

The Equation of Continuity and the Equation of Motion in Cartesian, cylindrical, and spherical coordinates

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Continuity Equation, Cartesian coordinates

$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$

Continuity Equation, cylindrical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r} \frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

Continuity Equation, spherical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial(\rho r^2 v_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\rho v_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial(\rho v_\phi)}{\partial \phi} = 0$$

Equation of Motion for an incompressible fluid, 3 components in Cartesian coordinates

$$\begin{aligned} \rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) &= -\frac{\partial P}{\partial x} + \left(\frac{\partial \tilde{\tau}_{xx}}{\partial x} + \frac{\partial \tilde{\tau}_{yx}}{\partial y} + \frac{\partial \tilde{\tau}_{zx}}{\partial z} \right) + \rho g_x \\ \rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) &= -\frac{\partial P}{\partial y} + \left(\frac{\partial \tilde{\tau}_{xy}}{\partial x} + \frac{\partial \tilde{\tau}_{yy}}{\partial y} + \frac{\partial \tilde{\tau}_{zy}}{\partial z} \right) + \rho g_y \\ \rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) &= -\frac{\partial P}{\partial z} + \left(\frac{\partial \tilde{\tau}_{xz}}{\partial x} + \frac{\partial \tilde{\tau}_{yz}}{\partial y} + \frac{\partial \tilde{\tau}_{zz}}{\partial z} \right) + \rho g_z \end{aligned}$$

Equation of Motion for an incompressible fluid, 3 components in cylindrical coordinates

$$\begin{aligned} \rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) &= -\frac{\partial P}{\partial r} + \left(\frac{1}{r} \frac{\partial(r \tilde{\tau}_{rr})}{\partial r} + \frac{1}{r} \frac{\partial \tilde{\tau}_{\theta r}}{\partial \theta} - \frac{\tilde{\tau}_{\theta\theta}}{r} + \frac{\partial \tilde{\tau}_{zr}}{\partial z} \right) + \rho g_r \\ \rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) &= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \left(\frac{1}{r^2} \frac{\partial(r^2 \tilde{\tau}_{r\theta})}{\partial r} + \frac{1}{r} \frac{\partial \tilde{\tau}_{\theta\theta}}{\partial \theta} + \frac{\partial \tilde{\tau}_{z\theta}}{\partial z} + \frac{\tilde{\tau}_{\theta r} - \tilde{\tau}_{r\theta}}{r} \right) + \rho g_\theta \\ \rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) &= -\frac{\partial P}{\partial z} + \left(\frac{1}{r} \frac{\partial(r \tilde{\tau}_{rz})}{\partial r} + \frac{1}{r} \frac{\partial \tilde{\tau}_{\theta z}}{\partial \theta} + \frac{\partial \tilde{\tau}_{zz}}{\partial z} \right) + \rho g_z \end{aligned}$$

Equation of Motion for an incompressible fluid, 3 components in spherical coordinates

$$\begin{aligned} &\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \right) \\ &= -\frac{\partial P}{\partial r} + \left(\frac{1}{r^2} \frac{\partial(r^2 \tilde{\tau}_{rr})}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\tilde{\tau}_{\theta r} \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \tilde{\tau}_{\phi r}}{\partial \phi} - \frac{\tilde{\tau}_{\theta\theta} + \tilde{\tau}_{\phi\phi}}{r} \right) + \rho g_r \\ &\rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2 \cot \theta}{r} \right) \\ &= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \left(\frac{1}{r^3} \frac{\partial(r^3 \tilde{\tau}_{r\theta})}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\tilde{\tau}_{\theta\theta} \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \tilde{\tau}_{\phi\theta}}{\partial \phi} + \frac{\tilde{\tau}_{\theta r} - \tilde{\tau}_{r\theta}}{r} - \frac{\tilde{\tau}_{\phi\phi} \cot \theta}{r} \right) + \rho g_\theta \\ &\rho \left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} + \frac{v_\theta v_\phi \cot \theta}{r} \right) \\ &= -\frac{1}{r \sin \theta} \frac{\partial P}{\partial \phi} + \left(\frac{1}{r^3} \frac{\partial(r^3 \tilde{\tau}_{r\phi})}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\tilde{\tau}_{\theta\phi} \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \tilde{\tau}_{\phi\phi}}{\partial \phi} + \frac{\tilde{\tau}_{\phi r} - \tilde{\tau}_{r\phi}}{r} + \frac{\tilde{\tau}_{\theta\theta} \cot \theta}{r} \right) + \rho g_\phi \end{aligned} \quad 3$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation) 3 components in Cartesian coordinates

