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FACTORS FOR UNIT CONVERSIONS

Quantity	Equivalent Values
Mass	1 kg = 1000 g = 0.001 metric ton = 2.20462 lb_m = 35.27392 oz
	1 lb_m = 16 oz = 5 x 10^{-4} ton = 453.593 g = 0.453593 kg
Length	1 m = 100 cm = 1000 mm = 10^6 microns (μm) = 10^{10} angstroms (Å)
	= 39.3701 in = 3.28084 ft = 1.09361 yd = 0.000621371 mile
	1 ft = 12 in. = 1/3 yd = 0.3048 m = 30.48 cm
Volume	1 m^3 = 1000 liters = 10^6 cm^3 = 10^6 ml
	= 35.31467 ft^3 = 219.969 imperial gallons = 264.172 gal
	= 1056.69 qt
	1 ft^3 = 1728 in^3 = 7.48052 gal = 0.028317 m^3 = 28.3168 liters
	= 28,316.8 cm^3
Force	1 N = 1 kg·m/s^2 = 10^5 dynes = 10^5 g·cm/s^2 = 0.22481 lb_f
	1 lb_f = 32.174 lb_m·ft/s^2 = 4.4482 N = 4.4482 x 10^5 dynes
Pressure	1 atm = 1.01325 x 10^5 N/m^2 (Pa) = 101.325 kPa = 1.01325 bars
	= 1.01325 x 10^6 dynes/cm^2
	= 760 mm Hg at 0˚ C (torr) = 10.333 m H_2O at 4˚ C
	= 14.696 lb_f/in^2 (psi) = 33.9 ft H_2O at 4˚C
	100 kPa = 1 bar
Energy	1 J = 1 N·m = 10^7 ergs = 10^7 dyne·cm
	= 2.778 x 10^{-7} kW·h = 0.23901 cal
	= 0.7376 ft·lb_f = 9.47817 x 10^{-4} Btu
Power	1 W = 1 J/s = 0.23885 cal/s = 0.7376 ft·lb_f/s = 9.47817 x 10^{-4} Btu/s = 3.4121 Btu/h
	= 1.341 x 10^{-3} hp (horsepower)
Viscosity	1 Pa·s = 1 N·s/m^2 = 1 kg/m·s
	= 10 poise = 10 dynes·s/cm^2 = 10 g/cm·s
	= 10^3 cp (centipoise)
	= 0.67197 lb_m/ft·s = 2419.088 lb_m/ft·h
Density	1 kg/m^3 = 10^{-3} g/cm^3
	= 0.06243 lb_m/ft^3
	10^3 kg/m^3 = 1 g/cm^3 = 62.428 lb_m/ft^3
Volumetric Flow	1 m^3/s= 35.31467 ft^3/s=15,850.32 gal/min (gpm)
	1 gpm = 6.30902 x 10^{-5} m^3/s=2.228009 x 10^{-3} ft^3/s=3.7854 liter/min
	1 liter/min=0.26417 gpm

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Temperature $T(^oC) = \frac{5}{9}\left[T(^oF) - 32\right]$

$T(^oF) = \frac{9}{5}T(^oC) + 32 = 1.8T(^oC) + 32$

Absolute Temperature T(K) = T(˚C) + 273.15

T(˚R) = T(˚F) + 459.67

Temperature Interval (ΔT) 1 C˚ = 1 K = 1.8 F˚ = 1.8 R˚

1F˚ = 1 R˚ = (5/9) C˚ = (5/9) K

USEFUL QUANTITIES

SG = ρ(20˚C)/ρ_{water} (4˚C)

ρ_{water}(4˚C) = 1000 kg/m^3 = 62.43 lb$_m$/ft^3 = 1.000 g/cm^3

ρ_{water}(25˚C) = 997.08 kg/m^3 = 62.25 lb$_m$/ft^3 = 0.99709 g/cm^3

g = 9.8066 m/s^2 = 980.66 cm/s^2 = 32.174 ft/s^2

μ_{water} (25˚C) = 8.937 x 10^{-4} Pa·s = 8.937 x 10^{-4} kg/m·s

= 0.8937 cp =0.8937 x 10^{-2} g/cm·s = 6.005 x 10^{-4} lb$_m$/ft·s

Composition of air:

N_2	78.03%
O_2	20.99%
Ar	0.94%
CO_2	0.03%
H_2, He, Ne, Kr, Xe	0.01%
	100.00%

M_{air} = 29 g/mol = 29 kg/kmol = 29 lb$_m$/lbmole

$\hat{C}_{p,water}$ (25˚C) = 4.182 kJ/kg K = 0.9989 cal/g˚C = 0.9997 Btu/lb$_m$˚F

R = 8.314 m^3·Pa/mol·K = 0.08314 liter·bar/mol·K = 0.08206 liter·atm/mol·K

= 62.36 liter·mm Hg/mol·K = 0.7302 ft^3·atm/lbmole·˚R

= 10.73 ft^3·psia/lbmole·˚R

= 8.314 J/mol·K

= 1.987 cal/mol·K = 1.987 Btu/lbmole·˚R

Ver. 30-Oct-2014

Energy Balance Notes CM2110/CM3110/CM3120

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Choosing the Right Energy Balance

