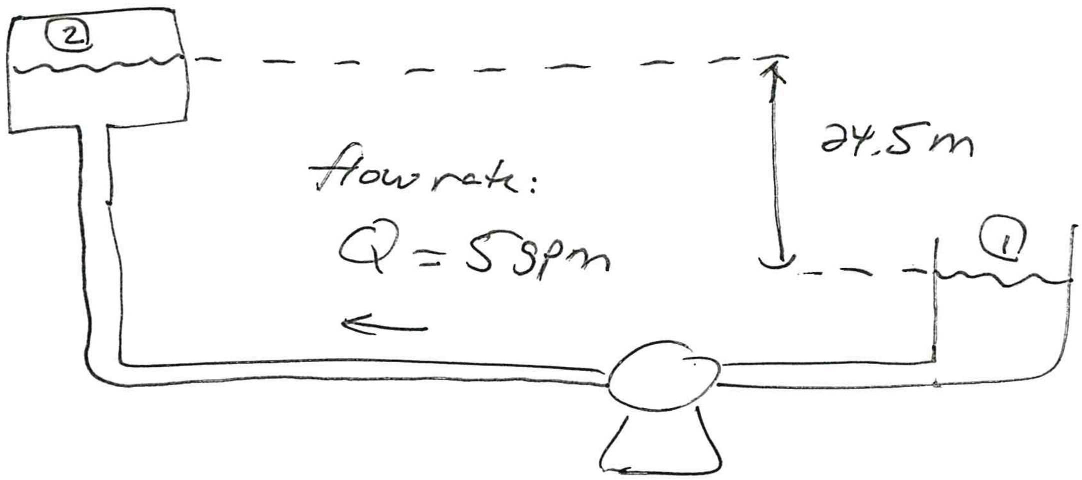
Final Exam

Soln

CM3120

30 April 2019

1.)



mechanical energy balance

$$\frac{\overset{2\,atm}{P_2} - \overset{1\,atm}{P_1}}{g\rho} + \cancel{\frac{\overset{0}{V_2^2} - \overset{0}{V_1^2}}{g2}} + \underbrace{(z_2 - z_1)}_{24.5\,m} + \frac{\overset{3.2\,ft}{F_{21}}}{g} = \frac{W_{s,on}}{g\dot{m}}$$

from Q

$$\dot{m} = Q\rho$$

$$= (5\ \cancel{gpm}) \frac{62.25\ lbm}{\cancel{ft^3}} \frac{2.228009 \times 10^{-3}\ \cancel{ft^3}/s}{1\ \cancel{gpm}}$$

$$\boxed{\dot{m} = 0.6934678\ \frac{lbm}{s}}$$

$$\frac{P_2 - P_1}{\rho g} = \frac{(1\ \cancel{atm})\ \cancel{ft^3}}{62.25\ \cancel{lb_m}}\ \frac{14.696\ \cancel{lb_f}}{\cancel{atm}\ (1\ \cancel{in^2})}\left(\frac{144\ \cancel{in^2}}{\cancel{ft^2}}\right)\left(\frac{\cancel{32.174}\ ft\ lb_m}{s^2\ \cancel{lb_f}}\right)$$

$$\times \frac{s^2}{\cancel{32.174}\ \cancel{ft}}$$

$$\boxed{\frac{P_2 - P_1}{\rho g} = 33.9956\ ft = \text{pressure head}}$$

$$z_2 - z_1 = (24.5\ m)\ \frac{ft}{0.3048\ m} = \boxed{80.380577\ ft}$$

= elevation head

$$\boxed{\frac{F_2}{g} = 3.2\ ft}$$ friction head.

$$\frac{W_{s,m}}{g\dot{m}} = (33.9956 + 0 + 80.380577 + 3.2)\ ft$$

$$= 117.576\ ft$$

$$\frac{W_{s,m}}{g} = \left(0.6934678 \frac{lb_m}{s}\right)(117.576\ ft)$$

$$W_{s,m} = 81.5 \frac{\cancel{ft}\ \cancel{lb_m}}{s} \frac{\cancel{32.174}\ ft}{\cancel{s^2}} \frac{\cancel{s^2}\ lb_f}{\cancel{32.174}\ \cancel{ft}\ \cancel{lb_m}}$$

