Module 1: Intro and Prerequisite Material

Steady Heat Transfer Review

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www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

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CM3120: Module 1

Introduction and Prereq Material

I. Introduction
II. Review of Prerequisite Material
   a. Microscopic energy balances
   b. Fourier's law of heat conduction (k, homogeneous)
   c. Newton's law of cooling (h, at a boundary)
   d. Resistances due to k and h
   e. Solving for the steady temperature field $T(x,y,z)$
   f. Dimensional analysis in heat transfer for h
   g. h Data correlations for forced and free convection
   h. h For radiation heat transfer

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Why study transport/unit ops?

Where are we in our study of transport/O.U.?

How far along did we get in CM3110 and other prerequisite courses?

Michigan Tech

CM3110
Transport Processes and Unit Operations I

Part 2:

Professor Faith Morrison
Department of Chemical Engineering
Michigan Technological University

CM3110 - Momentum and Heat Transport
CM3120 – Heat and Mass Transport

CM3120: Unsteady State Heat Transfer/Mass Transfer/Unit Operations

CM3120 builds on these topics from the prerequisites.

Module 1: Intro and Prerequisite Material

Steady Heat Transfer Review

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We begin with a review:

- Microscopic energy balance
- Fourier's law of heat conduction (k, homogeneous)
- Newton's law of cooling (h, at a boundary)
- Resistances due to k and h
- Solving for the *steady* temperature field $T(x, y, z)$
- Dimensional analysis in heat transfer for h
- h Data correlations for forced and free convection
- h For radiation heat transfer

Microscopic Energy Balance Review

CM3110 REVIEW

Unsteady State Heat Transfer

Module 1: Intro and Prerequisite Material

Steady Heat Transfer Review

We begin our prereq review here

We begin with a Review:

- Microscopic energy balance
- Fourier's law of heat conduction (k, homogeneous)
- Newton's law of cooling (h, at a boundary)
- Resistances due to k and h
- Solving for the *steady* temperature field $T(x,y,z)$
- Dimensional analysis in heat transfer
- Data correlations for forced and free convection
- Radiation heat transfer

Microscopic Energy Balance Review

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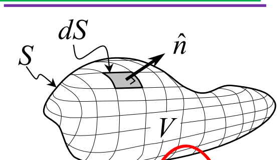
The **microscopic energy balance** is an expression of the law of **conservation of energy**.

It includes consideration of **unsteady** energy flows.

Microscopic Energy Balance:

Equation of Thermal Energy



Microscopic **energy** balance written on an arbitrarily shaped volume, *V*, enclosed by a surface, *S*

Gibbs notation: $\rho\left(\frac{\partial \hat{E}}{\partial t} + \underline{v}\cdot\nabla\hat{E}\right) = -\nabla\cdot\underline{\tilde{q}} + S_e$ **general conduction**

Gibbs notation: $\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = k\nabla^2 T + S_e$ **Only Fourier conduction**

(incompressible fluid, constant pressure, neglect $\hat{E}_k, \hat{E}_p$, viscous dissipation)

http://pages.mtu.edu/~fmorriso/cm310/energy_equation.html

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Microscopic Energy Balance Review

CM3110 REVIEW

What *physics* determines how rapidly (the *rate*) the heat transfers from one location to another?

Energy Transport law

Fourier's Law of Heat Conduction

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

(for a homogeneous phase)

$\frac{q_x}{A}$ –heat flux=energy/area time)

***k* – thermal conductivity**

$\frac{dT}{dx}$ –temperature gradient

(the driving physics of Fourier's law is Brownian motion: energy transports down ∇T due to Brownian motion)

Microscopic Energy Balance Review

CM3110 REVIEW

Heat Transfer Rate law:

Fourier's law of Heat Conduction

Makes reference to a coordinate system

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

Allows you to solve for temperature profiles (also known as temperature *distributions* or *fields*)

Gibbs notation: $\frac{q}{A} = -k\nabla T$

- Heat flows **down** a temperature gradient
- Flux is proportional to the magnitude of temperature gradient

Fourier's law in three dimensions

$$\underset{\sim}{\tilde{q}} = \frac{q}{A} = \begin{pmatrix} -k\frac{\partial T}{\partial x} \\ -k\frac{\partial T}{\partial y} \\ -k\frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$$

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Microscopic Energy Balance Review

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Equation of Energy

(microscopic energy balance)

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

- $\frac{\partial T}{\partial t}$: rate of change
- $\underline{v} \cdot \nabla T$: convection
- $k \nabla^2 T$: conduction (all directions)
- S_e: source (energy generated per unit volume per time)

velocity must satisfy equation of motion, equation of continuity

Due to: *electrical current; chemical reaction*

see handout for component notation

http://pages.mtu.edu/~fmorriso/cm310/energy_equation.html

The **Equation of Energy** in Cartesian, cylindrical, and spherical coordinates for Newtonian fluids of constant density, with source term S. Source could be electrical energy due to current flow, chemical energy, etc. Two cases are presented: the general case where thermal conductivity may be a function of temperature (vector flux $\underline{\tilde{q}} = \underline{q}/A$ appears in the equations); and the more usual case, where thermal conductivity is constant.

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Microscopic energy balance, in terms of flux; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = -\nabla \cdot \underline{\tilde{q}} + S$$

Microscopic energy balance, in terms of flux; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = -\left(\frac{\partial \tilde{q}_x}{\partial x} + \frac{\partial \tilde{q}_y}{\partial y} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = -\left(\frac{1}{r} \frac{\partial (r \tilde{q}_r)}{\partial r} + \frac{1}{r} \frac{\partial \tilde{q}_\theta}{\partial \theta} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right) = -\left(\frac{1}{r^2} \frac{\partial (r^2 \tilde{q}_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial (\tilde{q}_\theta sin\theta)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial \tilde{q}_\phi}{\partial \phi} \right) + S$$

Fourier's law of heat conduction, Gibbs notation: $\underline{\tilde{q}} = -k \nabla T$

Fourier's law of heat conduction, Cartesian coordinates: $\begin{pmatrix} \tilde{q}_x \\ \tilde{q}_y \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial x} \\ -k \frac{\partial T}{\partial y} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$

Fourier's law of heat conduction, cylindrical coordinates: $\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial r} \\ -\frac{k}{r} \frac{\partial T}{\partial \theta} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{r\theta z}$

Fourier's law of heat conduction, spherical coordinates: $\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_\phi \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial r} \\ -\frac{k}{r} \frac{\partial T}{\partial \theta} \\ -\frac{k}{r \sin\theta} \frac{\partial T}{\partial \phi} \end{pmatrix}_{r\theta\phi}$

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013. On the web at www.chem.mtu.edu/~fmorriso/IFM_WebAppendixCD2013.pdf