$$\begin{aligned}\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) &= -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) + \rho g_x \\ \rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) &= -\frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right) + \rho g_y \\ \rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) &= -\frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z\end{aligned}$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in cylindrical coordinates

$$\begin{aligned}\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) &= -\frac{\partial P}{\partial r} + \mu \left(\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (r v_r)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right) + \rho g_r \\ \rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) &= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \mu \left(\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (r v_\theta)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right) + \rho g_\theta \\ \rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) &= -\frac{\partial P}{\partial z} + \mu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z\end{aligned}$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in spherical coordinates

$$\begin{aligned}\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \right) &= -\frac{\partial P}{\partial r} + \mu \left(\frac{\partial}{\partial r} \left(\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v_r}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_r}{\partial \phi^2} \right. \\ &\quad \left. - \frac{2}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) - \frac{2}{r^2 \sin \theta} \frac{\partial v_\phi}{\partial \phi} \right) + \rho g_r \\ \rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2 \cot \theta}{r} \right) &= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \mu \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial v_\theta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_\theta}{\partial \phi^2} \right. \\ &\quad \left. + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} - \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial v_\phi}{\partial \phi} \right) + \rho g_\theta \\ \rho \left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} + \frac{v_\phi v_\theta \cot \theta}{r} \right) &= -\frac{1}{r \sin \theta} \frac{\partial P}{\partial \phi} + \mu \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial v_\phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (v_\phi \sin \theta) \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_\phi}{\partial \phi^2} \right. \\ &\quad \left. + \frac{2}{r^2 \sin \theta} \frac{\partial v_r}{\partial \phi} + \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial v_\theta}{\partial \phi} \right) + \rho g_\phi\end{aligned}$$

Note: the r -component of the Navier-Stokes equation in spherical coordinates may be simplified by adding $0 = \frac{2}{r} \nabla \cdot \underline{v}$ to the component shown above. This term is zero due to the continuity equation (mass conservation). See Bird et. al.

References:

1. R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena*, 2nd edition, Wiley: NY, 2002.
2. R. B. Bird, R. C. Armstrong, and O. Hassager, *Dynamics of Polymeric Fluids: Volume 1 Fluid Mechanics*, Wiley: NY, 1987.

The **Equation of Energy** in Cartesian, cylindrical, and spherical coordinates for Newtonian fluids of constant density, with source term S . Source could be electrical energy due to current flow, chemical energy, etc. Two cases are presented: the general case where thermal conductivity may be a function of temperature (vector flux $\underline{\tilde{q}} = \underline{q}/A$ appears in the equations); and the more usual case, where thermal conductivity is constant.

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Microscopic energy balance, in terms of flux; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = -\nabla \cdot \underline{\tilde{q}} + S$$

Microscopic energy balance, in terms of flux; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = - \left(\frac{\partial \tilde{q}_x}{\partial x} + \frac{\partial \tilde{q}_y}{\partial y} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = - \left(\frac{1}{r} \frac{\partial (r \tilde{q}_r)}{\partial r} + \frac{1}{r} \frac{\partial \tilde{q}_\theta}{\partial \theta} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) = - \left(\frac{1}{r^2} \frac{\partial (r^2 \tilde{q}_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (\tilde{q}_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \tilde{q}_\phi}{\partial \phi} \right) + S$$