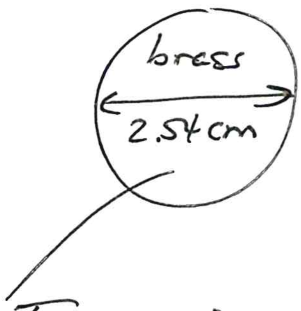
$$= 81.5 \frac{ft\ lb_f}{s}$$

$$= \boxed{82 \frac{ft\ lb_f}{s}} \quad (2\text{ sig figs})$$

$$\boxed{\text{also accept } 81 \frac{ft\ lb_f}{s}}$$

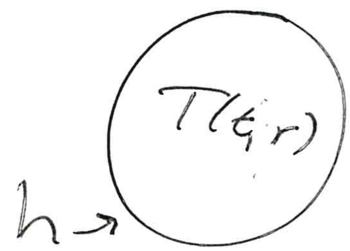
2)

initial



$T_0 = 25°C$

suddenly



$T_1 = T_b = 85°C$

see next page – micro-Energy balance on the sphere

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \right)$$

IC: $T = T_0 \quad \forall \ r$

BC: $r = R \qquad -k \frac{\partial T}{\partial r} = h \left(T\big|_R - T_{bulk} \right)$

$r = 0 \qquad \frac{q_r}{A} = 0$

use Heissler Chart,
since Heissler Chart is the
solution for this problem
(centerpoint temp. of sphere
with this Temp history)

Micro Energy balance on the sphere

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Solid sphere ($\underline{v} = 0$)

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \cancel{v_r \frac{\partial T}{\partial r}} + \cancel{\frac{v_\theta}{r} \frac{\partial T}{\partial \theta}} + \cancel{\frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi}} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \cancel{\frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right)} + \cancel{\frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2}} \right) + \cancel{S}$$

no electricity
no rxn

no θ, ϕ
variation

$$\frac{\partial T}{\partial t} = \underbrace{\frac{k}{\rho \hat{C}_p}}_{\alpha \text{ thermal diffusivity}} \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right)$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013. On the web at www.chem.mtu.edu/~fmorriso/IFM_WebAppendixCD2013.pdf

Heissler

$$Y_0 = \frac{T_1 - T}{T_1 - T_0} = \frac{\overset{5\,sec}{\widetilde{85 - 72}}}{85-25} = \frac{13}{60} = 0.22$$

$T_0 = 25°C$

$T_1 = T_b = 85°C$

$$\alpha_{brass} = 3.28 \times 10^{-5} \frac{m^2}{s}$$

$$X_1 = R = 1.27\ cm\ \frac{m}{100\,cm} = 1.27 \times 10^{-2}\ m$$

$$\underset{(5\,sec)}{F_0} = \frac{\alpha t}{X_1^2} = \frac{\left(3.28 \times 10^{-5} \frac{\cancel{m^2}}{\cancel{s}}\right)(5\,\cancel{s})}{\left(1.27 \times 10^{-2}\,\cancel{m}\right)^2}$$

$$= 1.02 \approx 1.0$$

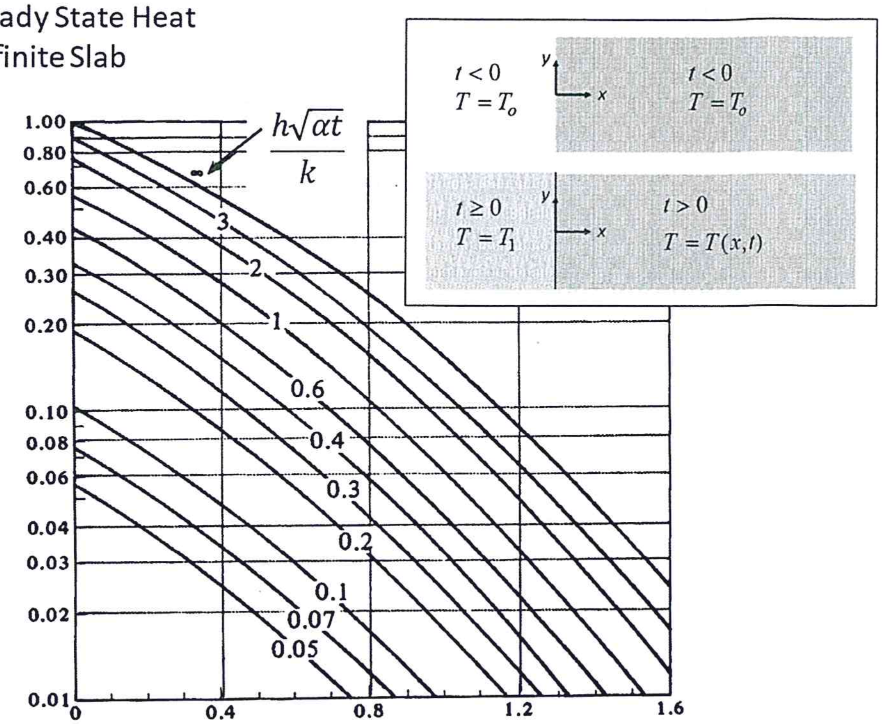
on Heissler:

$$\underset{(X,\ y)}{(1.0,\ 0.22)} = (F_0, Y_0) \Rightarrow \boxed{\frac{k}{hR} = 1.5}$$

$$= \frac{1}{Bi}$$

1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab

$$1 - Y = \left(\frac{T - T_0}{T_1 - T_0}\right)$$



$$\zeta = \frac{x}{2\sqrt{\alpha\, t}}$$

Geankoplis 4th ed.,
Figure 5.3-3, page 364

Heisler chart (sphere)

$$\text{label} = \frac{k}{hR} = \frac{1}{\text{Bi}}$$

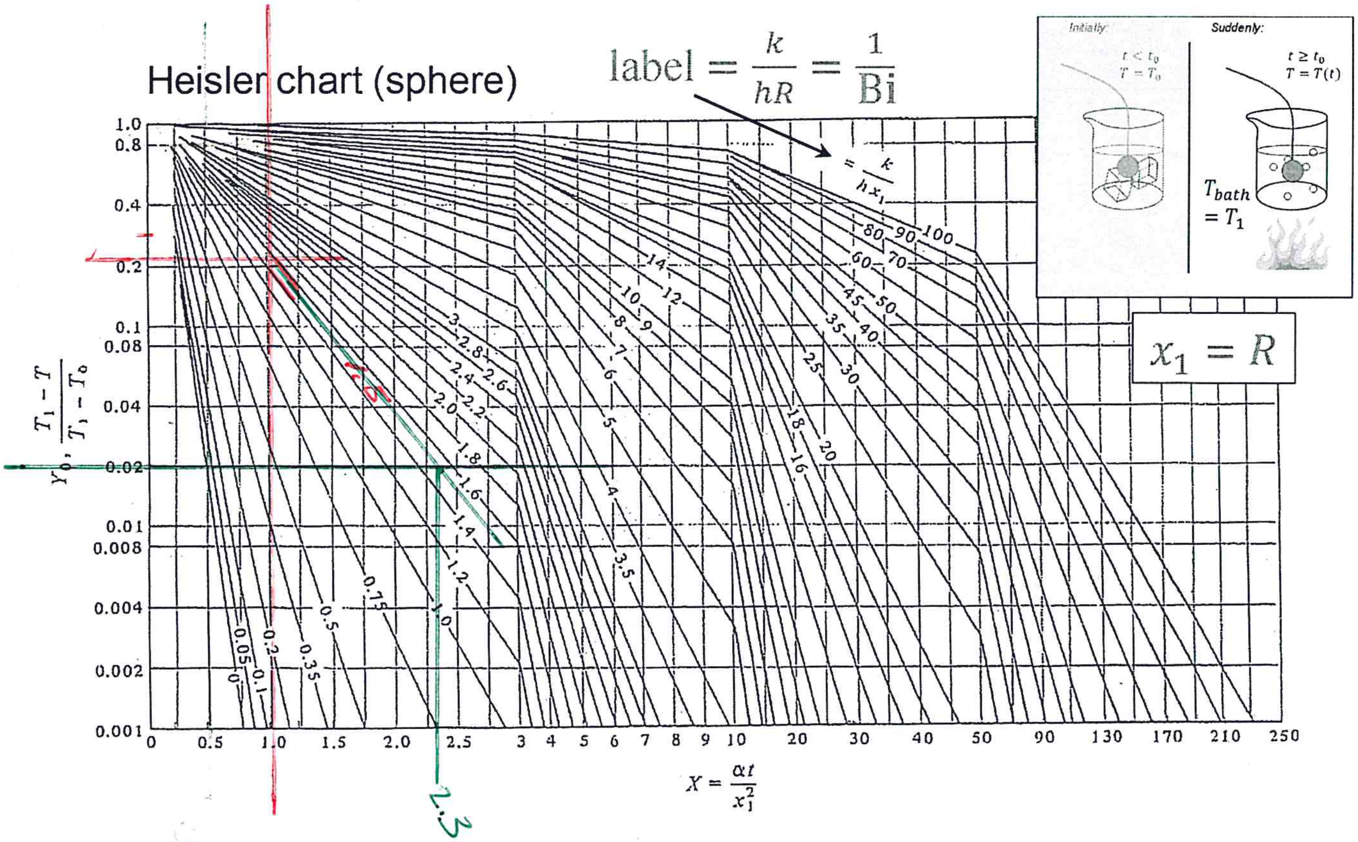


FIGURE 5.3-10. *Chart for determining the temperature at the center of a sphere for unsteady-state heat conduction. [From H. P. Heisler, Trans. A.S.M.E., 69, 227 (1947). With permission.]* *From Geankpolis, 4th edition, page 374*

How long does it take to reach 83.8°C at the center?