http://pages.mtu.edu/~fmorriso/cm310/energy_equation.html

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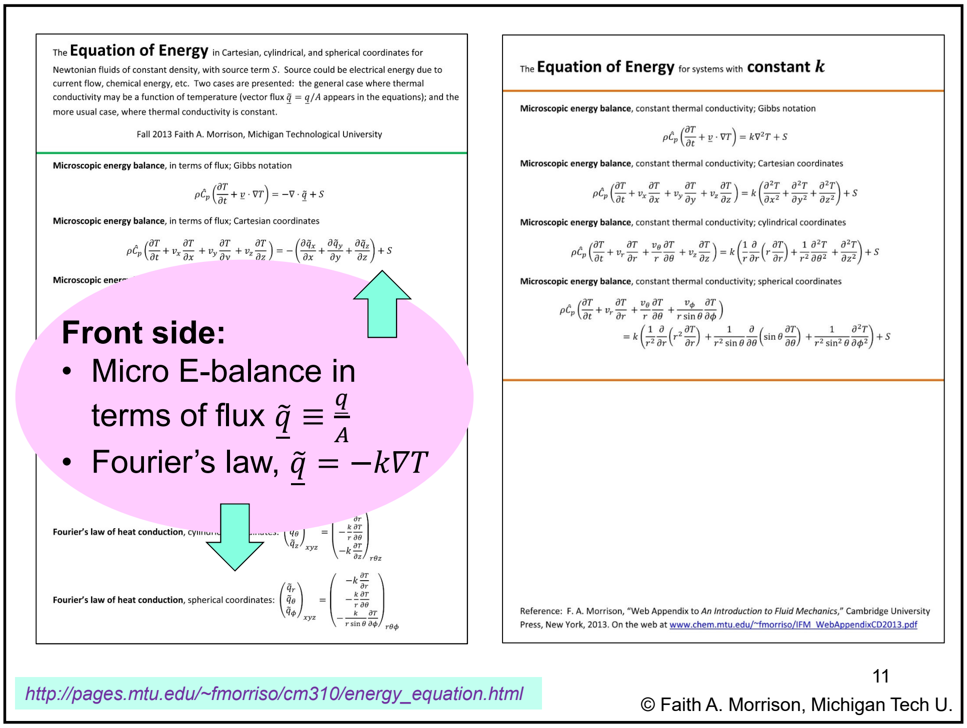
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Microscopic energy balance, in terms of flux; Gibbs notation
$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = -\nabla\cdot\underline{\tilde{q}} + S$
Microscopic energy balance, in terms of flux; Cartesian coordinates
Front side:
Micro E-balance in terms of flux $\underline{\tilde{q}} \equiv \frac{\underline{q}}{A}$
Fourier's law, $\underline{\tilde{q}} = -k\nabla T$
Fourier's law of heat conduction, spherical coordinates:
The Equation of Energy for systems with constant k
Microscopic energy balance, constant thermal conductivity; Gibbs notation
$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = k\nabla^2 T + S$
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Microscopic energy balance, constant thermal conductivity; cylindrical coordinates
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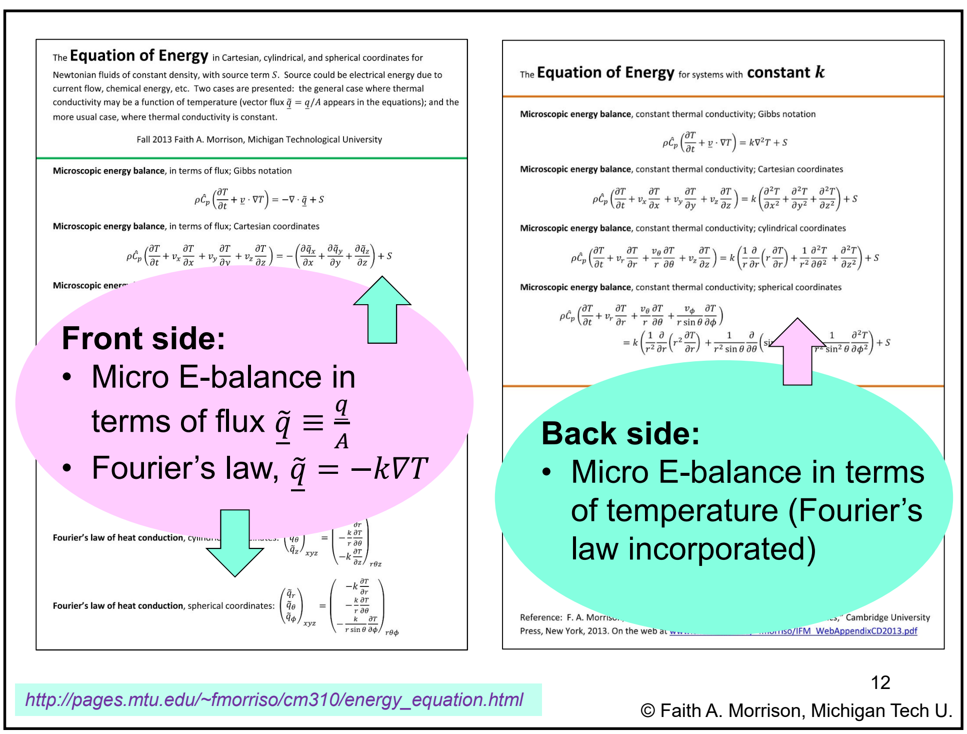
The Equation of Energy in Cartesian, cylindrical, and spherical coordinates for Newtonian fluids of constant density, with source term S.
Front side:
Micro E-balance in terms of flux $\underline{\tilde{q}} \equiv \frac{\underline{q}}{A}$
Fourier's law, $\underline{\tilde{q}} = -k\nabla T$
The Equation of Energy for systems with constant k
Back side:
Micro E-balance in terms of temperature (Fourier's law incorporated)
http://pages.mtu.edu/~fmorriso/cm310/energy_equation.html
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Microscopic Energy Balance

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The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right)$$
$$=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right)+S$$

http://pages.mtu.edu/~fmorriso/cm310/energy_equation.html

Fourier's Law of Heat Conduction

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Fourier's law of heat conduction, Gibbs notation: $\tilde{\underline{q}}=\underline{q}/A=-k\nabla T$

Fourier's law of heat conduction, Cartesian coordinates:
(constant thermal conductivity k)

$$\begin{pmatrix}\tilde{q}_x\\ \tilde{q}_y\\ \tilde{q}_z\end{pmatrix}_{xyz}=\begin{pmatrix}q_x/A\\ q_y/A\\ q_z/A\end{pmatrix}_{xyz}=\begin{pmatrix}-k\frac{\partial T}{\partial x}\\ -k\frac{\partial T}{\partial y}\\ -k\frac{\partial T}{\partial z}\end{pmatrix}_{xyz}$$

$$\tilde{q}_x=\frac{q_x}{A}$$

Fourier's law of heat conduction, cylindrical coordinates:
(constant thermal conductivity k)

$$\begin{pmatrix}\tilde{q}_r\\ \tilde{q}_\theta\\ \tilde{q}_z\end{pmatrix}_{r\theta z}=\begin{pmatrix}q_r/A\\ q_\theta/A\\ q_z/A\end{pmatrix}_{r\theta z}=\begin{pmatrix}-k\frac{\partial T}{\partial r}\\ -\frac{k}{r}\frac{\partial T}{\partial \theta}\\ -k\frac{\partial T}{\partial z}\end{pmatrix}_{r\theta z}$$

Fourier's law of heat conduction, spherical coordinates:
(constant thermal conductivity k)

$$\begin{pmatrix}\tilde{q}_r\\ \tilde{q}_\theta\\ \tilde{q}_\phi\end{pmatrix}_{r\theta\phi}=\begin{pmatrix}q_r/A\\ q_\theta/A\\ q_\phi/A\end{pmatrix}_{r\theta\phi}=\begin{pmatrix}-k\frac{\partial T}{\partial r}\\ -\frac{k}{r}\frac{\partial T}{\partial \theta}\\ -\frac{k}{r\sin\theta}\frac{\partial T}{\partial \phi}\end{pmatrix}_{r\theta\phi}$$

https://pages.mtu.edu/~fmorriso/cm310/energy.pdf

Microscopic Energy Balance Review

CM3110 REVIEW

Unsteady State Heat Transfer

Module 1: Intro and Prerequisite Material

Steady Heat Transfer Review

Now, *Boundary Conditions* and *Resistances*

We begin with a Review:

- ✓ Microscopic energy balance
- ✓ Fourier's law of heat conduction (k, homogeneous)
- Newton's law of cooling (h, at a boundary)
- Resistances due to k and h
- Solving for the *steady* temperature field $T(x, y, z)$
- Dimensional analysis in heat transfer
- Data correlations for forced and free convection
- Radiation heat transfer

Microscopic Energy Balance Review—Boundary Conditions

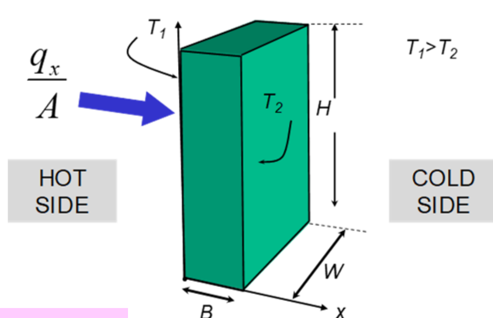
CM3110 REVIEW

We will need **boundary conditions** on temperature to solve the microscopic balances for the temperature distribution.