Fourier's law of heat conduction, Gibbs notation: $\underline{\tilde{q}} = -k \nabla T$

Fourier's law of heat conduction, Cartesian coordinates:

$$\begin{pmatrix} \tilde{q}_x \\ \tilde{q}_y \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial x} \\ -k \frac{\partial T}{\partial y} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$$

Fourier's law of heat conduction, cylindrical coordinates:

$$\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial r} \\ -\frac{k}{r} \frac{\partial T}{\partial \theta} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{r\theta z}$$

Fourier's law of heat conduction, spherical coordinates:

$$\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_\phi \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial r} \\ -\frac{k}{r} \frac{\partial T}{\partial \theta} \\ -\frac{k}{r \sin \theta} \frac{\partial T}{\partial \phi} \end{pmatrix}_{r\theta\phi}$$

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\begin{aligned} \rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) \\ = k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S \end{aligned}$$

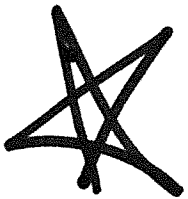
Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013. On the web at www.chem.mtu.edu/~fmorriso/IFM_WebAppendixCD2013.pdf

A.2-11 Heat-Transfer Properties of Liquid Water, SI Units

T (°C)	T (K)	ρ (kg/m ³)	c_p (kJ/kg·K)	$\mu \times 10^3$ (Pa·s, or kg/m·s)	k (W/m·K)	N_{Pr}	$\beta \times 10^4$ (1/K)	$(g\beta\rho^2/\mu^2)$ $\times 10^{-8}$ (1/K·m ³)
0	273.2	999.6	4.229	1.786	0.5694	13.3	-0.630	
15.6	288.8	998.0	4.187	1.131	0.5884	8.07	1.44	10.93
26.7	299.9	996.4	4.183	0.860	0.6109	5.89	2.34	30.70
37.8	311.0	994.7	4.183	0.682	0.6283	4.51	3.24	68.0
65.6	338.8	981.9	4.187	0.432	0.6629	2.72	5.04	256.2
93.3	366.5	962.7	4.229	0.3066	0.6802	1.91	6.66	642
121.1	394.3	943.5	4.271	0.2381	0.6836	1.49	8.46	1300
148.9	422.1	917.9	4.312	0.1935	0.6836	1.22	10.08	2231
204.4	477.6	858.6	4.522	0.1384	0.6611	0.950	14.04	5308
260.0	533.2	784.9	4.982	0.1042	0.6040	0.859	19.8	11 030
315.6	588.8	679.2	6.322	0.0862	0.5071	1.07	31.5	19 260

A.2-11 Heat-Transfer Properties of Liquid Water, English Units

T (°F)	ρ ($\frac{lb_m}{ft^3}$)	c_p ($\frac{btu}{lb_m \cdot ^\circ F}$)	$\mu \times 10^3$ ($\frac{lb_m}{ft \cdot s}$)	k ($\frac{btu}{h \cdot ft \cdot ^\circ F}$)	N_{Pr}	$\beta \times 10^4$ (1/°R)	$(g\beta\rho^2/\mu^2)$ $\times 10^{-6}$ (1/°R·ft ³)
32	62.4	1.01	1.20	0.329	13.3	-0.350	
60	62.3	1.00	0.760	0.340	8.07	0.800	17.2
80	62.2	0.999	0.578	0.353	5.89	1.30	48.3
100	62.1	0.999	0.458	0.363	4.51	1.80	107
150	61.3	1.00	0.290	0.383	2.72	2.80	403
200	60.1	1.01	0.206	0.393	1.91	3.70	1010
250	58.9	1.02	0.160	0.395	1.49	4.70	2045
300	57.3	1.03	0.130	0.395	1.22	5.60	3510
400	53.6	1.08	0.0930	0.382	0.950	7.80	8350
500	49.0	1.19	0.0700	0.349	0.859	11.0	17 350
600	42.4	1.51	0.0579	0.293	1.07	17.5	30 300