1. Closed System (note: $\Delta = \Sigma_{final} - \Sigma_{initial}$)
   - $\Delta E_k + \Delta E_p + \Delta U = Q_{in} + W_{on}$ (FR)
   - Is it adiabatic? (if yes, Q_{in}=0)
   - Are there moving parts, e.g. do the walls move? (if no, W_{on}=0)
   - Is the *system* moving? (if no, ΔE_k=0)
   - Is there a change in elevation of the *system*? (if no, $\Delta E_p = 0$)
   - Does T, phase, or chemical composition change? (if no to all, $\Delta U = 0$)

2. Open System (the fluid is the system) (note: $\Delta = \Sigma_{out} - \Sigma_{in}$)
   1. Is it a Mechanical Energy Balance (MEB) problem? (turbulent, α=1; laminar, α=0.5; *F*= total frictional loss between inlet and outlet, $W_{s,on} = -W_{s,by}$)
      - $\frac{\Delta P}{\rho} + \frac{1}{2\alpha}\Delta v^2 + g\,\Delta z + F = \frac{W_{s,on\,fluid}}{\dot{m}}$ (FR)
      - $\frac{(P_{out}-P_{in})}{\rho} + \frac{(v_{out}^2-v_{in}^2)}{2\alpha} + g(z_{out}-z_{in}) + F = \frac{W_{s,on\,fluid}}{\dot{m}}$ (FR)

      The mechanical energy balance is only valid for systems for which the following is true:

      i. single-input, single output
      ii. small or zero Q_{in}
      iii. incompressible fluid (ρ = constant)
      iv. small or zero ΔT
      v. no reaction, no phase change
   2. Is it a *regular* open system balance?
      - $\Delta E_k + \Delta E_p + \Delta H = Q_{in} + W_{s,on}$ (FR)
      - Is it adiabatic? (if yes, $Q_{in} = 0$)
      - Are there moving parts, e.g. pump, turbine, mixing shaft? (if no, $W_{s,on} = 0$)
      - Does the average velocity of the fluid change between the input and the output? (if no, $\Delta E_k = 0$); remember $\langle v\rangle = v_{av} = v = \frac{volumetric\ flow\ rate}{area}$
      - Is there a change in elevation of the system between input and output? (if no, $\Delta E_p = 0$)
      - Does T, phase, chemical composition, or ***P* change**? (if no to all, $\Delta H = 0$)

Calculating Internal Energy

1. Constant T, **P changes only**
   (a) real gases => look it up in a table (e.g. **steam**, Tables B5, B6, B7)
   (b) ideal gases => $\Delta\hat{U} = 0$
   (c) liquids, solids => $\Delta\hat{U} = 0$
2. Constant P, **T changes only**
   (a) real gases => look it up in a table (e.g. **steam**), or, if *V* is constant, $\Delta\hat{U} = \int_{T_1}^{T_2}\hat{C}_v(T)dT$
   (b) ideal gases => $\Delta\hat{U} = \int_{T_1}^{T_2}\hat{C}_v(T)dT$; also, $\hat{C}_p = \hat{C}_v + R$
   (c) liquids, solids=> $\Delta\hat{U} = \int_{T_1}^{T_2}\hat{C}_v(T)dT$; also $\hat{C}_p \approx \hat{C}_v$
3. Constant T, P, **phase changes**
   (a) real gases => look it up in a table (e.g. **steam**)
   (b) liquid to vapor => $\Delta\hat{U} = \Delta\hat{H}_{vap}(T) - P\Delta\hat{V}_{vap} \approx \Delta\hat{H}_{vap} - RT$
   (c) solid to vapor => $\Delta\hat{U} = \Delta\hat{H}_{sub}(T) - P\Delta\hat{V}_{sub} \approx \Delta\hat{H}_{sub} - RT$
   (d) solid to liquid => $\Delta\hat{U} = \Delta\hat{H}_{melt}(T) - P\Delta\hat{V}_{melt} \approx \Delta\hat{H}_{melt}$
4. Constant T, P, **mixing occurs**
   (a) gases => $\Delta\hat{U} = 0$

(b) similar liquids => $\Delta\widehat{U} = 0$

(c) dissimilar liquids/solids => $\Delta\widehat{U} = \Delta\widehat{H}_{solution}$, see WT or Perry's Handbook

Note: be careful with units, $\Delta\widehat{H}_{solution}[=]\frac{kJ}{mole\ solute}$

5. Constant T, P, **reaction occurs:** $\Delta\widehat{U} = \Delta\widehat{H}_{rxn}$

Calculating Enthalpy

1. Constant T, **P changes only** (Note: Since T is constant, $\widehat{U}$ does not change.)
   (a) real gases - look it up in a table (e.g. **steam**, Tables B5, B6, B7)
   (b) ideal gases

$$\widehat{H} = \widehat{U} + P\widehat{V}$$
$$= \widehat{U} + RT$$
$$\left(\widehat{H}_2 - \widehat{H}_1\right) = \left(\widehat{U}_2 - \widehat{U}_1\right) + R(T_2 - T_1)$$
$$\Delta\widehat{H} = \Delta\widehat{U} = 0$$

   (c) liquids, solids

$$\widehat{H} = \widehat{U} + P\widehat{V}$$
$$\Delta\widehat{H} = \Delta\left(P\widehat{V}\right)$$
$$\widehat{V} \approx \text{constant with respect to } P$$
$$\Delta\widehat{H} = \widehat{V}(\Delta P)$$

