CM3120: Module 2

Unsteady State Heat Transfer

I. Introduction
II. Unsteady Microscopic Energy Balance—(slash and burn)
III. Unsteady Macroscopic Energy Balance
IV. Dimensional Analysis (unsteady)—Biot number, Fourier number
V. Low Biot number solutions—Lumped parameter analysis
VI. Short Cut Solutions—(initial temperature T_0; finite h), Gurney and Lurie charts (as a function of position, $m = \frac{1}{\mathrm{Bi}}$, and Fo); Heissler charts (center point only, as a function of $m = 1/\mathrm{Bi}$, and Fo)
VII. Full Analytical Solutions (stretch)

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CM3120: Module 2

Lecture I: Introduction to Unsteady State Heat Transfer

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www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

 2

Heat Transfer: Steady vs. Unsteady

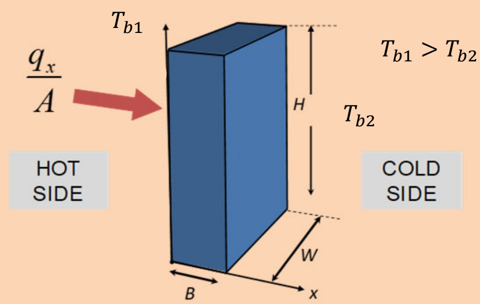
To get started, let's contrast the **steady** and **unsteady** cases in a familiar problem:

HEAT TRANSFER

Steady vs. Unsteady

1D, heat conduction, in the x-direction, in a slab

Newton's law of Cooling boundary conditions



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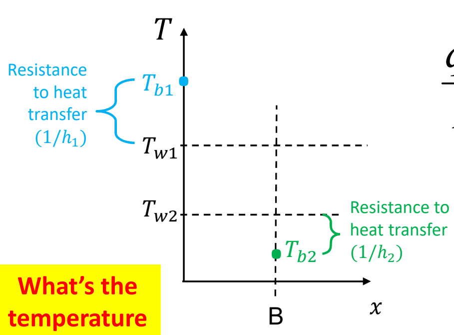
Heat Transfer: Steady vs. Unsteady

Heat Transfer at Steady State

(Newton's law of cooling BCs)

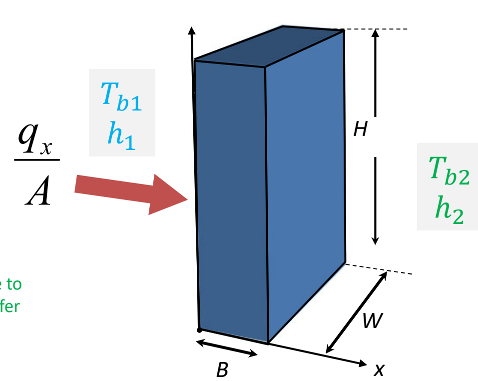
1D, rectangular geometry:

- Independent of time
- Flux $\frac{q_x}{A}$ = **constant**
- **linear** temperature profile
- Steady resistance to heat transfer at both boundaries



What's the temperature profile?

$T_{b1} > T_{b2}$



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Heat Transfer: Steady vs. Unsteady

Heat Transfer at **Steady** State

(Newton's law of cooling BCs)

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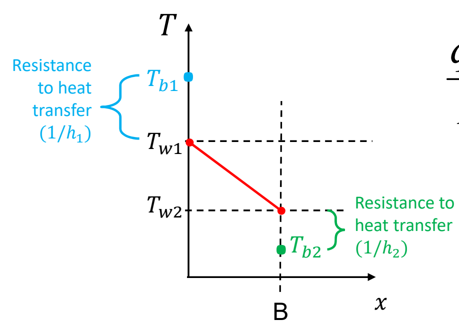
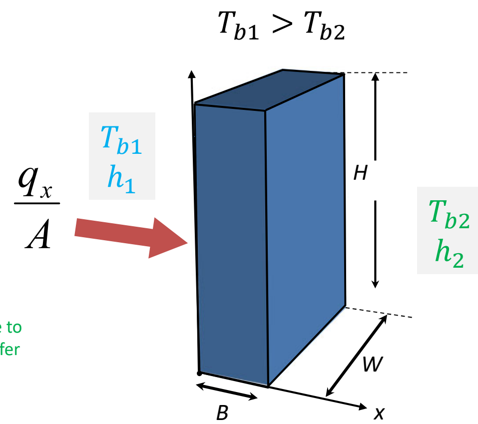