Note: $\mu \times 10^3 = 1.786 \text{ Pa s}$
 $\Rightarrow \mu = 1.786 \times 10^{-3} \text{ Pa s}$

A.3-3 Physical Properties of Air at 101.325 kPa (1 Atm Abs), SI Units

T (°C)	T (K)	ρ (kg/m ³)	c_p (kJ/kg·K)	$\mu \times 10^5$ (Pa·s, or kg/m·s)	k (W/m·K)	N_{Pr}	$\beta \times 10^3$ (1/K)	$g\beta\rho^2/\mu^2$ (1/K·m ³)
-17.8	255.4	1.379	1.0048	1.62	0.02250	0.720	3.92	2.79×10^8
0	273.2	1.293	1.0048	1.72	0.02423	0.715	3.65	2.04×10^8
10.0	283.2	1.246	1.0048	1.78	0.02492	0.713	3.53	1.72×10^8
37.8	311.0	1.137	1.0048	1.90	0.02700	0.705	3.22	1.12×10^8
65.6	338.8	1.043	1.0090	2.03	0.02925	0.702	2.95	0.775×10^8
93.3	366.5	0.964	1.0090	2.15	0.03115	0.694	2.74	0.534×10^8
121.1	394.3	0.895	1.0132	2.27	0.03323	0.692	2.54	0.386×10^8
148.9	422.1	0.838	1.0174	2.37	0.03531	0.689	2.38	0.289×10^8
176.7	449.9	0.785	1.0216	2.50	0.03721	0.687	2.21	0.214×10^8
204.4	477.6	0.740	1.0258	2.60	0.03894	0.686	2.09	0.168×10^8
232.2	505.4	0.700	1.0300	2.71	0.04084	0.684	1.98	0.130×10^8
260.0	533.2	0.662	1.0341	2.80	0.04258	0.680	1.87	0.104×10^8

A.3-3 Physical Properties of Air at 101.325 kPa (1 Atm Abs), English Units

T (°F)	ρ ($\frac{lb_m}{ft^3}$)	c_p ($\frac{btu}{lb_m \cdot ^\circ F}$)	μ (centipoise)	k ($\frac{btu}{h \cdot ft \cdot ^\circ F}$)	N_{Pr}	$\beta \times 10^3$ (1/°R)	$g\beta\rho^2/\mu^2$ (1/°R·ft ³)
0	0.0861	0.240	0.0162	0.0130	0.720	2.18	4.39×10^6
32	0.0807	0.240	0.0172	0.0140	0.715	2.03	3.21×10^6
50	0.0778	0.240	0.0178	0.0144	0.713	1.96	2.70×10^6
100	0.0710	0.240	0.0190	0.0156	0.705	1.79	1.76×10^6
150	0.0651	0.241	0.0203	0.0169	0.702	1.64	1.22×10^6
200	0.0602	0.241	0.0215	0.0180	0.694	1.52	0.840×10^6
250	0.0559	0.242	0.0227	0.0192	0.692	1.41	0.607×10^6
300	0.0523	0.243	0.0237	0.0204	0.689	1.32	0.454×10^6
350	0.0490	0.244	0.0250	0.0215	0.687	1.23	0.336×10^6
400	0.0462	0.245	0.0260	0.0225	0.686	1.16	0.264×10^6
450	0.0437	0.246	0.0271	0.0236	0.674	1.10	0.204×10^6
500	0.0413	0.247	0.0280	0.0246	0.680	1.04	0.163×10^6

Source: National Bureau of Standards, Circular 461C, 1947; 564, 1955; NBS-NACA, Tables of Thermal Properties of Gases, 1949; F. G. Keyes, Trans. A.S.M.E., 73, 590, 597 (1951); 74, 1303 (1952); D. D. Wagman, Selected Values of Chemical Thermodynamic Properties, Washington, D.C.: National Bureau of Standards, 1953.