2. Constant P, **T changes only**
   (a) real gases => look it up in a table to be most accurate (e.g. **steam**), otherwise $\Delta\widehat{H} = \int_{T_1}^{T_2} \hat{C}_p(T)dT$
   (b) ideal gases => $\Delta\widehat{H} = \int_{T_1}^{T_2} \hat{C}_p(T)dT$
   (c) liquids, solids => $\Delta\widehat{H} = \int_{T_1}^{T_2} \hat{C}_p(T)dT$
3. Constant T, P, **phase changes**
   (a) liquid to vapor => $\Delta\hat{H} = \Delta\hat{H}_{vap}(T)$ Note: $\frac{d\ lnP^*}{d\ln(1/T)} = \frac{\Delta\ \hat{H}vap}{R}$ (Clapeyron equation)
   (b) solid to vapor => $\Delta\widehat{H} = \Delta\widehat{H}_{sub}(T)$
   (c) solid to liquid => $\Delta\widehat{H} = \Delta\widehat{H}_{melt}(T)$
4. Constant T, P, **mixing occurs**
   (a) gases => $\Delta\widehat{H} = 0$
   (b) similar liquids => $\Delta\widehat{H} = 0$
   (c) dissimilar liquids/solids => $\Delta\widehat{H} = \Delta\widehat{H}_{solution}$, see WT or Perry's; Note: be careful with units, $\Delta\widehat{H}_{solution}[=]\frac{J}{mole\ solute}$
5. Constant T, P, **reaction occurs:** $\Delta\widehat{U} = \Delta\widehat{H}_{rxn}$

Problem-Solving Strategies for Energy Balances

1. Write down *neatly* everything you are doing so that you and the grader both understand better what you are thinking.
2. Draw your flow sheet Large. Leave yourself plenty of room to add information to the drawing. Draw it over if it becomes too crowded. Do not erase; lightly cross out and start a new sheet. You may want to come back to the original information.
3. Always write complete units with quantities, e.g., for mole fraction A the units are $\frac{moles\ A}{moles\ total}$; quantities are meaningless without the units.

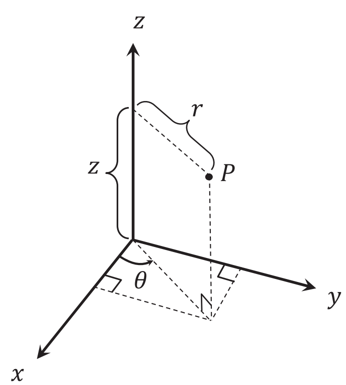
4. What are you looking for? What balance can help you find it?
    a. Is a mass balance necessary?
    b. Is it an open or a closed system?
    c. Is it a mechanical energy balance problem?
5. Convert inconvenient units, e.g., convert volumes and volume fractions into moles or masses, since mass is conserved and volume is not. Also, convert dew points and other similar information (e.g. percent humidity, molal saturation, etc.) to compositions if possible.
6. Do you have a piece of information you do not know what to do with? What is its definition? Look it up in the index, if necessary. Write it with its proper units and try to interpret how it impacts the problem.
7. What has remained constant in the problem? Is it isothermal (T constant)? Isobaric (P constant)? Constant V or $\hat{V}$? Adiabatic ($Q_{in} = 0$)? Is the mass flow constant? Is the volumetric flow constant? Is the heat flow constant or known?
8. Remember that if a system is ***saturated***, you know a great deal about it:
    a. If it is a *pure component*, you only need to know the phase (i.e. solid, liquid, vapor) and *one* of the following to know everything about the stream: T, P, $\hat{V}$, $\hat{U}$, $\hat{H}$.
    b. If it is a *mixture*, Raoult's law applies to each component, $y_i P = x_i P_i^*(T)$.
9. When looking for $\hat{U}$, $\hat{H}$, or $\hat{V}$, always take it from a table, if it is available. It *is* available in a table for water/steam:
    a. Table B.5, FR page 642, "Properties of *Saturated* Steam," sorted by temperature
    b. Table B.6, FR page 644, "Properties of *Saturated* Steam,"' sorted by pressure
    c. Table B.7, FR page 650, "Properties of *Superheated* Steam," presented in a grid of pressure and temperature. Saturated steam properties are also presented in the first two columns of Table B.7, but the steps in pressure are large, and therefore Tables B.5 and B.6 are more accurate for saturated steam properties at lower T and P.
10. If the problem is complex, break it down into smaller pieces and draw separate flow sheets that correspond to the smaller pieces.
11. Name unknown streams, compositions, and enthalpies or internal energies. See if there are few unknowns which can be solved for. Try different methods of naming the unknowns if the first way you think of does not turn out to be convenient.
12. Check for forgotten relations:
    a. mass balance
    b. mole fractions and mass fractions sum to 1.
    c. If a stream is just split, with no special process unit present, the mole or mass fractions are the same in all streams before and after the split.
    d. For a fixed, closed system, V, $\hat{V}$, and mass are constant.
13. The last step is to answer the question. Always present your answer with the correct number of significant figures and a box around it.