$$Y_0 = \frac{T_1 - T}{T_1 - T_0} = \frac{85 - 83.8}{85 - 25} = \frac{1.2}{60} = 0.02$$

0.02 intersects $\frac{k}{hR} = 1.5$ at $F_0 = 2.3$

see chart, (F_0, Y_0)

$$F_0 = 2.3 = \frac{\alpha t}{X_1^2}$$

$$= \frac{\left(3.28\times10^{-5}\ \frac{m^2}{s}\right)(t)}{\left(1.27\times10^{-2}\ m\right)^2}$$

$$\boxed{t = 11 \text{ Seconds}}$$

3. a) diffusion vs. linear driving force model

k_c = mass xfer coef

use when bulk convection is significant

b) $Sh = f(Re, Sc)$

Sherwood #

Reynolds #

Schmidt #

$$Sh = \frac{k_c D}{D_{AB}}$$

D — mass xfer lengthscale

$$Re = \frac{\rho V D}{\mu}$$

V — air velocity

D — momentum lengthscale

μ — viscosity

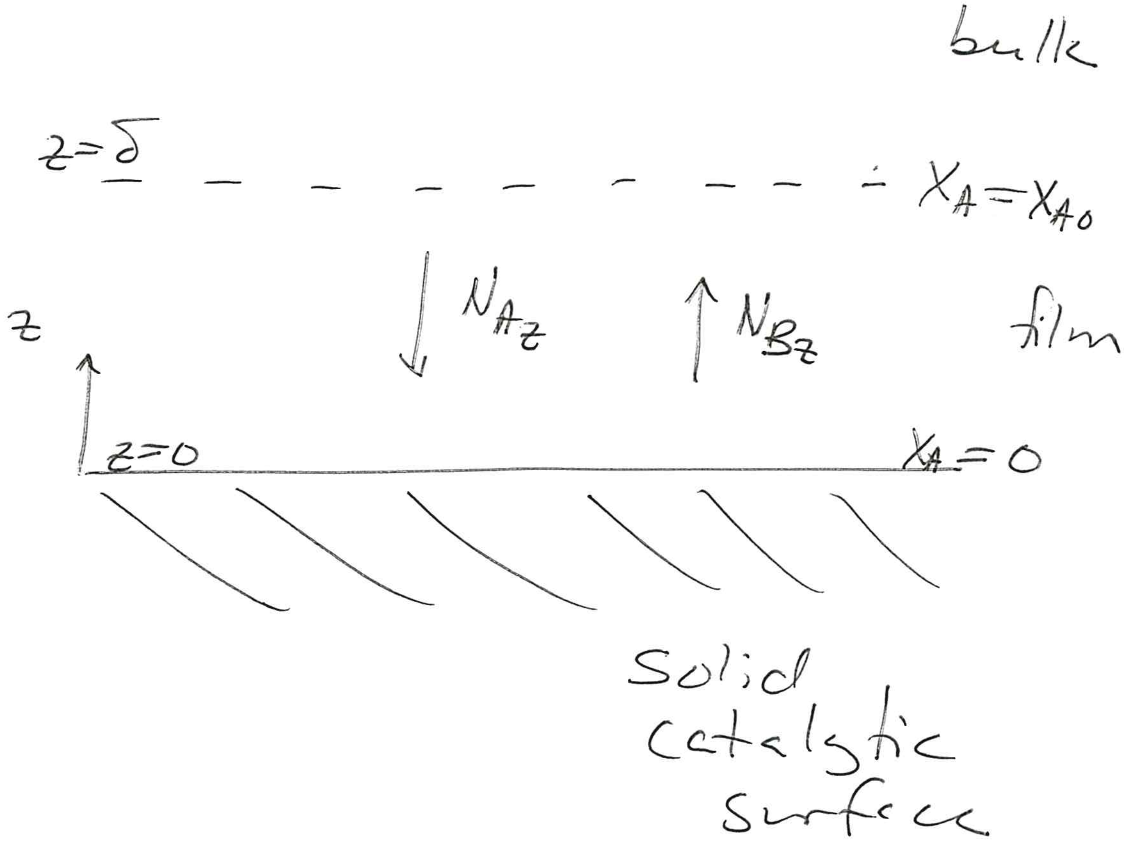
$$Sc = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}}$$