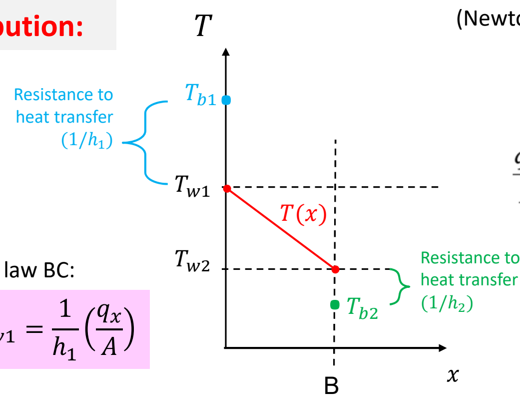


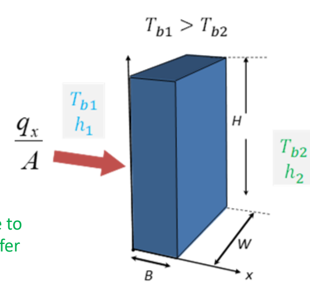
Heat Transfer: Steady vs. Unsteady

Heat Transfer at **Steady** State

(Newton's law of cooling BCs)

Temperature distribution:





Newton's law BC:

$$T_{b1} - T_{w1} = \frac{1}{h_1}\left(\frac{q_x}{A}\right)$$

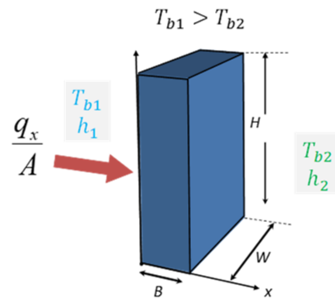
$$\frac{q_x}{A} = h_1(T_{b1} - T_{w1}) = -k\frac{dT}{dx} = h_2\,(T_{w2} - T_{b2})$$

Flux $\frac{q_x}{A} =$ **constant**

Independent of time

Heat Transfer: Steady vs. Unsteady

Unsteady Heat Transfer



There are many circumstances that cause unsteady heat transfer.

To imagine a case where heat transfer is unsteady:

- We must specify the state of the system at some point in time (***initial conditions***)
- We must specify what then happens to cause heat to start to transfer (**the scenario**).

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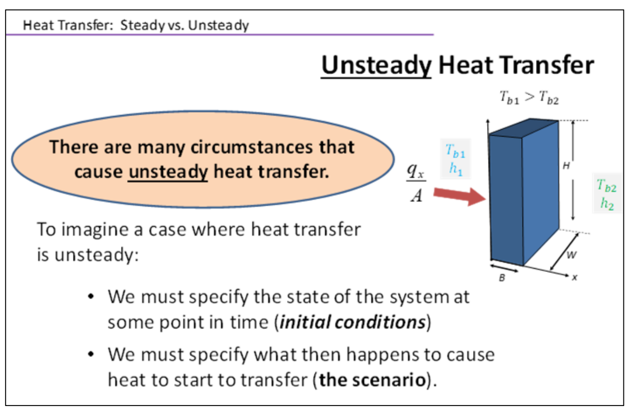

Can you think of any real engineering situations (unsteady heat xfer)?
Can you write them in terms of:

- **initial conditions and**
- **a modeling scenario?**

You try.