References

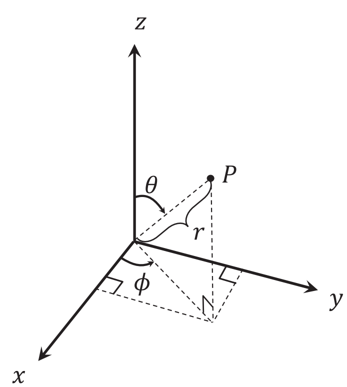
1. (FR) R. M. Felder, and R. W. Rousseau, *Elementary Principles of Chemical Processes,* 3rd Edition (Wiley, NY: 2000).
2. (G) C. J. Geankoplis, *Transport Processes and Unit Operations*, 4th Edition (Prentice Hall: Englewood Cliffs, NJ, 2003).
3. (WT) J. C. Whitwell and R. K. Toner, *Conservation of Mass and Energy*, pp 344-346, McGraw-Hill, Inc., 1969.

The equations in F. A. Morrison, ***An Introduction to Fluid Mechanics*** (Cambridge, 2013) assume the following definitions of the cylindrical and spherical coordinate systems.

Cylindrical Coordinate System: Note that the θ-coordinate swings around the z-axis



Spherical Coordinate System: Note that the θ-coordinate swings down from the z-axis; this is different from its definition in the cylindrical system above.



Typical values of the convection heat transfer coefficient. From Incropera et al., Fundamentals of Heat and Mass Transfer, 6th edition, Wiley, 2007.

Process		$h\left(\frac{W}{m^2K}\right)$
Free convection	Gases	2-25
	Liquids	50-1000
Forced convection	Gases	25-250
	Liquids	100-20,000
Convection with phase change	Boiling or condensation	2500-10^5

The Equation of Continuity and the Equation of Motion in Cartesian, cylindrical, and spherical coordinates

CM3110 Fall 2011 Faith A. Morrison

Continuity Equation, Cartesian coordinates

$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$

Continuity Equation, cylindrical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho v_\theta)}{\partial \theta} + \frac{\partial (\rho v_z)}{\partial z} = 0$$

Continuity Equation, spherical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial (\rho r^2 v_r)}{\partial r} + \frac{1}{r \sin\theta}\frac{\partial (\rho v_\theta \sin\theta)}{\partial \theta} + \frac{1}{r \sin\theta}\frac{\partial (\rho v_\phi)}{\partial \phi} = 0$$

Equation of Motion for an incompressible fluid, 3 components in Cartesian coordinates

$$\begin{aligned}
\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) &= -\frac{\partial P}{\partial x} + \left(\frac{\partial \tilde{\tau}_{xx}}{\partial x} + \frac{\partial \tilde{\tau}_{yx}}{\partial y} + \frac{\partial \tilde{\tau}_{zx}}{\partial z} \right) + \rho g_x \\
\rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) &= -\frac{\partial P}{\partial y} + \left(\frac{\partial \tilde{\tau}_{xy}}{\partial x} + \frac{\partial \tilde{\tau}_{yy}}{\partial y} + \frac{\partial \tilde{\tau}_{zy}}{\partial z} \right) + \rho g_y \\
\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) &= -\frac{\partial P}{\partial z} + \left(\frac{\partial \tilde{\tau}_{xz}}{\partial x} + \frac{\partial \tilde{\tau}_{yz}}{\partial y} + \frac{\partial \tilde{\tau}_{zz}}{\partial z} \right) + \rho g_z
\end{aligned}$$

Equation of Motion for an incompressible fluid, 3 components in cylindrical coordinates

$$\begin{aligned}
\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) &= -\frac{\partial P}{\partial r} + \left(\frac{1}{r}\frac{\partial (r \tilde{\tau}_{rr})}{\partial r} + \frac{1}{r}\frac{\partial \tilde{\tau}_{\theta r}}{\partial \theta} - \frac{\tilde{\tau}_{\theta\theta}}{r} + \frac{\partial \tilde{\tau}_{zr}}{\partial z} \right) + \rho g_r \\
\rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) &= -\frac{1}{r}\frac{\partial P}{\partial \theta} + \left(\frac{1}{r^2}\frac{\partial (r^2 \tilde{\tau}_{r\theta})}{\partial r} + \frac{1}{r}\frac{\partial \tilde{\tau}_{\theta\theta}}{\partial \theta} + \frac{\partial \tilde{\tau}_{z\theta}}{\partial z} + \frac{\tilde{\tau}_{\theta r} - \tilde{\tau}_{r\theta}}{r} \right) + \rho g_\theta \\
\rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) &= -\frac{\partial P}{\partial z} + \left(\frac{1}{r}\frac{\partial (r \tilde{\tau}_{rz})}{\partial r} + \frac{1}{r}\frac{\partial \tilde{\tau}_{\theta z}}{\partial \theta} + \frac{\partial \tilde{\tau}_{zz}}{\partial z} \right) + \rho g_z
\end{aligned}$$