Example 1: Heat flux in a rectangular solid – Temperature BC

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?

Assumptions:
- wide, tall slab
- steady state



We may know the temperature at the boundary.

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Microscopic Energy Balance Review—Boundary Conditions

CM3110 REVIEW

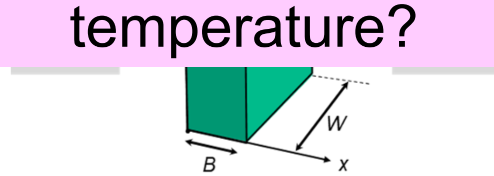
We will need **boundary conditions** on temperature to solve the microscopic balances for the temperature distribution.

Example 1: Heat flux in a rectangular solid – Temperature BC

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?

Assumptions:
- wide, tall slab
- steady state

What if we don't know the wall temperature?



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Microscopic Energy Balance Review—Boundary Conditions

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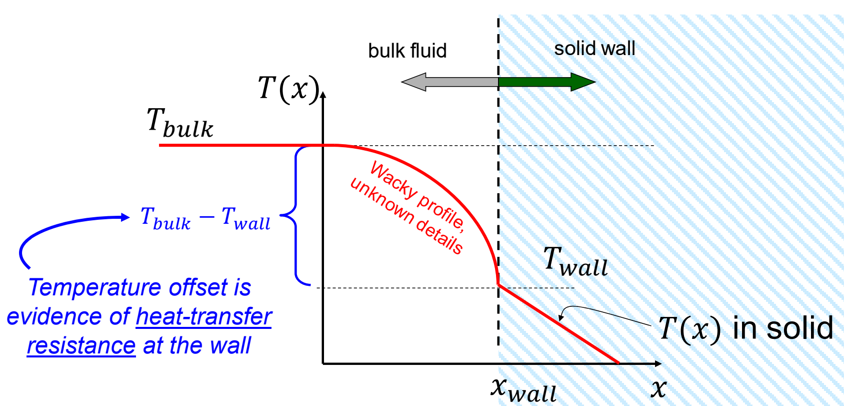
The interface between the solid and the fluid calls for a new type of **boundary condition**, *Newton's Law of Cooling*.



There may exist a *resistance* to heat transfer at the boundary, due to fluid characteristics

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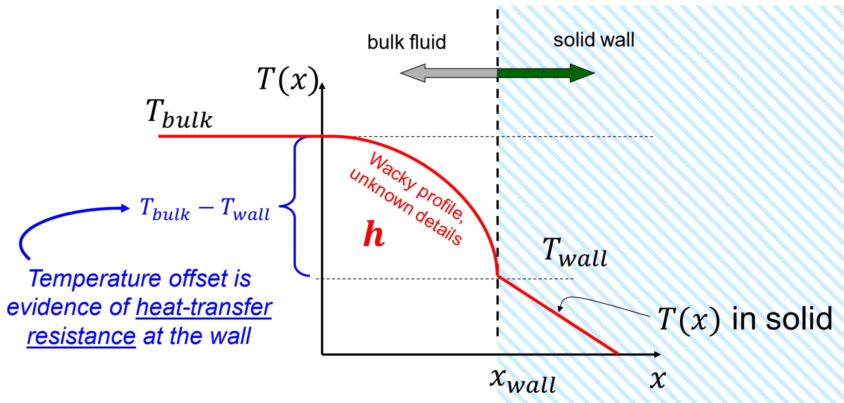
Microscopic Energy Balance Review—Boundary Conditions

CM3110 REVIEW

The interface between the solid and the fluid calls for a new type of **boundary condition**, *Newton's Law of Cooling*.



The temperature difference at the fluid-wall interface is caused by complex fluid phenomena that are lumped together into the heat transfer coefficient, h

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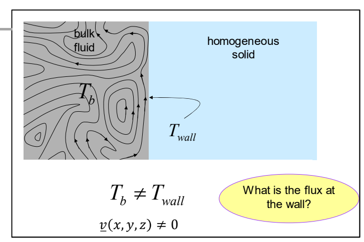
Microscopic Energy Balance Review



The heat flux at the wall is given by the empirical expression known as

Newton's Law of Cooling

(a *linear driving-force* model for interphase heat transport)

This expression serves as the definition of the **heat transfer coefficient**.

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

***h* depends on:**

- geometry
- **fluid** velocity field
- **fluid** properties
- temperature

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Microscopic Energy Balance Review

Review so far…

- **Microscopic energy balance**

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = k\nabla^2 T + S_e$$

rate of change ($\frac{\partial T}{\partial t}$); convection ($\underline{v}\cdot\nabla T$); conduction (all directions) ($k\nabla^2 T$); source (S_e)

- **Fourier's law of heat conduction**

Fourier's Law of Heat Conduction

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

(for a homogeneous phase)

- **Newton's law of cooling** (h, at the phase boundary)

This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

Next

- ***Resistances*** due to k and h

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Microscopic Energy Balance Review—Resistance to Heat Transfer

$\mathcal{R}$ = Resistance to Heat Transfer

The language of ***resistance*** to describe the physics of heat transfer will be handy in our study of unsteady state temperature profiles. We encountered this language in CM3110, and we review and summarize now.

$$\frac{q_x}{A} = \frac{\text{driving force}}{\sum \text{resistances}}$$

Two limitations create resistance:

1. *Limited* conductivity within the homogeneous phase (k)
2. *Limited* heat transfer between phases at a boundary (h)

Also, resistances:

1. Are affected by geometry (rectangular versus radial)
2. Can be stacked (that is, added together like electrical resistances)

Note: Geankoplis uses a slightly different definition of resistance; we follow Bird et al. 2002.

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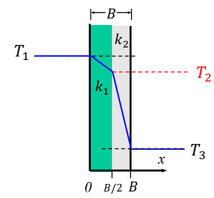
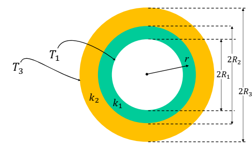
1D Heat Transfer – Resistance



Thermal conductivity k and heat transfer coefficient h may be thought of as sources of ***resistance*** $\mathcal{R}$ to heat transfer.

These resistances $\mathcal{R}$ ***stack up*** in a logical way, allowing us to quickly and accurately determine the effect of adding insulating layers, encountering pipe fouling, and other applications.



Using the microscopic energy balance on a test problem, we can solve for the temperature profile and then the heat flux, which is the driving force/resistance.

We can then identify the resistances for each test case considered.

$$\frac{q_x}{A} = \frac{\text{driving force}}{\sum \text{resistances}}$$

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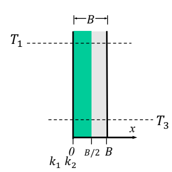
1D Heat Transfer – Resistance

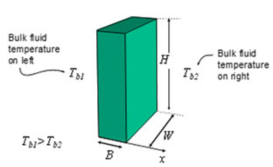
RESISTANCE SUMMARY:

1D Rectangular: Door ($k_1 = k_2$), and Composite Door



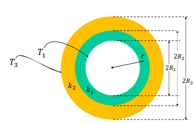
$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\left(\frac{B/2}{k_1} + \frac{B/2}{k_2}\right)}$$

1D Rectangular: Slab with Newton's law BC



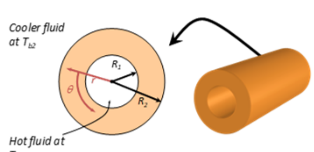
$$\frac{q_x}{A} = \frac{T_{b1} - T_{b2}}{\left(\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}\right)}$$

1D Radial: Pipe ($k_1 = k_2$) and Composite Pipe



$$\frac{q_r}{A} = \frac{(T_1 - T_3)}{\frac{1}{k_1}\ln\frac{R_2}{R_1} + \frac{1}{k_2}\ln\frac{R_3}{R_2}}\left(\frac{1}{r}\right)$$