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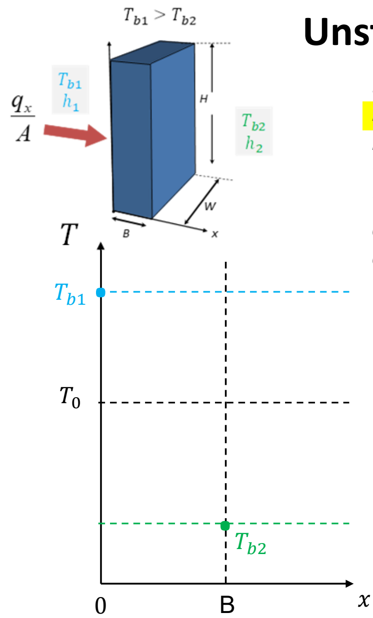
Heat Transfer: Steady vs. Unsteady

Unsteady Heat Transfer



Example: A wide, tall slab initially uniformly at T_0 is suddenly subjected to flowing fluid on its two broad faces. The left fluid is at T_{b1} and its heat transfer to the wall is characterized by heat transfer coefficient h_1, while the right side is at T_{b2} and characterized by h_2. What is the temperature distribution across the slab as a function of time?

What do we think will happen?

- Will there be heat transfer resistance at the boundaries?
- Will there be a linear temperature profile in the slab?
- Femtoseconds after the change, what does the profile look like?
- What will the solution trend towards as time goes on ($\rightarrow \infty$)?

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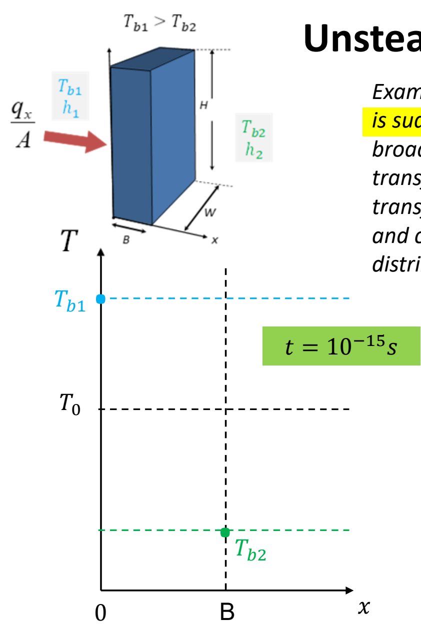
Heat Transfer: Steady vs. Unsteady

Unsteady Heat Transfer



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What do we think will happen?

You try.

$femtosecond = 10^{-15} s$

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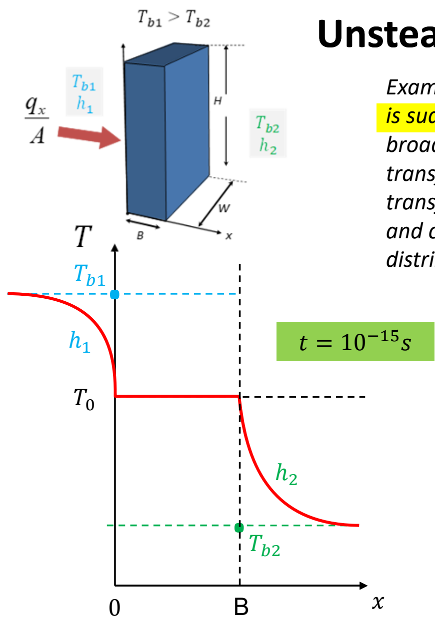
Heat Transfer: Steady vs. Unsteady

Unsteady Heat Transfer



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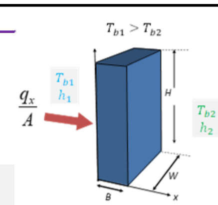
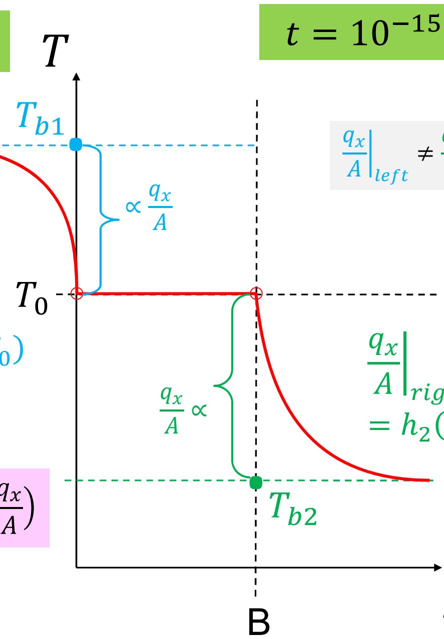
Heat Transfer: Steady vs. Unsteady