D_{AB} — diffusion coef (air–water vapor)

ρ — air density

physical properties of AIR or water diffusing in air

4)



Reaction: $2A \rightarrow B$

2 "A" approach the surface
1 "B" leaves the surface
) Steady State

$$\underbrace{N_{A_z}}_{\text{"A" must approach twice as fast}} = \overset{\text{opposite dir}}{-}2 \underbrace{N_{B_z}}_{\text{as "B" leaves + in the opposite direction}}$$

Micro Species "A" mass bal (11)

The **Equation of Species Mass Balance in Terms of Combined Molar quantities** in Cartesian, cylindrical, and spherical coordinates for binary mixtures of A and B.

The general case, where the combined molar flux with respect to molar velocity ($\underline{N}_A$), is given on page 1.

Spring 2019 Faith A. Morrison, Michigan Technological University

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In terms of total molar flux, $\underline{N}_A$

Microscopic species mass balance, in terms of molar flux; Gibbs notation

$$\frac{\partial c_A}{\partial t} = -\nabla \cdot \underline{N}_A + R_A$$

i.e. 1D diffusion

WRF 25-11

Microscopic species mass balance, in terms of combined molar flux; Cartesian coordinates

wide, clip

$$\cancel{\frac{\partial c_A}{\partial t}} = -\left(\cancel{\frac{\partial N_{A,x}}{\partial x}} + \cancel{\frac{\partial N_{A,y}}{\partial y}} + \frac{\partial N_{A,z}}{\partial z}\right) + \cancel{R_A}$$

no homogeneous rxn

Study

Microscopic species mass balance, in terms of combined molar flux; cylindrical coordinates

$$\frac{\partial c_A}{\partial t} = -\left(\frac{1}{r}\frac{\partial (rN_{A,r})}{\partial r} + \frac{1}{r}\frac{\partial N_{A,\theta}}{\partial \theta} + \frac{\partial N_{A,z}}{\partial z}\right) + R_A$$

Microscopic species mass balance, in terms of combined molar flux; spherical coordinates

$$\frac{\partial c_A}{\partial t} = -\left(\frac{1}{r^2}\frac{\partial (r^2 N_{A,r})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (N_{A,\theta}\sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial N_{A,z}}{\partial \phi}\right) + R_A$$

$$\frac{dN_{A,z}}{dz} = 0$$

$$\boxed{N_{A,z} = C_1}$$

Fick's law of diffusion, Gibbs notation: $\underline{N}_A = x_A(\underline{N}_A + \underline{N}_B) - cD_{AB}\nabla x_A$

WRF 24-22

$$= c_A \underline{v}^* - cD_{AB}\nabla x_A$$

Fick's law of diffusion, Cartesian coordinates:

$$\begin{pmatrix} N_{A,x} \\ N_{A,y} \\ N_{A,z} \end{pmatrix}_{xyz} = \begin{pmatrix} \cancel{x_A(N_{A,x} + N_{B,x}) - cD_{AB}\frac{\partial x_A}{\partial x}} \\ \cancel{x_A(N_{A,y} + N_{B,y}) - cD_{AB}\frac{\partial x_A}{\partial y}} \\ x_A(N_{A,z} + N_{B,z}) - cD_{AB}\frac{\partial x_A}{\partial z} \end{pmatrix}_{xyz}$$