1D Radial: Pipe with Newton's law BC



$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\left(\frac{1}{r}\right)$$

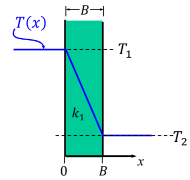
1D Heat Transfer – Resistance

Temperature Boundary Conditions

1D Rectangular

Let: $\mathcal{R} \equiv \frac{B}{k}$

$$\frac{q_x}{A} = \frac{(T_1 - T_2)}{\mathcal{R}} = \frac{\text{driving force}}{\text{resistance}}$$

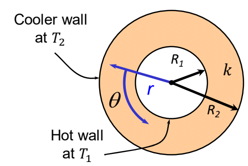


Note: Geankoplis uses a different resistance. For rectangular heat flux:
$R_{Geankoplis} = \mathcal{R}/LW$

1D Radial

Let: $\mathcal{R} \equiv \frac{1}{k}\ln\frac{R_2}{R_1}$

$$\frac{q_r}{A} = \left(\frac{(T_1 - T_2)}{\mathcal{R}}\right)\frac{1}{r} = \frac{\text{driving force}}{\text{resistance}}$$



Note: Geankoplis uses a different resistance. For radial heat flux:
$R_{Geankoplis} = \mathcal{R}/2\pi L$

1D Heat Transfer – Resistance

Newton's Law of Cooling Boundary Conditions

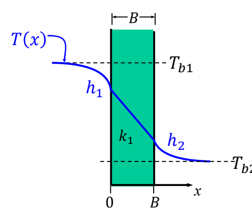
1D Rectangular

Let:

$\mathcal{R}_i \equiv \frac{1}{h_i}$ for $i = 1,2$

$\mathcal{R}_3 \equiv \frac{B}{k}$

$$\frac{q_x}{A} = \frac{(T_{b1} - T_{b2})}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3} = \frac{\text{driving force}}{\text{resistance}}$$



1D Radial

Let:

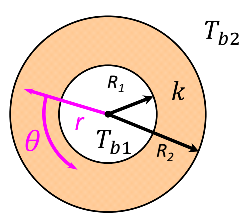
$\mathcal{R}_i \equiv \frac{1}{R_i h_i}$ for $i = 1,2$

$\mathcal{R}_3 \equiv \frac{1}{k}\ln\frac{R_2}{R_1}$

h_1 = inside

h_2 = outside

$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3}\left(\frac{1}{r}\right) = \frac{\text{driving force}}{\text{resistance}}$$



Microscopic Energy Balance Review

CM3110 REVIEW

Unsteady State Heat Transfer

Module 1: Intro and Prerequisite Material

Steady Heat Transfer Review

We begin with a Review:

Now, *"Slash and Burn"*

- ✓ Microscopic energy balance
- ✓ Fourier's law of heat conduction (k, homogeneous)
- ✓ Newton's law of cooling (h, at a boundary)
- ✓ Resistances due to k and h
- Solving for the *steady* temperature field $T(x, y, z)$
- Dimensional analysis in heat transfer
- Data correlations for forced and free convection
- Radiation heat transfer

Microscopic Energy Balance—Solve for Temperature Field

CM3110 REVIEW

For review, let's carry out an example of 1D, steady heat transfer

CM3110
Transport I
Part II: Heat Transfer

MichiganTech

One-Dimensional Heat Transfer
(part 1: rectangular slab)

Simple problems that allow us to identify the physics

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

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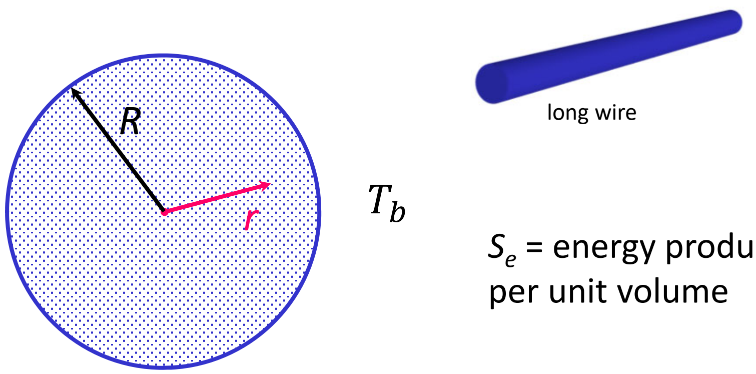
Microscopic Energy Balance—Solve for Temperature Field

Example 3: Heat Conduction with Generation

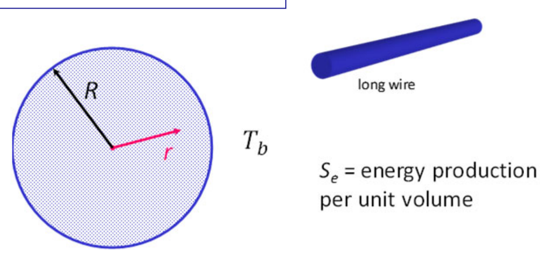
What is the steady state temperature profile in a wire if heat is generated uniformly throughout the wire at a rate of S_e W/m^3 and the bulk fluid surrounding the wire is at T_b? What is the heat flux?



S_e = energy production per unit volume

Microscopic Energy Balance—Solve for Temperature Field

Example: Heat conduction with generation



h = heat transfer coefficient

Let's try.

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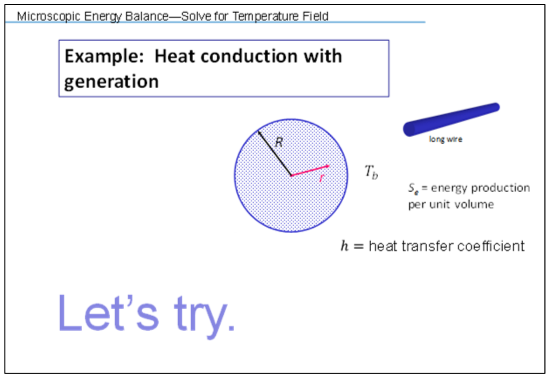

In class solution will be posted with the other "hand notes."

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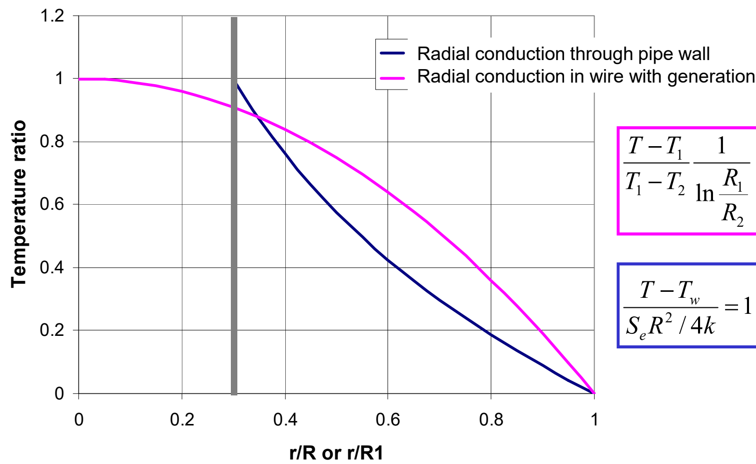
Microscopic Energy Balance—Solve for Temperature Field

Compare radial conduction solutions



$$\frac{T-T_1}{T_1-T_2}\frac{1}{\ln\frac{R_1}{R_2}}=\ln\frac{r}{R_1}$$

$$\frac{T-T_w}{S_eR^2/4k}=1-\left(\frac{r}{R}\right)^2$$

Unsteady State Heat Transfer

CM3110 REVIEW

Steady Heat-Transfer Review Summary (thus far):

- Microscopic energy balance
- Fourier's law of heat conduction (k, homogeneous)
- Newton's law of cooling (h, at the boundary between two phases)
- **Resistances** due to k and h; vary with boundary conditions (BC) and geometry

	***T* BC**	***h* BC**
1D rectangular	$\frac{B}{k}$	$\frac{1}{h}$
1D radial	$\frac{1}{k}\ln\frac{R_2}{R_1}$	$\frac{1}{Rh}$

- Solving for the *steady* temperature field $T(x,y,z)$, a.k.a. "*Slash and Burn*"

Unsteady State Heat Transfer

CM3110 REVIEW

Steady Heat-Transfer Review Summary (thus far):

- Microscopic energy balance
- Fourier's law of heat conduction (k, homogeneous)
- Newton's law of cooling (h, at the boundary between two phases)
- **Resistances** due to k and h; vary with boundary conditions (BC) and geometry

	***T* BC**	***h* BC**
1D rectangular	$\frac{B}{k}$	$\frac{1}{h}$
1D radial	$\frac{1}{k}\ln\frac{R_2}{R_1}$	$\frac{1}{Rh}$

Sneak peak: The ratio of T (*internal*) and h (*external*) resistances is the **Biot** number:

$$\boldsymbol{Bi} = \frac{B/k}{1/h} = \frac{\boldsymbol{hB}}{\boldsymbol{k}}$$

This is important in unsteady heat transfer.