Equation of Motion for an incompressible fluid, 3 components in spherical coordinates

$$\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \right)$$

$$= -\frac{\partial P}{\partial r} + \left(\frac{1}{r^2}\frac{\partial (r^2 \tilde{\tau}_{rr})}{\partial r} + \frac{1}{r \sin\theta}\frac{\partial (\tilde{\tau}_{\theta r}\sin\theta)}{\partial \theta} + \frac{1}{r \sin\theta}\frac{\partial \tilde{\tau}_{\phi r}}{\partial \phi} - \frac{\tilde{\tau}_{\theta\theta} + \tilde{\tau}_{\phi\phi}}{r} \right) + \rho g_r$$

$$\rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2 \cot\theta}{r} \right)$$

$$= -\frac{1}{r}\frac{\partial P}{\partial \theta} + \left(\frac{1}{r^3}\frac{\partial (r^3 \tilde{\tau}_{r\theta})}{\partial r} + \frac{1}{r \sin\theta}\frac{\partial (\tilde{\tau}_{\theta\theta}\sin\theta)}{\partial \theta} + \frac{1}{r \sin\theta}\frac{\partial \tilde{\tau}_{\phi\theta}}{\partial \phi} + \frac{\tilde{\tau}_{\theta r} - \tilde{\tau}_{r\theta}}{r} - \frac{\tilde{\tau}_{\phi\phi}\cot\theta}{r} \right) + \rho g_\theta$$

$$\rho \left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} + \frac{v_\phi v_\theta \cot\theta}{r} \right)$$

$$= -\frac{1}{r \sin\theta}\frac{\partial P}{\partial \phi} + \left(\frac{1}{r^3}\frac{\partial (r^3 \tilde{\tau}_{r\phi})}{\partial r} + \frac{1}{r \sin\theta}\frac{\partial \left(\tilde{\tau}_{\theta\phi}\sin\theta\right)}{\partial \theta} + \frac{1}{r \sin\theta}\frac{\partial \tilde{\tau}_{\phi\phi}}{\partial \phi} + \frac{\tilde{\tau}_{\phi r} - \tilde{\tau}_{r\phi}}{r} + \frac{\tilde{\tau}_{\phi\theta}\cot\theta}{r} \right) + \rho g_\phi$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation) 3 components in Cartesian coordinates

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+\rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in cylindrical coordinates

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial P}{\partial r}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right)+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial P}{\partial \theta}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right)+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in spherical coordinates

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi}-\frac{v_\theta^2+v_\phi^2}{r}\right)$$
$$= -\frac{\partial P}{\partial r}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right)\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial\phi^2}\right.$$
$$\left.-\frac{2}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(v_\theta\sin\theta\right)-\frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}\right)+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}+\frac{v_r v_\theta}{r}-\frac{v_\phi^2\cot\theta}{r}\right)$$
$$= -\frac{1}{r}\frac{\partial P}{\partial \theta}+\mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(v_\theta\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial\phi^2}\right.$$
$$\left.+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}-\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}\right)+\rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t}+v_r\frac{\partial v_\phi}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}+\frac{v_r v_\phi}{r}+\frac{v_\phi v_\theta\cot\theta}{r}\right)$$
$$= -\frac{1}{r\sin\theta}\frac{\partial P}{\partial \phi}+\mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(v_\phi\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial\phi^2}\right.$$
$$\left.+\frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial\phi}+\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right)+\rho g_\phi$$

Note: the r-component of the Navier-Stokes equation in spherical coordinates may be simplified by adding $0 = \frac{2}{r}\nabla\cdot\underline{v}$ to the component shown above. This term is zero due to the continuity equation (mass conservation). See Bird et. al.

References:

1. R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena*, 2^nd edition, Wiley: NY, 2002.
2. R. B. Bird, R. C. Armstrong, and O. Hassager, *Dynamics of Polymeric Fluids: Volume 1 Fluid Mechanics*, Wiley: NY, 1987.

The **Equation of Energy** in Cartesian, cylindrical, and spherical coordinates for Newtonian fluids of constant density, with source term S. Source could be electrical energy due to current flow, chemical energy, etc. Two cases are presented: the general case where thermal conductivity may be a function of temperature (vector flux $\underline{\tilde{q}} = \underline{q}/A$ appears in the equations); and the more usual case, where thermal conductivity is constant.

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Microscopic energy balance, in terms of flux; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = -\nabla \cdot \underline{\tilde{q}} + S$$

Microscopic energy balance, in terms of flux; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = -\left(\frac{\partial \tilde{q}_x}{\partial x} + \frac{\partial \tilde{q}_y}{\partial y} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = -\left(\frac{1}{r} \frac{\partial (r \tilde{q}_r)}{\partial r} + \frac{1}{r} \frac{\partial \tilde{q}_\theta}{\partial \theta} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right) = -\left(\frac{1}{r^2} \frac{\partial (r^2 \tilde{q}_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial (\tilde{q}_\theta sin\theta)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial \tilde{q}_\phi}{\partial \phi} \right) + S$$

Fourier's law of heat conduction, Gibbs notation: $\underline{\tilde{q}} = -k \nabla T$

Fourier's law of heat conduction, Cartesian coordinates: $\begin{pmatrix} \tilde{q}_x \\ \tilde{q}_y \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial x} \\ -k \frac{\partial T}{\partial y} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$

Fourier's law of heat conduction, cylindrical coordinates: $\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial r} \\ -\frac{k}{r} \frac{\partial T}{\partial \theta} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{r\theta z}$

Fourier's law of heat conduction, spherical coordinates: $\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_\phi \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial r} \\ -\frac{k}{r} \frac{\partial T}{\partial \theta} \\ -\frac{k}{r \sin\theta} \frac{\partial T}{\partial \phi} \end{pmatrix}_{r\theta\phi}$