Unsteady

$t = 10^{-15} s$





$$\left.\frac{q_x}{A}\right|_{left} \neq \left.\frac{q_x}{A}\right|_{right}$$

$$\left.\frac{q_x}{A}\right|_{slab} = 0$$

$$\left.\frac{q_x}{A}\right|_{left} = h_1(T_{b1} - T_0)$$

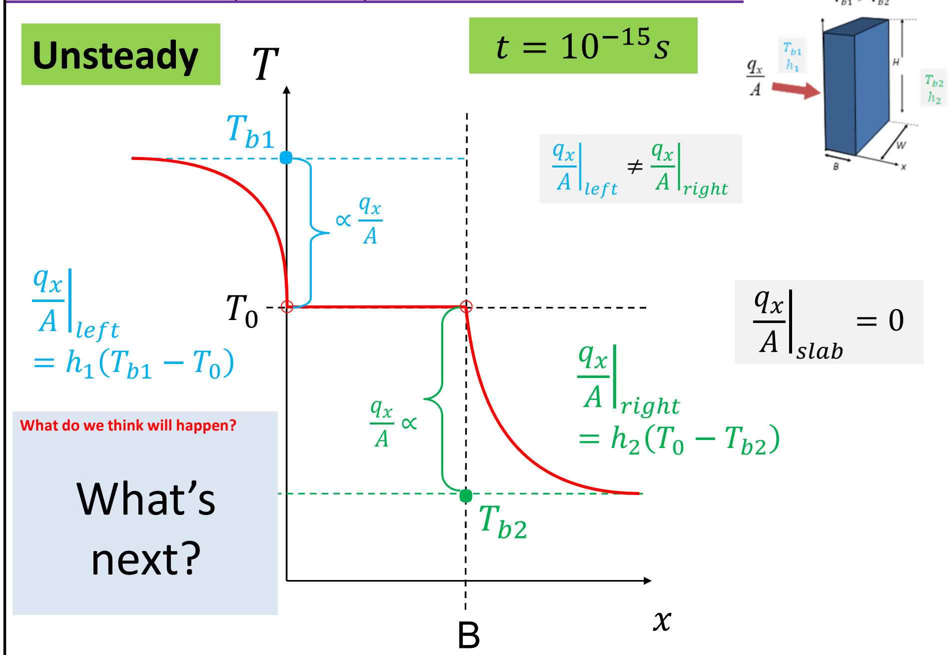
$$\left.\frac{q_x}{A}\right|_{right} = h_2(T_0 - T_{b2})$$

Newton's law BC:

$$T_{b1} - T_{w1} = \frac{1}{h_1}\left(\frac{q_x}{A}\right)$$

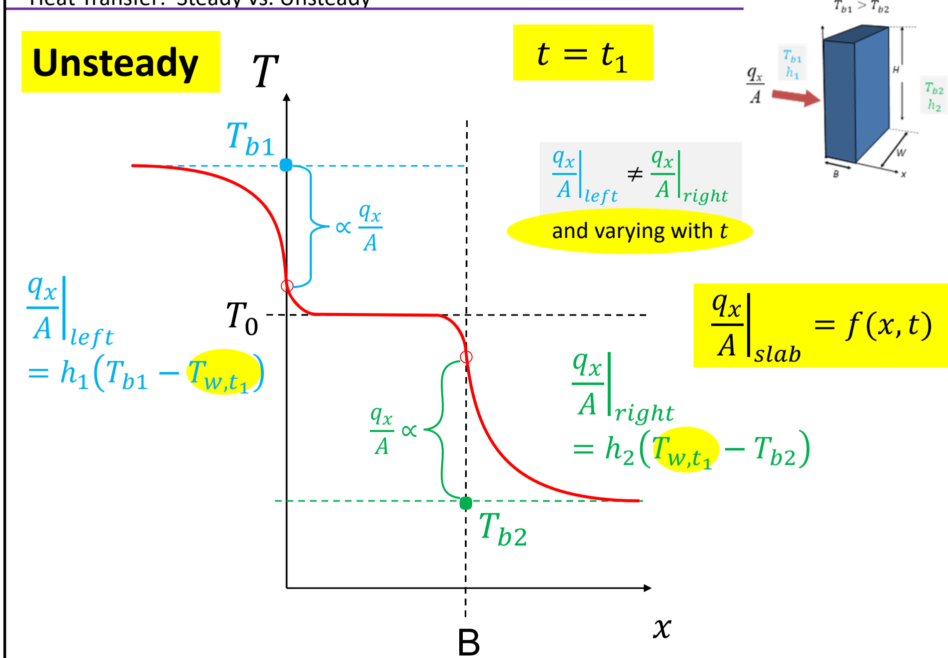
Heat flows in the positive x- direction →