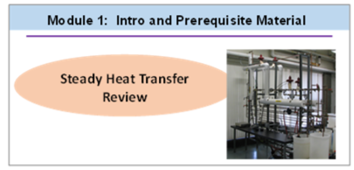
- Solving for the *steady* temperature field $T(x,y,z)$, a.k.a. "*Slash and Burn*"

Microscopic Energy Balance Review

CM3110 REVIEW

Unsteady State Heat Transfer

Finally, Dimensional analysis and data correlations for heat transfer coefficient h

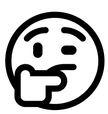


We begin with a Review:

- ✓ Microscopic energy balance
- ✓ Fourier's law of heat conduction (k, homogeneous)
- ✓ Newton's law of cooling (h, at a boundary)
- ✓ Resistances due to k and h
- ✓ Solving for the *steady* temperature field $T(x, y, z)$
- Dimensional analysis in heat transfer
- Data correlations for forced and free convection
- Radiation heat transfer

Microscopic Energy Balance Review

CM3110 REVIEW



Dimensional Analysis?

For complex systems, we turn to data correlations based on dimensional analysis

The engineering quantity of interest in heat transfer is the amount of heat $\dot{Q}$ transferred

Nusselt number, a dimensionless heat transfer coefficient, is a dimensionless amount of heat $\dot{Q}$ transferred

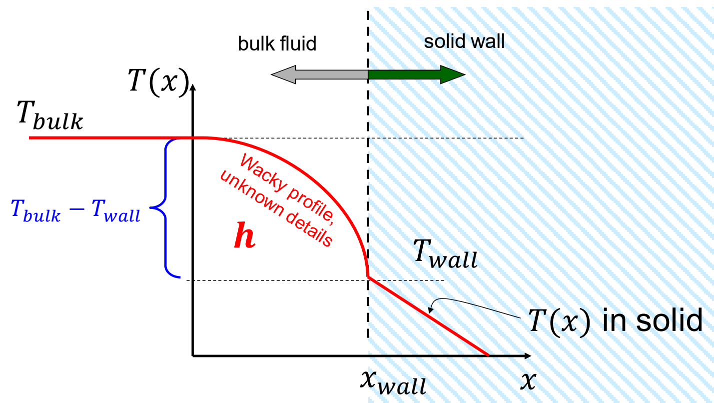
Microscopic Energy Balance Review—Dimensional Analysis

CM3110 REVIEW

Heat Transfer Coefficient:

- Linear driving force model
- Heat transfer between phases

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$



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Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Review: What is *Dimensional Analysis*?

- Flow in pipes at all flow rates (laminar and turbulent)
  Solution: Navier-Stokes, Re, Fr, L/D, dimensionless wall force $= f$; $f = f(\mathrm{Re}, L/D)$
- Rough pipes
  Solution: add additional length scale; then nondimensionalize
- Non-circular conduits
  Solution: Use hydraulic diameter as the length scale of the flow to nondimensionalize
- Flow around obstacles (spheres, other complex shapes
  Solution: Navier-Stokes, Re, dimensionless drag $= C_D$; $C_D = C_D(\mathrm{Re})$
- Boundary layers
  Solution: Two components of velocity need independent lengthscales

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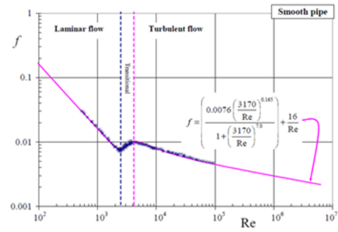
Complex Heat Transfer – Dimensional Analysis

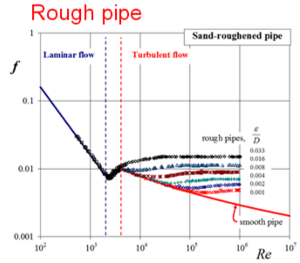
CM3110 REVIEW

Data Correlations:

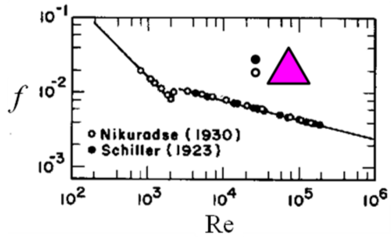
Turbulent flow (smooth pipe)



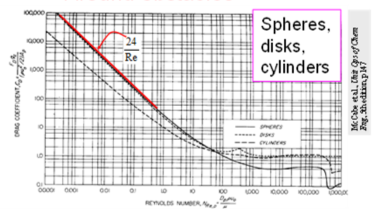
Rough pipe



Noncircular cross section



Around obstacles



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Complex Heat Transfer – Dimensional Analysis

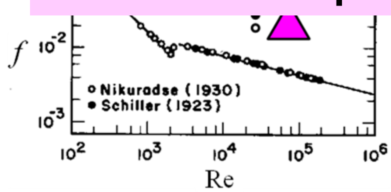
CM3110 REVIEW

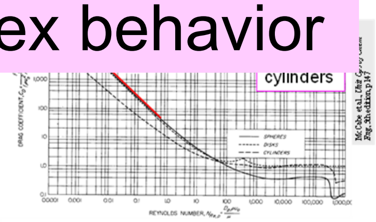
Data Correlations:

Turbulent flow (smooth pipe)

Rough pipe

Dimensional analysis allows us to capture and engineer ***around*** or ***with*** complex behavior





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How does Dimensional Analysis work in Heat Transfer?

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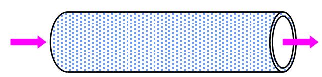
First try: **Forced Convection**

Complex Heat Transfer – Dimensional Analysis

Chosen problem: Forced Convection Heat Transfer
Solution: Dimensional Analysis



Following procedure familiar from pipe flow,

- **What are governing equations?**
- **Scale factors (dimensionless numbers)?**
- **Quantity of interest?**
  Answer: Heat flux at the wall

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Dimensional Analysis in Forced Convection Heat Transfer

Pipe flow

CM3110 REVIEW

z-component of the Navier-Stokes Equation:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$$

$$=-\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Choose:

D = characteristic length
V = characteristic velocity
D/V = characteristic time
ρV^2 = characteristic pressure

- Choose “typical” values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

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Dimensional Analysis in Forced Convection Heat Transfer

FORCED CONVECTION

Pipe flow

CM3110 REVIEW

non-dimensional variables:

time:

$$t^* \equiv \frac{tV}{D}$$

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z}{V}$$

$$v_r^* \equiv \frac{v_r}{V}$$

$$v_\theta^* \equiv \frac{v_\theta}{V}$$

driving force:

$$P^* \equiv \frac{P}{\rho V^2}$$

$$g_z^* \equiv \frac{g_z}{g}$$

- Choose "typical" values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

Dimensional Analysis in Forced Convection Heat Transfer

FORCED CONVECTION

Energy

Microscopic energy balance:

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

non-dimensional variables:

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

temperature:

$$T^* \equiv \frac{T-T_o}{(T_1-T_o)}$$

source:

$$S^* \equiv \frac{S}{S_0}$$

Choose:
T – use a characteristic **interval** (since distance from absolute zero is not part of this physics)
S – use a reference source, S_0

$$S_0 \equiv \frac{(T_1-T_0)V\rho\hat{C}_p}{D}\ [=]\ \frac{W}{m^2}$$

T_0 = surface
T_1 = bulk

Complex Heat Transfer – Dimensional Analysis Forced Convection

CM3110 REVIEW

Forced Convection Heat Transfer

FORCED CONVECTION

Pipe flow

non-dimensional variables:

time: $t^* \equiv \frac{tV}{D}$

position: $r^* \equiv \frac{r}{D}$, $z^* \equiv \frac{z}{D}$

velocity: $v_z^* \equiv \frac{v_z}{V}$, $v_r^* \equiv \frac{v_r}{V}$, $v_\theta^* \equiv \frac{v_\theta}{V}$

driving force: $P^* \equiv \frac{P}{\rho V^2}$, $g_z^* \equiv \frac{g_z}{g}$

- Choose "typical" values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

Forced Convection Heat Transfer

FORCED CONVECTION

Energy

Microscopic energy balance:

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right) = k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right) + S$$

non-dimensional variables:

position: $r^* \equiv \frac{r}{D}$, $z^* \equiv \frac{z}{D}$

temperature: $T^* \equiv \frac{T - T_o}{(T_1 - T_o)}$

source: $S^* \equiv \frac{S}{S_0}$

Choose:
T – use a characteristic **interval** (since distance from absolute zero is not part of this physics)
S – use a reference source, S_0

$S_0 \equiv \frac{(T_1 - T_0)V\rho \hat{C}_p}{D} [=] \frac{W}{m^3}$

Substitute all these definitions,

$$(t^*, r^*, z^*, p^*, g^*, v_r^*, v_\theta^*, v_z^*, T^*, S^*)$$

into the ***governing equations*** and simplify…

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Complex Heat Transfer – Dimensional Analysis Forced Convection

CM3110 REVIEW

FORCED CONVECTION
Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + v_r^*\frac{\partial T^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial T^*}{\partial \theta} + v_z^*\frac{\partial T^*}{\partial z^*}\right) = \frac{1}{\mathrm{Pe}}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial T^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 T^*}{\partial \theta^2} + \frac{\partial^2 T^*}{\partial z^2}\right) + S^*$$

$$\mathrm{Pe} = \mathrm{Pr}\,\mathrm{Re} = \frac{\hat{C}_p\mu}{k}\frac{\rho VD}{\mu}$$

FORCED CONVECTION
Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt^*} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}\left(\nabla^2 v_z\right)^* + \frac{1}{Fr}g^*$$

$$\mathrm{Pr} \equiv \frac{\hat{C}_p\mu}{k}$$

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0$$

$$T^* = T^*(\mathrm{Re}, \mathrm{Pr})$$
$$\underline{v}^* = \underline{v}^*(\mathrm{Re}, \mathrm{Fr})$$

$$\frac{Dv_z}{Dt} \equiv \frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

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Next?

Microscopic Energy Balance Review

CM3110 REVIEW

Dimensional Analysis?

For complex systems, we turn to data correlations for heat transfer coefficients based on dimensional analysis

The engineering quantity of interest is the amount of heat transferred $\dot{Q}$

Nusselt number, a dimensionless heat transfer coefficient, is also a dimensionless amount of heat transferred $\dot{Q}$

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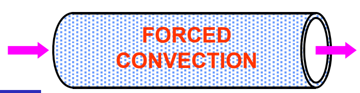
Forced Convection Heat Transfer



CM3110 REVIEW

Linear driving force model

$$\left|\frac{q_x}{A}\right| = h|T_1 - T_0|$$

Apply in the fluid, at the surface:

$$(2\pi RL)(h)(T_1 - T_0) = \dot{Q} = \iint_S \left[-\hat{e}_r \cdot \underset{\sim}{\tilde{q}}\right]_{surface} dS$$

$$(2\pi RL)(h)(T_1 - T_0) = \dot{Q} = \int_0^{2\pi}\int_0^{L} +k\left.\frac{\partial T}{\partial r}\right|_{r=R} R\,dz\,d\theta$$

Now, non-dimensionalize this expression as well.

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Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Non-dimensionalize

non-dimensional variables:

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

temperature:

$$T^* \equiv \frac{T - T_o}{(T_1 - T_o)}$$

T_0 = surface
T_1 = bulk

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

$$h(\pi D L)(T_1 - T_o) = \int_0^{2\pi}\int_0^{L/D} -k \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{(T_1 - T_o)}{D}\frac{D^2}{2} dz^* d\theta$$

$$2\pi\left(\frac{hD}{k}\right)\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^* d\theta$$

Nusselt number, Nu (dimensionless heat-transfer coefficient; dimensionless amount of heat transferred)

$$Nu = Nu\left(T^*, \frac{L}{D}\right)$$

Complex Heat Transfer – Dimensional Analysis

FORCED CONVECTION
Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + v_r^*\frac{\partial T^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial T^*}{\partial\theta} + v_z^*\frac{\partial T^*}{\partial z^*}\right) = \frac{1}{\mathrm{Pe}}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r\frac{\partial T^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 T^*}{\partial\theta^2} + \frac{\partial^2 T^*}{\partial z^2}\right) + S^*$$

$$\mathrm{Pe} = \mathrm{Pr}\,\mathrm{Re} = \frac{\hat{C}_p\mu}{k}\frac{\rho VD}{\mu}$$

FORCED CONVECTION
Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}\left(\nabla^2 v_z\right)^* + \frac{1}{Fr}g^*$$

$$\mathrm{Pr} \equiv \frac{\hat{C}_p\mu}{k}$$

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0$$

Quantity of interest

$$Nu = \frac{1}{2\pi L/D}\int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^*\,d\theta$$

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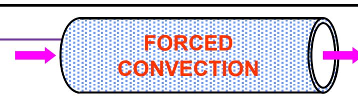
Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW



According to our **forced convection dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of four dimensionless groups:

Peclet number

$$\mathrm{Pe} \equiv \frac{\rho\hat{C}_p VD}{k} = \frac{\hat{C}_p\mu}{k}\frac{\rho VD}{\mu}$$

Prandtl number

$$\mathrm{Pr} \equiv \frac{\hat{C}_p\mu}{k}$$ (fluid properties)

$$\mathrm{Nu} = \mathrm{Nu}\left(\mathrm{Re}, \mathrm{Pr}, \mathrm{Fr}, \frac{L}{D}\right)$$

(tentative)

Now, do the experiments.

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Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

According to our **forced convection** **dimensional analysis** calculations and follow-up experiments, the dimensionless heat transfer coefficient should be found to be a function of *these* four dimensionless groups:

no free surfaces

$$\mathrm{Nu} = \mathrm{Nu}\left(\mathrm{Re}, \mathrm{Pr}, \mathrm{Fr}, \frac{L}{D}, \frac{\mu_b}{\mu_w}\right)$$

FORCED CONVECTION

Peclet number

$$\mathrm{Pe} \equiv \frac{\rho \hat{C}_p V D}{k} = \frac{\hat{C}_p \mu}{k} \frac{\rho V D}{\mu}$$

Prandtl number

$$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$ (fluid properties)

"it turns out…"

Sometimes $\mu(T)$ seen to be important

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Data Correlations for **Forced Convection** Heat Transfer

CM3110 REVIEW

Forced convection

Heat Transfer in Laminar flow in pipes

$$\mathrm{Nu}_a = \frac{h_a D}{k} = 1.86\left(\mathrm{RePr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder-Tate equation (laminar flow)

$$q = h_a A \Delta T_a$$

$$\Delta T_a = \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2}$$

Forced convection

Heat Transfer in Turbulent flow in pipes

$$\mathrm{Nu}_{lm} = \frac{h_{lm} D}{k} = 0.027\mathrm{Re}^{0.8}\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder-Tate equation (turbulent flow)

$$q = h_{lm} A \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{\Delta T_{w-bi} - \Delta T_{w-bo}}{\ln\left(\frac{\Delta T_{w-bo}}{\Delta T_{w-bo}}\right)}$$

Physical Properties (except μ_w) evaluated at:

$$\frac{T_{b,in} + T_{b,out}}{2}$$

*May have to be estimated

$$\frac{T_{b,in} + T_{b,out}}{2}$$

Bulk mean temperature

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Complex Heat Transfer – Dimensional Analysis—Forced Convection

Exam 1 Handout, Forced Convection Data Correlations

Heat Transfer Data Correlations for Examinations

CM3110 Transport Phenomena I
Michigan Technological University
Professor Faith A. Morrison
1 December 2020

I. Forced Convection Through Pipes

In forced convection, we determined from dimensional analysis that the Nusselt number is a function of at most Re, Pr, L/D, and viscosity ratio.