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013. On the web at www.chem.mtu.edu/~fmorriso/IFM_WebAppendixCD2013.pdf

A.2-9 Properties of Saturated Steam and Water (Steam Table), SI Units

Temperature (°C)	Vapor Pressure (kPa)	Specific Volume (m^3/kg)		Enthalpy (kJ/kg)		Entropy (kJ/kg · K)	
		Liquid	Sat'd Vapor	Liquid	Sat'd Vapor	Liquid	Sat'd Vapor
0.01	0.6113	0.0010002	206.136	0.00	2501.4	0.0000	9.1562
3	0.7577	0.0010001	168.132	12.57	2506.9	0.0457	9.0773
6	0.9349	0.0010001	137.734	25.20	2512.4	0.0912	9.0003
9	1.1477	0.0010003	113.386	37.80	2517.9	0.1362	8.9253
12	1.4022	0.0010005	93.784	50.41	2523.4	0.1806	8.8524
15	1.7051	0.0010009	77.926	62.99	2528.9	0.2245	8.7814
18	2.0640	0.0010014	65.038	75.58	2534.4	0.2679	8.7123
21	2.487	0.0010020	54.514	88.14	2539.9	0.3109	8.6450
24	2.985	0.0010027	45.883	100.70	2545.4	0.3534	8.5794
25	3.169	0.0010029	43.360	104.89	2547.2	0.3674	8.5580
27	3.567	0.0010035	38.774	113.25	2550.8	0.3954	8.5156
30	4.246	0.0010043	32.894	125.79	2556.3	0.4369	8.4533
33	5.034	0.0010053	28.011	138.33	2561.7	0.4781	8.3927
36	5.947	0.0010063	23.940	150.86	2567.1	0.5188	8.3336
40	7.384	0.0010078	19.523	167.57	2574.3	0.5725	8.2570
45	9.593	0.0010099	15.258	188.45	2583.2	0.6387	8.1648
50	12.349	0.0010121	12.032	209.33	2592.1	0.7038	8.0763
55	15.758	0.0010146	9.568	230.23	2600.9	0.7679	7.9913
60	19.940	0.0010172	7.671	251.13	2609.6	0.8312	7.9096
65	25.03	0.0010199	6.197	272.06	2618.3	0.8935	7.8310
70	31.19	0.0010228	5.042	292.98	2626.8	0.9549	7.7553
75	38.58	0.0010259	4.131	313.93	2635.3	1.0155	7.6824
80	47.39	0.0010291	3.407	334.91	2643.7	1.0753	7.6122
85	57.83	0.0010325	2.828	355.90	2651.9	1.1343	7.5445
90	70.14	0.0010360	2.361	376.92	2660.1	1.1925	7.4791
95	84.55	0.0010397	1.9819	397.96	2668.1	1.2500	7.4159
100	101.35	0.0010435	1.6729	419.04	2676.1	1.3069	7.3549
105	120.82	0.0010475	1.4194	440.15	2683.8	1.3630	7.2958
110	143.27	0.0010516	1.2102	461.30	2691.5	1.4185	7.2387
115	169.06	0.0010559	1.0366	482.48	2699.0	1.4734	7.1833
120	198.53	0.0010603	0.8919	503.71	2706.3	1.5276	7.1296
125	232.1	0.0010649	0.7706	524.99	2713.5	1.5813	7.0775
130	270.1	0.0010697	0.6685	546.31	2720.5	1.6344	7.0269
135	313.0	0.0010746	0.5822	567.69	2727.3	1.6870	6.9777
140	316.3	0.0010797	0.5089	589.13	2733.9	1.7391	6.9299
145	415.4	0.0010850	0.4463	610.63	2740.3	1.7907	6.8833
150	475.8	0.0010905	0.3928	632.20	2746.5	1.8418	6.8379
155	543.1	0.0010961	0.3468	653.84	2752.4	1.8925	6.7935
160	617.8	0.0011020	0.3071	675.55	2758.1	1.9427	6.7502
165	700.5	0.0011080	0.2727	697.34	2763.5	1.9925	6.7078
170	791.7	0.0011143	0.2428	719.21	2768.7	2.0419	6.6663
175	892.0	0.0011207	0.2168	741.17	2773.6	2.0909	6.6256
180	1002.1	0.0011274	0.19405	763.22	2778.2	2.1396	6.5857
190	1254.4	0.0011414	0.15654	807.62	2786.4	2.2359	6.5079
200	1553.8	0.0011565	0.12736	852.45	2793.2	2.3309	6.4323
225	2548	0.0011992	0.07849	966.78	2803.3	2.5639	6.2503
250	3973	0.0012512	0.05013	1085.36	2801.5	2.7927	6.0730
275	5942	0.0013168	0.03279	1210.07	2785.0	3.0208	5.8938
300	8581	0.0010436	0.02167	1344.0	2749.0	3.2534	5.7045

Welty, Rorrer, Foster, "Fundamentals of Momentum, Heat, and Mass Transfer, 6th ed., Wiley 2015

Typo in value of α_{Cu} corrected, 24Feb2019.