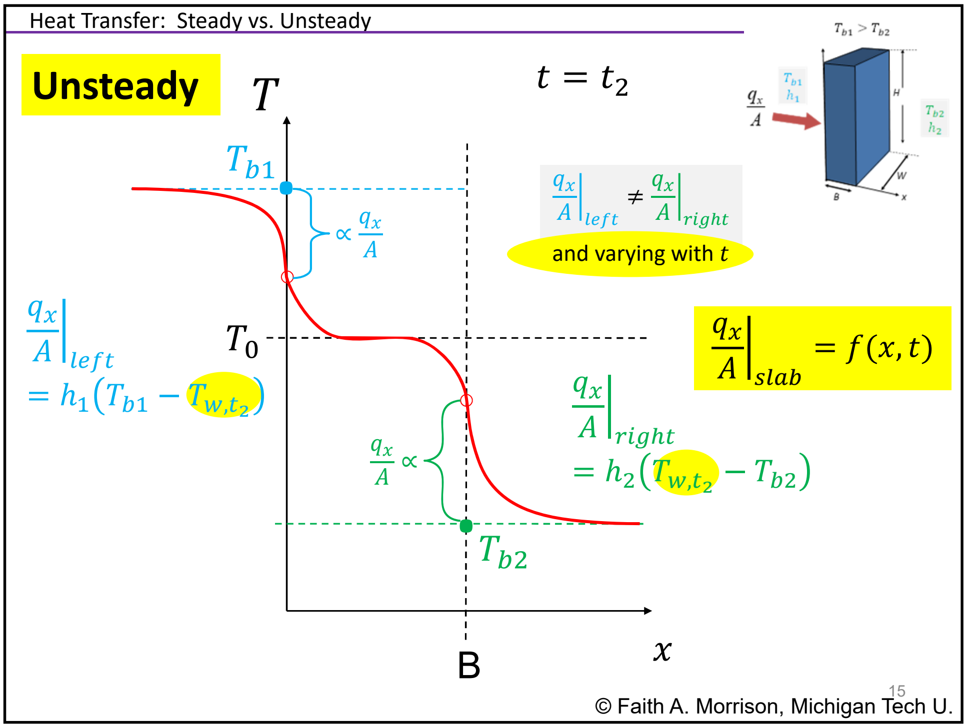
Heat Transfer: Steady vs. Unsteady



Heat flows in the positive x- direction →

Heat Transfer: Steady vs. Unsteady




Heat Transfer: Steady vs. Unsteady
Unsteady
$t = t_2$
$T_{b1} > T_{b2}$
T
T_{b1}
$\propto \frac{q_x}{A}$
$\left.\frac{q_x}{A}\right|_{left} \neq \left.\frac{q_x}{A}\right|_{right}$
and varying with t
$\left.\frac{q_x}{A}\right|_{left} = h_1(T_{b1} - T_{w,t_2})$
T_0
$\left.\frac{q_x}{A}\right|_{slab} = f(x,t)$
$\left.\frac{q_x}{A}\right|_{right} = h_2(T_{w,t_2} - T_{b2})$
$\frac{q_x}{A} \propto$
T_{b2}
x
B
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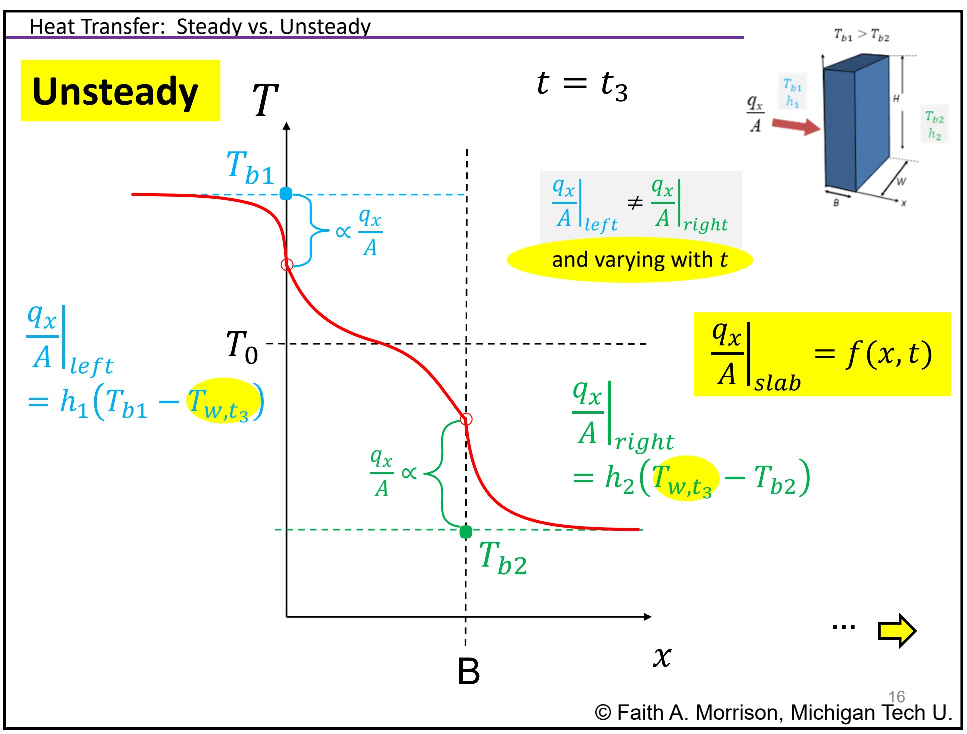
Heat Transfer: Steady vs. Unsteady
Unsteady
$t = t_3$
$T_{b1} > T_{b2}$
T
T_{b1}
$\propto \frac{q_x}{A}$