Prandtl number (fluid properties) $$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k} \quad (1)$$

In pipe flow with heat transfer taking place, the fluid enters at bulk fluid temperature T_{bi} and exits at T_{bo}. T_w is the temperature of the wall. For Nu data correlations in forced convection through pipes, all fluid material properties except $\mu_w = \mu(T_w)$ are evaluated at the mean bulk temperature. The mean bulk temperature is given by

Mean bulk temperature $$\bar{T}_b \equiv \frac{T_{bi} + T_{bo}}{2} \quad (2)$$

A. Laminar Flow in Pipes

Sieder and Tate's correlation (Geankoplis, p260) for laminar flow is

Laminar flow $$\mathrm{Nu}_a = \frac{h_a D}{k} = 1.86\left(\mathrm{RePr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} \quad (3)$$

$$q = h_a A \Delta T_a \quad (4)$$

Arithmetic mean driving force $$\Delta T_a = \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2} \quad (5)$$

B. Turbulent Flow in Pipes

Sieder and Tate's correlation (Geankoplis, p261) for turbulent flow is

Turbulent flow $$\mathrm{Nu}_{lm} = \frac{h_{lm} D}{k} = 0.027\mathrm{Re}^{0.8}\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} \quad (6)$$

$$q = h_{lm} A \Delta T_{lm} \quad (7)$$

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Log mean driving force $$\Delta T_{lm} = \frac{\Delta T_{w-bi} - \Delta T_{w-bo}}{ln\left(\frac{\Delta T_{w-bi}}{\Delta T_{w-bo}}\right)} \quad (8)$$

II. Forced Convection Around the Outside of a Cylinder

In heat transfer taking place between a fluid at bulk temperature T_b flowing perpendicular to a cylinder with wall temperature T_w, the material properties are evaluated at the film temperature.

Film temperature $$T_f \equiv \frac{T_w + T_b}{2} \quad (9)$$

The data correlation for Nusselt number in this case is

Outside Cylinder $$\mathrm{Nu} = \frac{hD}{k} = C\mathrm{Re}^m\mathrm{Pr}^{\frac{1}{3}} \quad (10)$$

Wall-bulk driving force $$q = hA(T_w - T_b) \quad (11)$$

The values of C and m depend on the Reynolds number (Geankoplis, Table 4.6-1, p272). These values are valid for $\mathrm{Pr} > 0.6$.

Re	m	C
$1-4$	0.330	0.989
$4-40$	0.385	0.911
$40-4{,}000$	0.466	0.683
$4{,}000-4\times10^4$	0.618	0.193
$4\times10^4-2.5\times10^5$	0.805	0.0266

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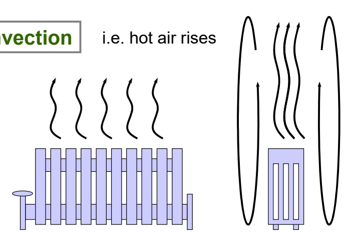
When the physics of the heat transfer changes, the correlations and dimensionless numbers change.

CM3110 REVIEW

new physics: **Natural Convection**

Complex Heat Transfer – Dimensional Analysis—Free Convection

Free Convection i.e. hot air rises



The methods don't change, however

- heat moves from hot surface to cold air (fluid) by radiation and conduction
- increase in fluid temperature decreases fluid density
- recirculation flow begins
- **recirculation adds to the heat-transfer** from conduction and radiation

$\Rightarrow$ coupled heat and momentum transport

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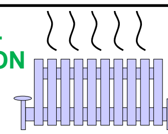
Complex Heat Transfer – Dimensional Analysis

NATURAL CONVECTION



NATURAL(FREE) CONVECTION

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + v_x^* \frac{\partial T^*}{\partial x^*} + v_y^* \frac{\partial T^*}{\partial y^*} + v_z^* \frac{\partial T^*}{\partial z^*}\right) = \frac{1}{\mathrm{Pr}}\left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right)$$

CM3110 REVIEW

NATURAL(FREE) CONVECTION

Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt} = (\nabla^2 v_z)^* + \left[\frac{gL^3\bar{\rho}^2\bar{\beta}(T_2 - \bar{T})}{\mu^2}\right] T^*$$

$$\mathrm{Gr} \equiv \frac{gL^3\bar{\rho}^2\bar{\beta}(T_2 - \bar{T})}{\mu^2}$$

≡Grashof number

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0$$

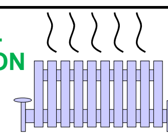
Quantity of interest

$$\mathrm{Nu} = \int_0^1 \int_0^1 + \left.\frac{dT^*}{dy^*}\right|_{y^*=0} dx^* dz^*$$

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

NATURAL CONVECTION



According to our **natural convection dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of two dimensionless groups:

Grashof number

$$\mathrm{Gr} \equiv \frac{gL^3\bar{\rho}^2\bar{\beta}(T_2 - \bar{T})}{\mu^2}$$

Prandtl number

$$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$ (fluid properties)

$$\mathrm{Nu} = \mathrm{Nu}(\mathrm{Gr}, \mathrm{Pr})$$

NATURAL(FREE) CONVECTION

Now, do the experiments.

Complex Heat Transfer – Dimensional Analysis—Free Convection

Experimental Results:

$$\mathrm{Gr} \equiv \frac{gD^3\bar{\rho}^2\bar{\beta}\Delta T}{\mu^2}$$

Example: Natural convection from vertical planes and cylinders

$$\mathrm{Nu} = \frac{hL}{k} = a\mathrm{Gr}^m\,\mathrm{Pr}^m$$

- *a*,*m* are given in Table 4.7-1, page 278 Geankoplis for several cases
- *L* is the height of the plate
- all physical properties evaluated at the **film temperature**, T_f

$$T_f = \frac{T_w + T_b}{2}$$

Free convection correlations use the film temperature for calculating the physical properties

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Complex Heat Transfer – Dimensional Analysis—Free Convection

Exam 1 Handout, Natural Convection Data Correlations

III. Natural Convection from Various Geometries

Natural convection heat transfer coefficients from various surfaces have been found by dimensional analysis and experimentally to correlate as follows:

Natural convection (various geometries) $$\mathrm{Nu} = \frac{hL}{k} = a(\mathrm{Gr}\,\mathrm{Pr})^m \quad (12)$$

Grashof number $$\mathrm{Gr} \equiv \frac{L^3\rho^2 g\beta\Delta T}{\mu^2} \quad (13)$$

The values for a and m depend on the geometry; values may be found in Geankoplis in Table 4.7-1 (p278, shown below). Table 4.7-2 (p280, next pages) provides simplified versions of the correlations specialized to common fluids (air, water, organic liquids).