Appendix H

Physical Properties of Solids

Material	ρ (lb_m/ft^3) (68°F)	ρ (kg/m^3) (293 K)	c_p (Btu/lb_m°F) (293 K)	c_p (J/kg · 1K) $\times 10^{-2}$ (293K)	α (ft^2/h) (68°F)	α (m^2/s) · 10^5 (293k)	k (Btu/h ft°F) °F (68)	k (Btu/h ft°F) °F (212)	k (Btu/h ft°F) °F (572)	k (W/m · K) K (293)	k (W/m · K) K (373)	k (W/m · K) K (573)
Metals												
Aluminum	168.6	2,701.1	0.224	9.383	3.55	9.16	132	133	133	229	229	230
Copper	555	8,890	0.092	3.854	3.98	11.27	223	219	213	386	379	369
Gold	1206	19,320	0.031	1.299	4.52	11.66	169	170	172	293	294	298
Iron	492	7,880	0.122	5.110	0.83	2.14	42.3	39	31.6	73.2	68	54
Lead	708	11,300	0.030	1.257	0.80	2.06	20.3	19.3	17.2	35.1	33.4	29.8
Magnesium	109	1,750	0.248	10.39	3.68	9.50	99.5	96.8	91.4	172	168	158
Nickel	556	8,910	0.111	4.560	0.87	2.24	53.7	47.7	36.9	93.0	82.6	63.9
Platinum	1340	21,500	0.032	1.340	0.09	0.23	40.5	41.9	43.5	70.1	72.5	75.3
Silver	656	10,500	0.057	2.388	6.42	16.57	240	237	209	415	410	362
Tin	450	7,210	0.051	2.136	1.57	4.05	36	34	—	62	59	—
Tungsten	1206	19,320	0.032	1.340	2.44	6.30	94	87	77	160	150	130
Uranium	1167	18,700	0.027	1.131	0.53	1.37	16.9	17.2	19.6	29.3	29.8	33.9
Zinc	446	7,150	0.094	3.937	1.55	4.00	65	63	58	110	110	100
Alloys												
Aluminum 2024	173	2,770	0.230	9.634	1.76	4.54	70.2			122		
Brass (70% Cu, 30% Ni)	532	8,520	0.091	3.812	1.27	3.28	61.8	73.9	85.3	107	128	148
Constantan (60% Cu, 40% Ni)	557	8,920	0.098	4.105	0.24	0.62	13.1	15.4		22.7	26.7	
Iron, cast	455	7,920	0.100	4.189	0.65	1.68	29.6	26.8		51.2	46.4	
Nichrome V	530	8,490	0.106	4.440	0.12	0.31	7.06	7.99	9.94	12.2	13.8	17.2
Stainless steel	488	7,820	0.110	4.608	0.17	0.44	9.4	10.0	13	16	17.3	23
Steel, mild (1% C)	488	7,820	0.113	4.733	0.45	1.16	24.8	24.8	22.9	42.9	42.9	39.0
Nonmetals												
Asbestos	36	580	0.25	10.5			0.092	0.11	0.125	0.159	0.190	0.21
Brick (fire clay)	144	2,310	0.22	9.22				0.65			1.13	
Brick (masonry)	106	1,670	0.20	8.38			0.38			0.66		
Brick (chrome)	188	3,010	0.20	8.38				0.67			1.16	
Concrete	144	2,310	0.21	8.80			0.70			1.21		
Corkboard	10	160	0.4	17			0.025			0.043		
Diatomaceous earth, powdered	14	220	0.2	8.4			0.03			0.05		
Glass, window	170	2,720	0.2	8.4			0.45			0.78		
Glass, Pyrex	140	2,240	0.2	8.4			0.63	0.67	0.84	1.09	1.16	1.45
Kaolin firebrick	19	300							0.052			0.09
85% Magnesia	17	270					0.038	0.041		0.066	0.071	
Sandy loam, 4% H_2O	104	1,670	0.4	17			0.54			0.94		
Sandy loam, 10% H_2O	121	1,940					1.08			1.87		
Rock wool	10	160	0.2	8.4			0.023	0.033		0.040	0.057	
Wood, oak ⊥ to grain	51	820	0.57	23.9			0.12			0.21		
Wood, oak ‖ to grain	51	820	0.57	23.9			0.23			0.40		

A.2-11 Heat-Transfer Properties of Liquid Water, SI Units

T (°C)	T (K)	ρ (kg/m³)	c_p (kJ/kg · K)	μ × 10³ (Pa · s, or kg/m · s)	k (W/m · K)	N_{Pr}	β × 10⁴ (1/K)	$(g\beta\rho^2/\mu^2) \times 10^{-8}$ (1/K · m³)
0	273.2	999.6	4.229	1.786	0.5694	13.3	−0.630	
15.6	288.8	998.0	4.187	1.131	0.5884	8.07	1.44	10.93
26.7	299.9	996.4	4.183	0.860	0.6109	5.89	2.34	30.70
37.8	311.0	994.7	4.183	0.682	0.6283	4.51	3.24	68.0
65.6	338.8	981.9	4.187	0.432	0.6629	2.72	5.04	256.2
93.3	366.5	962.7	4.229	0.3066	0.6802	1.91	6.66	642
121.1	394.3	943.5	4.271	0.2381	0.6836	1.49	8.46	1300
148.9	422.1	917.9	4.312	0.1935	0.6836	1.22	10.08	2231
204.4	477.6	858.6	4.522	0.1384	0.6611	0.950	14.04	5308
260.0	533.2	784.9	4.982	0.1042	0.6040	0.859	19.8	11 030
315.6	588.8	679.2	6.322	0.0862	0.5071	1.07	31.5	19 260