1D

related by rxn stoichiometry

Fick's law of diffusion, cylindrical coordinates:

$$\begin{pmatrix} N_{A,r} \\ N_{A,\theta} \\ N_{A,z} \end{pmatrix}_{r\theta z} = \begin{pmatrix} x_A(N_{A,r} + N_{B,r}) - cD_{AB}\frac{\partial x_A}{\partial r} \\ x_A(N_{A,\theta} + N_{B,\theta}) - \frac{cD_{AB}}{r}\frac{\partial x_A}{\partial \theta} \\ x_A(N_{A,z} + N_{B,z}) - cD_{AB}\frac{\partial x_A}{\partial z} \end{pmatrix}_{r\theta z}$$

Fick's law of diffusion, spherical coordinates:

$$\begin{pmatrix} N_{A,r} \\ N_{A,\theta} \\ N_{A,\phi} \end{pmatrix}_{r\theta\phi} = \begin{pmatrix} x_A(N_{A,r} + N_{B,r}) - cD_{AB}\frac{\partial x_A}{\partial r} \\ x_A(N_{A,\theta} + N_{B,\theta}) - \frac{cD_{AB}}{r}\frac{\partial x_A}{\partial \theta} \\ x_A(N_{A,\phi} + N_{B,\phi}) - \frac{cD_{AB}}{r\sin\theta}\frac{\partial x_A}{\partial \phi} \end{pmatrix}_{r\theta\phi}$$

FICK'S LAW:

$$N_{Az} = x_A (N_{Az} + \underbrace{N_{Bz}}_{= -\frac{1}{2} N_{Az} \text{ (stoichiometry)}}) - c D_{AB} \frac{dx_A}{dz}$$

$$N_{Az} = x_A \underbrace{\left(N_{Az} - \frac{1}{2} N_{Az}\right)}_{\frac{1}{2} N_{Az}} - c D_{AB} \frac{dx_A}{dz}$$

From MASS BAL, species "A":

$$\underbrace{N_{Az}}_{C_1} \left(1 - \frac{1}{2} x_A\right) = - c D_{AB} \frac{dx_A}{dz}$$

$$(-2) \int \frac{dx_A \left(-\frac{1}{2}\right)}{\left(1 - \frac{1}{2} x_A\right)} = \int \left(\frac{C_1}{-c D_{AB}}\right) dz$$

$$-2 \ln\left(1 - \frac{1}{2} x_A\right) = \left(\frac{C_1}{-c D_{AB}}\right) z + C_2$$

BC: $z = 0 \quad x_A = 0$ (fast rxn at surface)

$z = \delta \quad x_A = x_{A0}$ (given)

BC1:

$$-2\underbrace{\cancel{\ln(1)}}_{0} = C_2$$

$$\Rightarrow \boxed{C_2 = 0}$$

BC2:

$$-2\ln\left(1-\frac{1}{2}x_{A0}\right) = \frac{c_1}{-cD_{AB}}\,\delta$$

$$N_{A_z} = c_1 = \ln\left(1-\frac{1}{2}x_{A0}\right)\frac{\cancel{2}\,cD_{AB}}{\delta} = -\cancel{2}N_{B_z}$$

$$\boxed{N_{B_z} = -\frac{cD_{AB}}{\delta}\ln\left(1-\frac{1}{2}x_{A0}\right)}$$

(14)

5. (20 points) The 8 Friday topics are listed below. Answer the following questions.
   a. What was your Friday project UO? In one sentence, what is the engineering purpose of that unit operation and what physics is exploited to achieve that purpose? Answer depends on your topic

   For which of the 8 Friday unit operations is the separation ***significantly*** driven by:

   *(**note** that you can put a unit into more than one slot;*
   One unit is not used; please write the unit name, not the number):

   b. Pressure: (1) MEMBRANE SEPARATION

   (2) FILTRATION ADSORPTION

   c. Heat: MULTI-EFFECT

   (1) EVAPORATORS

   (2) DRYERS

   (3) DISTILLATION

   d. Rotary motion: (1) DRYERS or FILTRATION

   (2) CENTRIFUGATION or MEMBRANE SEP

   e. Diffusion: (1) ABSORPTION OR MEMBRANE SEP'N

   OR ADSORPTION

1. **Evaporators** (single, multiple effect)	2. **Membrane Separation** (reverse osmosis, microfiltration, ultrafiltration)
3. **Dryers** (batch continuous	4. **Filtration** (conventional, continuous)
5. **Absorption** with a "**B**"	6. **Distillation**(conventional, azeotropic, multicomponent)
7. **Adsorption** with a "**D**"	8. **Centrifugation** (batch, continuous)

not used.