Geankoplis, 4th Edition

Table 1: Emissivity ϵ of solids (300K)

Material	ϵ
Aluminum foil	0.04
Asbestos board	0.96
Polished brass	0.03
Cast iron, turned and heated	0.60-0.70
Concrete	0.85
Ice, smooth	0.966
Ice, rough	0.985
Plaster	0.98
Roofing paper	0.91
Sand	0.76
Steel, Oxidized	0.79
Wrought Iron	0.94

Stephan-Boltzman Constant:

$$\sigma = 0.1712 \times 10^{-8} \frac{BTU}{h ft^2 R^4}$$

$$\sigma = 5.676 \times 10^{-8} \frac{W}{m^2 K^4}$$

Reference: Engineering Toolbox, www.engineeringtoolbox.com/emissivity-coefficients-d_447.html**Table 2: Thermal diffusivity $\alpha = k/\rho\hat{C}_p$ of solids**

Material	Thermal diffusivity (m²/s)	Thermal diffusivity (mm²/s)
Silver, pure (99.9%)	1.6563×10^{-4}	165.63
Gold	1.27×10^{-4}	127
Copper at 25°C	1.11×10^{-4}	111
Aluminum	8.418×10^{-5}	84.18
Steel, stainless 304A at 27°C	4.2×10^{-6}	4.2
Steel, stainless 310 at 25°C	3.352×10^{-6}	3.352
Iron	2.3×10^{-5}	23
Silicon	8.8×10^{-5}	88
Quartz	1.4×10^{-6}	1.4
Water at 25°C	0.143×10^{-6}	0.143
Water vapor (1 atm, 400 K)	2.338×10^{-5}	23.38
Air (300 K)	1.9×10^{-5}	19

Reference: Wikipedia, en.wikipedia.org/wiki/Thermal_diffusivity

FACTORS FOR UNIT CONVERSIONS

Quantity	Equivalent Values
Mass	1 kg = 1000 g = 0.001 metric ton = 2.20462 lb _m = 35.27392 oz 1 lb _m = 16 oz = 5 x 10 ⁻⁴ ton = 453.593 g = 0.453593 kg
Length	1 m = 100 cm = 1000 mm = 10 ⁶ microns (μm) = 10 ¹⁰ angstroms (Å) = 39.37 in = 3.2808 ft = 1.0936 yd = 0.0006214 mile 1 ft = 12 in. = 1/3 yd = 0.3048 m = 30.48 cm
Volume	1 m ³ = 1000 liters = 10 ⁶ cm ³ = 10 ⁶ ml = 35.3145 ft ³ = 220.83 imperial gallons = 264.17 gal = 1056.68 qt 1 ft ³ = 1.728 m ³ = 7.4805 gal = 0.028317 m ³ = 28.317 liters = 28.317 cm ³
Force	1 N = 1 kg m/s ² = 10 ⁵ dynes = 10 ⁵ g cm/s ² = 0.22481 lb _f 1 lb _f = 32.174 lb _m ft/s ² = 4.4482 N = 4.4482 x 10 ⁵ dynes
Pressure	1 atm = 1.01325 x 10 ⁵ N/m ² (Pa) = 101.325 kPa = 1.01325 bars = 1.01325 x 10 ⁶ dynes/cm ² = 760 mm Hg at 0° C (torr) = 10.333 m H ₂ O at 4° C = 14.696 lb _f /in ² (psi) = 33.9 ft H ₂ O at 4° C 100 kPa = 1 bar
Energy	1 J = 1 N m = 10 ⁷ ergs = 10 ⁷ dyne cm = 2.778 x 10 ⁷ kWh = 0.23901 cal = 0.7376 ft lb _f = 9.47817 x 10 ⁴ Btu
Power	1 W = 1 J/s = 0.23885 cal/s = 0.7376 ft lb _f /s = 9.47817 x 10 ⁴ Btu/s = 3.4121 Btu/h = 1.341 x 10 ³ hp (horsepower)
Viscosity	1 Pa s = 1 N s/m ² = 1 kg/ms = 10 poise = 10 dynes s/cm ² = 10 g/cm s = 10 ³ cp (centipoise) = 0.67197 lb _m /fts = 2419.088 lb _m /ft h
Density	1 kg/m ³ = 10 ⁻³ g/cm ³ = 0.06243 lb _m /ft ³ 10 ³ kg/m ³ = 1 g/cm ³ = 62.428 lb _m /ft ³
Volumetric Flow	1 m ³ /s = 35.3145 ft ³ /s = 15,850.2 gal/min (gpm) 1 gpm = 6.30907 x 10 ⁻⁵ m ³ /s = 2.2802 x 10 ³ ft ³ /s = 3.7854 liter/min 1 liter/min = 0.26417 gpm