A.2-11 Heat-Transfer Properties of Liquid Water, English Units

T (°F)	ρ $\left(\frac{lb_m}{ft^3}\right)$	c_p $\left(\frac{btu}{lb_m \cdot °F}\right)$	μ × 10³ $\left(\frac{lb_m}{ft \cdot s}\right)$	k $\left(\frac{btu}{h \cdot ft \cdot °F}\right)$	N_{Pr}	β × 10⁴ (1/°R)	$(g\beta\rho^2/\mu^2) \times 10^{-6}$ (1/°R · ft³)
32	62.4	1.01	1.20	0.329	13.3	−0.350	
60	62.3	1.00	0.760	0.340	8.07	0.800	17.2
80	62.2	0.999	0.578	0.353	5.89	1.30	48.3
100	62.1	0.999	0.458	0.363	4.51	1.80	107
150	61.3	1.00	0.290	0.383	2.72	2.80	403
200	60.1	1.01	0.206	0.393	1.91	3.70	1010
250	58.9	1.02	0.160	0.395	1.49	4.70	2045
300	57.3	1.03	0.130	0.395	1.22	5.60	3510
400	53.6	1.08	0.0930	0.382	0.950	7.80	8350
500	49.0	1.19	0.0700	0.349	0.859	11.0	17 350
600	42.4	1.51	0.0579	0.293	1.07	17.5	30 300

Geankoplis, 4th ed

A.3-3 Physical Properties of Air at 101.325 kPa (1 Atm Abs), SI Units

T (°C)	T (K)	ρ (kg/m^3)	c_p ($kJ/kg \cdot K$)	$\mu \times 10^5$ ($Pa \cdot s$, or $kg/m \cdot s$)	k ($W/m \cdot K$)	N_{Pr}	$\beta \times 10^3$ ($1/K$)	$g\beta\rho^2/\mu^2$ ($1/K \cdot m^3$)
−17.8	255.4	1.379	1.0048	1.62	0.02250	0.720	3.92	2.79×10^8
0	273.2	1.293	1.0048	1.72	0.02423	0.715	3.65	2.04×10^8
10.0	283.2	1.246	1.0048	1.78	0.02492	0.713	3.53	1.72×10^8
37.8	311.0	1.137	1.0048	1.90	0.02700	0.705	3.22	1.12×10^8
65.6	338.8	1.043	1.0090	2.03	0.02925	0.702	2.95	0.775×10^8
93.3	366.5	0.964	1.0090	2.15	0.03115	0.694	2.74	0.534×10^8
121.1	394.3	0.895	1.0132	2.27	0.03323	0.692	2.54	0.386×10^8
148.9	422.1	0.838	1.0174	2.37	0.03531	0.689	2.38	0.289×10^8
176.7	449.9	0.785	1.0216	2.50	0.03721	0.687	2.21	0.214×10^8
204.4	477.6	0.740	1.0258	2.60	0.03894	0.686	2.09	0.168×10^8
232.2	505.4	0.700	1.0300	2.71	0.04084	0.684	1.98	0.130×10^8
260.0	533.2	0.662	1.0341	2.80	0.04258	0.680	1.87	0.104×10^8

A.3-3 Physical Properties of Air at 101.325 kPa (1 Atm Abs), English Units

T (°F)	ρ $\left(\frac{lb_m}{ft^3}\right)$	c_p $\left(\frac{btu}{lb_m \cdot °F}\right)$	μ (centipoise)	k $\left(\frac{btu}{h \cdot ft \cdot °F}\right)$	N_{Pr}	$\beta \times 10^3$ ($1/°R$)	$g\beta\rho^2/\mu^2$ ($1/°R \cdot ft^3$)
0	0.0861	0.240	0.0162	0.0130	0.720	2.18	4.39×10^6
32	0.0807	0.240	0.0172	0.0140	0.715	2.03	3.21×10^6
50	0.0778	0.240	0.0178	0.0144	0.713	1.96	2.70×10^6
100	0.0710	0.240	0.0190	0.0156	0.705	1.79	1.76×10^6
150	0.0651	0.241	0.0203	0.0169	0.702	1.64	1.22×10^6
200	0.0602	0.241	0.0215	0.0180	0.694	1.52	0.840×10^6
250	0.0559	0.242	0.0227	0.0192	0.692	1.41	0.607×10^6
300	0.0523	0.243	0.0237	0.0204	0.689	1.32	0.454×10^6
350	0.0490	0.244	0.0250	0.0215	0.687	1.23	0.336×10^6
400	0.0462	0.245	0.0260	0.0225	0.686	1.16	0.264×10^6
450	0.0437	0.246	0.0271	0.0236	0.674	1.10	0.204×10^6
500	0.0413	0.247	0.0280	0.0246	0.680	1.04	0.163×10^6

Source: National Bureau of Standards, *Circular* 461C, 1947; 564, 1955; NBS–NACA, *Tables of Thermal Properties of Gases*, 1949; F. G. Keyes, *Trans. A.S.M.E.*, **73**, 590, 597 (1951); **74**, 1303 (1952); D. D. Wagman, *Selected Values of Chemical Thermodynamic Properties*. Washington, D.C.: National Bureau of Standards, 1953.

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