TABLE 4.7-1. *Constants for Use with Eq. (4.7-4) for Natural Convection*

Physical Geometry	$N_{Gr}N_{Pr}$	*a*	*m*	*Ref.*
Vertical planes and cylinders [vertical height $L < 1$ m (3 ft)]				
	$<10^4$	1.36	$\frac{1}{5}$	(P3)
	10^4–10^9	0.59	$\frac{1}{4}$	(M1)
	$>10^9$	0.13	$\frac{1}{3}$	(M1)
Horizontal cylinders [diameter D used for L and $D < 0.20$ m (0.66 ft)]				
	$<10^{-5}$	0.49	0	(P3)
	10^{-5}–10^{-3}	0.71	$\frac{1}{25}$	(P3)
	10^{-3}–1	1.09	$\frac{1}{10}$	(P3)
	1–10^4	1.09	$\frac{1}{5}$	(P3)
	10^4–10^9	0.53	$\frac{1}{4}$	(M1)
	$>10^9$	0.13	$\frac{1}{3}$	(P3)
Horizontal plates				
Upper surface of heated plates or lower surface of cooled plates	10^5–2×10^7	0.54	$\frac{1}{4}$	(M1)
	2×10^7–3×10^{10}	0.14	$\frac{1}{3}$	(M1)
Lower surface of heated plates or upper surface of cooled plates	10^5–10^{11}	0.58	$\frac{1}{5}$	(F1)

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TABLE 4.7-2. *Simplified Equations for Natural Convection from Various Surfaces*

		Equation		
Physical Geometry	$N_{Gr}N_{Pr}$	h = btu/h·ft²·°F, L = ft, ΔT = °F, D = ft	h = W/m²·K, L = m, ΔT = K, D = m	*Ref.*
Air at 101.32 kPa (1 atm) abs pressure				
Vertical planes and cylinders	10^4–10^9	$h = 0.28(\Delta T/L)^{1/4}$	$h = 1.37(\Delta T/L)^{1/4}$	(P1)
	$>10^9$	$h = 0.18(\Delta T)^{1/3}$	$h = 1.24\,\Delta T^{1/3}$	(P1)
Horizontal cylinders	10^3–10^9	$h = 0.27(\Delta T/D)^{1/4}$	$h = 1.32(\Delta T/D)^{1/4}$	(M1)
	$>10^9$	$h = 0.18(\Delta T)^{1/3}$	$h = 1.24\,\Delta T^{1/3}$	(M1)
Horizontal plates				
Heated plate facing upward or cooled plate facing downward	10^5–2×10^7	$h = 0.27(\Delta T/L)^{1/4}$	$h = 1.32(\Delta T/L)^{1/4}$	(M1)
	2×10^7–3×10^{10}	$h = 0.22(\Delta T)^{1/3}$	$h = 1.52\,\Delta T^{1/3}$	(M1)
Heated plate facing downward or cooled plate facing upward	3×10^5–3×10^{10}	$h = 0.12(\Delta T/L)^{1/4}$	$h = 0.59(\Delta T/L)^{1/4}$	(M1)
Water at 70°F (294 K)				
Vertical planes and cylinders	10^4–10^9	$h = 26(\Delta T/L)^{1/4}$	$h = 127(\Delta T/L)^{1/4}$	(P1)
Organic liquids at 70°F (294 K)				
Vertical planes and cylinders	10^4–10^9	$h = 12(\Delta T/L)^{1/4}$	$h = 59(\Delta T/L)^{1/4}$	(P1)

Reference: C. J. Geankoplis, Transport Processes and Separation Process Principles, 4th Edition, Prentice Hall, 2003.

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Dimensional analysis will be a key tool in the third transport field, diffusion/mass transfer (Module 3)

Dimensional Analysis

These numbers tell us about the relative importance of the terms they precede.

Dimensionless numbers from the **Equations of Change** *(microscopic balances)*

momentum — Non-dimensional Navier-Stokes Equation

$$\left(\frac{\partial v_z^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* v_z^*\right) = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\text{Re}}(\nabla^{*2} v_z^*) + \frac{1}{\text{Fr}} g^*$$

Re — Reynolds
Fr — Froude

energy — Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\text{RePr}}(\nabla^{*2} T^*) + S^*$$

Pe — Péclet$_h$ = RePr
Pr — Prandtl

mass — Non-dimensional Continuity Equation (species A)

$$\left(\frac{\partial x_A^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* x_A^*\right) = \frac{1}{\text{ReSc}}(\nabla^{*2} x_A^*)$$

Pe — Péclet$_m$ = ReSc
Sc — Schmidt

ref: BSL1, p581, 644

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Microscopic Energy Balance Review

CM3110 REVIEW

Unsteady State Heat Transfer

Module 1: Intro and Prerequisite Material

Steady Heat Transfer Review

We begin with a Review:

- ✓ Microscopic energy balance
- ✓ Fourier's law of heat conduction (k, homogeneous)
- ✓ Newton's law of cooling (h, at a boundary)
- ✓ Resistances due to k and h
- ✓ Solving for the *steady* temperature field $T(x, y, z)$
- ✓ Dimensional analysis in heat transfer
- ✓ Data correlations for forced and free convection
- Radiation heat transfer

Radiation

Radiation Summary:

CM3110 REVIEW

Always use absolute temperature (Kelvin) in radiation calculations.

General properties:

- Absorptivity, α
  - gray body: $\alpha = \text{constant}$
  - black body: $\alpha = 1$
- Emissivity, ε
  $$q_{emit} = \varepsilon q_{emit,blackbody}$$
- Kirchoff's law: $\alpha = \varepsilon$
- Stefan-Boltzman law
  $$\frac{q_{emit,blackbody}}{A} = \sigma T^4$$

NET Radiation energy going from surface 1 to surface 2:

$$\frac{q_{1-2} - q_{2-1}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Heat shields:

$$\frac{q}{A} = \left(\frac{1}{N+1}\right)\frac{\sigma(T_1^4 - T_3^4)}{\left(\frac{2}{\varepsilon} - 1\right)}$$

Net heat transfer to a body:

$$\frac{q}{A} = \varepsilon\Big|_{T_s} \sigma\left(T_s^4 - T_{body}^4\right)$$

Heat transfer coefficient:

$$h_{rad} = \frac{\varepsilon|_{T_s}\sigma\left(T_s^4 - T_{body}^4\right)}{\left(T_s - T_{body}\right)}$$

Geankoplis 4th ed., eqn 4.10-10 p304

Stefan-Boltzman constant:

$$\sigma = 5.676 \times 10^{-8} \frac{W}{m^2K^4}$$

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Radiation Summary:

CM3110 REVIEW

Always use absolute temperature (Kelvin) in radiation calculations.

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- Absorptivity, α
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- Emissivity, ε
  $$q_{emit} = \varepsilon q_{emit,blackbody}$$
- Kirchoff's law: $\alpha = \varepsilon$
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Microscopic Energy Balance Review

CM3110 REVIEW

Module 1: Intro and Prerequisite material

Unsteady State Heat Transfer

Module 1: Intro and Prerequisite Material

Steady Heat Transfer Review

DONE!

We begin with a Review:

- ✓ Microscopic energy balance
- ✓ Fourier's law of heat conduction (k, homogeneous)
- ✓ Newton's law of cooling (h, at a boundary)
- ✓ Resistances due to k and h
- ✓ Solving for the *steady* temperature field $T(x, y, z)$
- ✓ Dimensional analysis in heat transfer
- ✓ Data correlations for forced and free convection
- ✓ Radiation heat transfer

CM3120: Module 1

✓ Introduction and Prereq Material

I. Introduction

II. Review of Prerequisite Material

  a. Microscopic energy balances
  b. Fourier's law of heat conduction (k, homogeneous)
  c. Newton's law of cooling (h, at a boundary)
  d. Resistances due to k and h
  e. Solving for the steady temperature field $T(x,y,z)$
  f. Dimensional analysis in heat transfer for h
  g. h Data correlations for forced and free convection
  h. h For radiation heat transfer

DONE!

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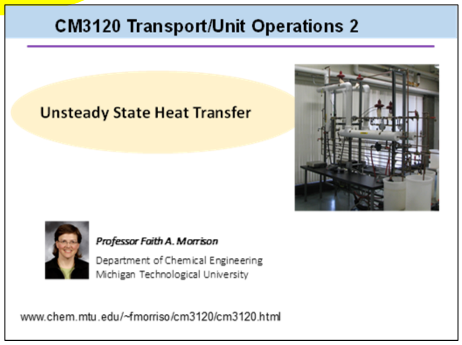
NEXT:

Module 2: Unsteady State Heat Transfer



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