Temperature	$T(^{\circ}\text{C}) = \frac{5}{9}[T(^{\circ}\text{F}) - 32]$ $T(^{\circ}\text{F}) = \frac{9}{5}T(^{\circ}\text{C}) + 32 = 1.8T(^{\circ}\text{C}) + 32$
Absolute Temperature	$T(\text{K}) = T(^{\circ}\text{C}) + 273.15$ $T(^{\circ}\text{R}) = T(^{\circ}\text{F}) + 459.67$
Temperature Interval (ΔT)	$1^{\circ}\text{C} = 1\text{ K} = 1.8^{\circ}\text{F} = 1.8\text{ R}^{\circ}$ $1\text{F}^{\circ} = 1\text{ R}^{\circ} = (5/9)^{\circ}\text{C} = (5/9)\text{ K}$

USEFUL QUANTITIES

	SG = ρ(20°C)/ρ _{water} (4°C)
	ρ _{water} (4°C) = 1000 kg/m ³ = 62.43 lb _m /ft ³ = 1.000 g/cm ³ ρ _{water} (25°C) = 997.08 kg/m ³ = 62.25 lb _m /ft ³ = 0.99709 g/cm ³ g = 9.8066 m/s ² = 980.66 cm/s ² = 32.174 ft/s ² μ _{water} (25°C) = 8.937 x 10 ⁻⁴ Pa s = 8.937 x 10 ⁻⁴ kg/ms = 0.8937 cp = 0.8937 x 10 ⁻² g/cms = 6.005 x 10 ⁻⁴ lb _m /fts
Composition of air:	N ₂ 78.03% O ₂ 20.99% Ar 0.94% CO ₂ 0.03% H ₂ , He, Ne, Kr, Xe 0.01% 100.00%
	M _{air} = 29 g/mol = 29 kg/kmol = 29 lb _m /lbmole
	Ĉ _{p,water} (25°C) = 4.182 kJ/kg K = 0.9989 cal/g°C = 0.9997 Btu/lb _m °F
	R = 8.314 m ³ Pa/mol K = 0.08314 liter bar/mol K = 0.08206 liter atm/mol K = 62.36 liter mm Hg/mol K = 0.7302 ft ³ atm/lbmole °R = 10.73 ft ³ psia/lbmole °R = 8.314 J/mol K = 1.987 cal/mol K = 1.987 Btu/lbmole °R