$\left.\frac{q_x}{A}\right|_{left} \neq \left.\frac{q_x}{A}\right|_{right}$
and varying with t
$\left.\frac{q_x}{A}\right|_{left} = h_1(T_{b1} - T_{w,t_3})$
T_0
$\left.\frac{q_x}{A}\right|_{slab} = f(x,t)$
$\left.\frac{q_x}{A}\right|_{right} = h_2(T_{w,t_3} - T_{b2})$
$\frac{q_x}{A} \propto$
T_{b2}
x
B
…
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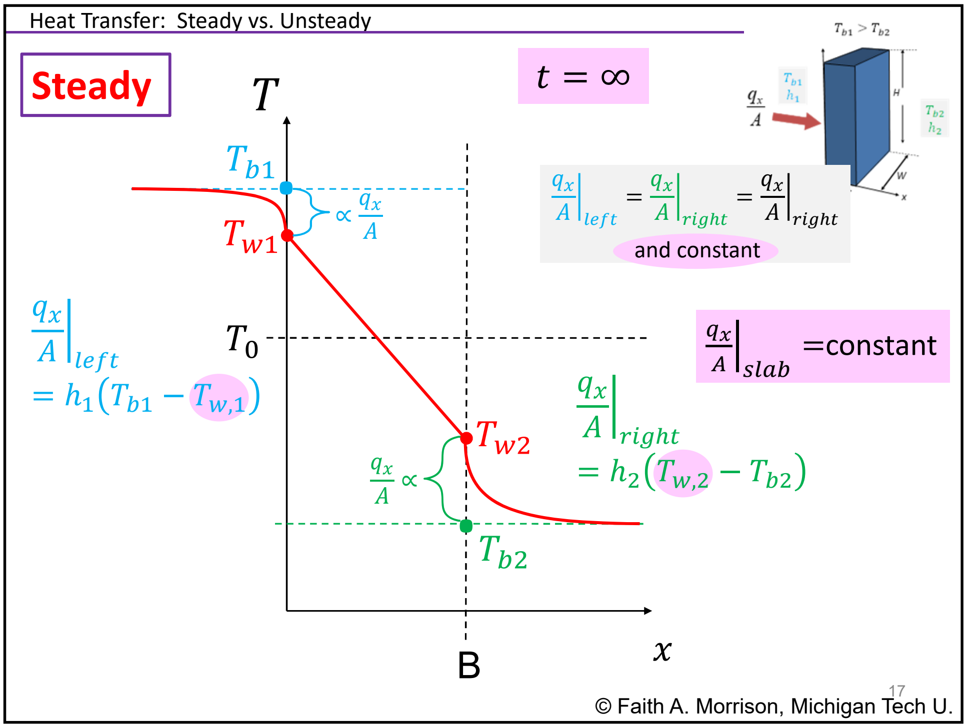
Heat Transfer: Steady vs. Unsteady
Steady
$t = \infty$
$T_{b1} > T_{b2}$
$\frac{q_x}{A}$
T_{b1} h_1
T_{b2} h_2
H
W
x
T
T_{b1}
T_{w1}
$\propto \frac{q_x}{A}$
$\frac{q_x}{A}\Big|_{left} = \frac{q_x}{A}\Big|_{right} = \frac{q_x}{A}\Big|_{right}$
and constant
$\frac{q_x}{A}\Big|_{left} = h_1(T_{b1} - T_{w,1})$
T_0
$\frac{q_x}{A}\Big|_{slab}$ =constant
$\frac{q_x}{A}\Big|_{right} = h_2(T_{w,2} - T_{b2})$