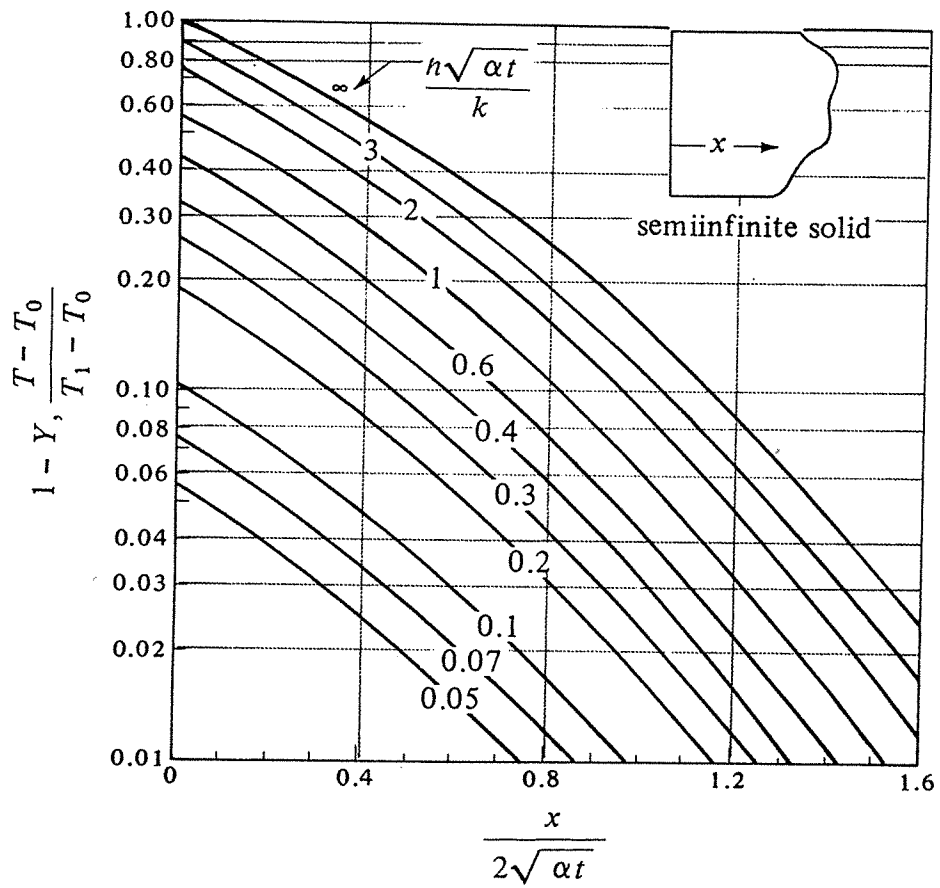


FIGURE 5.3-3. Unsteady-state heat conducted in a semi-infinite solid with surface convection. Calculated from Eq. (5.3-7)(S1).

Geankoplis, 4th edition

Unsteady Macroscopic Energy Balance

$accumulation = input - output$

$Q_{in} = \text{Heat } \mathbf{into} \text{ the chosen macroscopic control volume}$

$$\frac{d}{dt}(U_{sys} + E_{k,sys} + E_{p,sys}) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$$

$Q_{in} = \sum_i q_{in,i}$ comes from a variety of sources:

- Thermal conduction: $q_{in} = -kA \frac{dT}{dx}$
- Convection heat xfer: $q_{in} = hA(T_b - T)$
- Radiation: $q_{in} = \epsilon\sigma A(T_{surroundings}^4 - T_{surface}^4)$
- Electric current: $q_{in} = I^2 R_{elec} L$
- Chemical Reaction: $q_{in} = S_{rxn} V_{sys}$

$S [=] \frac{\text{energy}}{\text{time volume}}$

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Unsteady Macroscopic Energy Balance

$\frac{d}{dt}(U_{sys} + E_{k,sys} + E_{p,sys}) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$

$Q_{in} = \sum_i q_{in,i}$ comes from a variety of sources:

- **Thermal conduction:** $q_{in} = -kA \frac{dT}{dx}$
e.g. device held by bracket; a solid phase that extends through boundaries of control volume
- **Convection heat xfer:** $q_{in} = hA(T_b - T)$
e.g. device dropped in stirred liquid; forced air stream flows past, natural convection occurs outside system; phase change at boundary
- **Radiation:** $q_{in} = \epsilon\sigma A(T_{surroundings}^4 - T_{surface}^4)$
e.g. device at high temp. exposed to a gas/vacuum; hot enough to produce nat. conv.=possibly hot enough for radiation
- **Electric current:** $q_{in} = I^2 R_{elec} L$
e.g. if electric current is flowing within the device/control volume/system
- **Chemical Reaction:** $q_{in} = S_{rxn} V_{sys}$
e.g. if a homogeneous reaction is taking place throughout the device/ control volume/system

S-B constant:
 $\sigma = 5.676 \times 10^{-8} \frac{W}{m^2 K^4}$

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