T_{w2}
$\frac{q_x}{A} \propto$
T_{b2}
B
x
17
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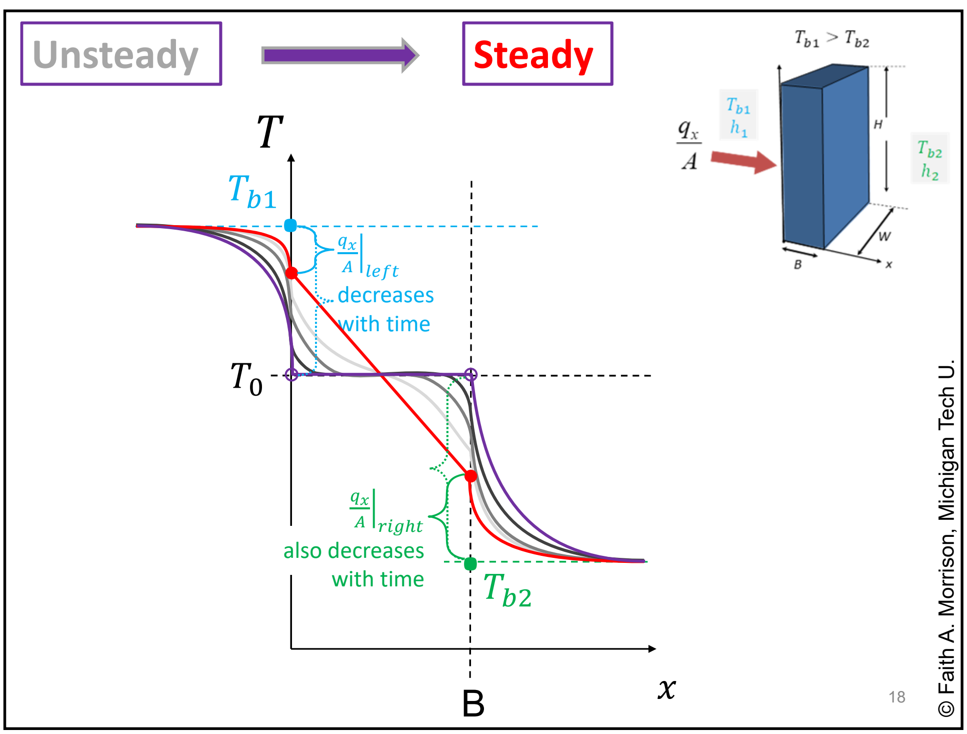
Unsteady
Steady
$T_{b1} > T_{b2}$
$\frac{q_x}{A}$
T_{b1} h_1
T_{b2} h_2
H
W
B
x
T
T_{b1}
$\frac{q_x}{A}\Big|_{left}$ decreases with time
T_0
$\frac{q_x}{A}\Big|_{right}$ also decreases with time
T_{b2}
B
x
18
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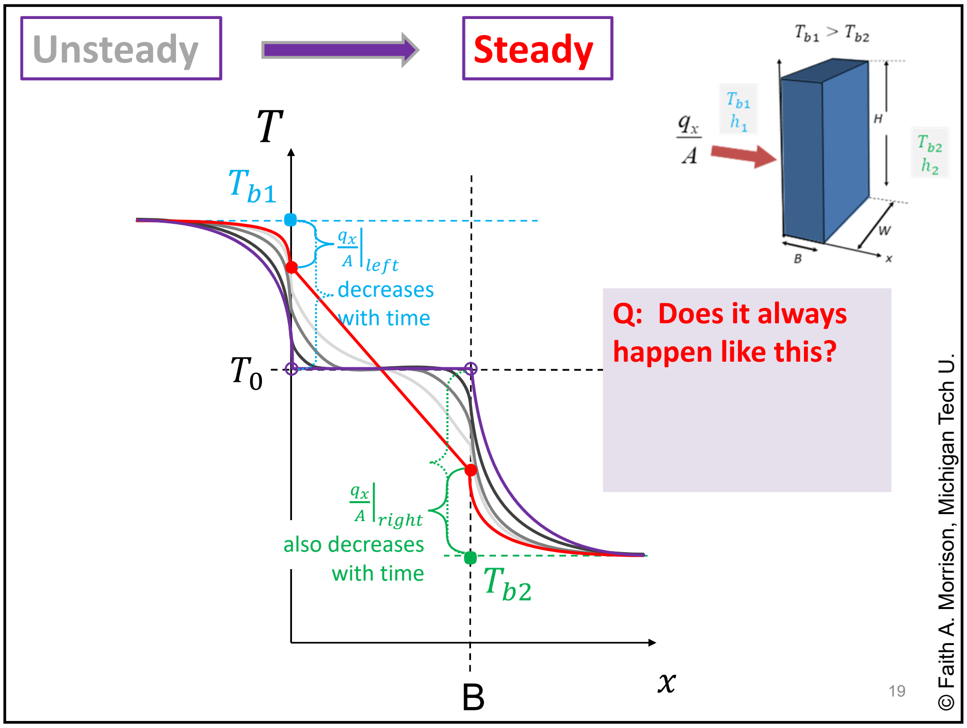
Unsteady
Steady
$T_{b1} > T_{b2}$
$\frac{q_x}{A}$
T_{b1} h_1
T_{b2} h_2
H
W
B
x
T
T_{b1}
$\left.\frac{q_x}{A}\right|_{left}$ decreases with time
T_0
$\left.\frac{q_x}{A}\right|_{right}$ also decreases with time
T_{b2}
B
x
Q: Does it always happen like this?
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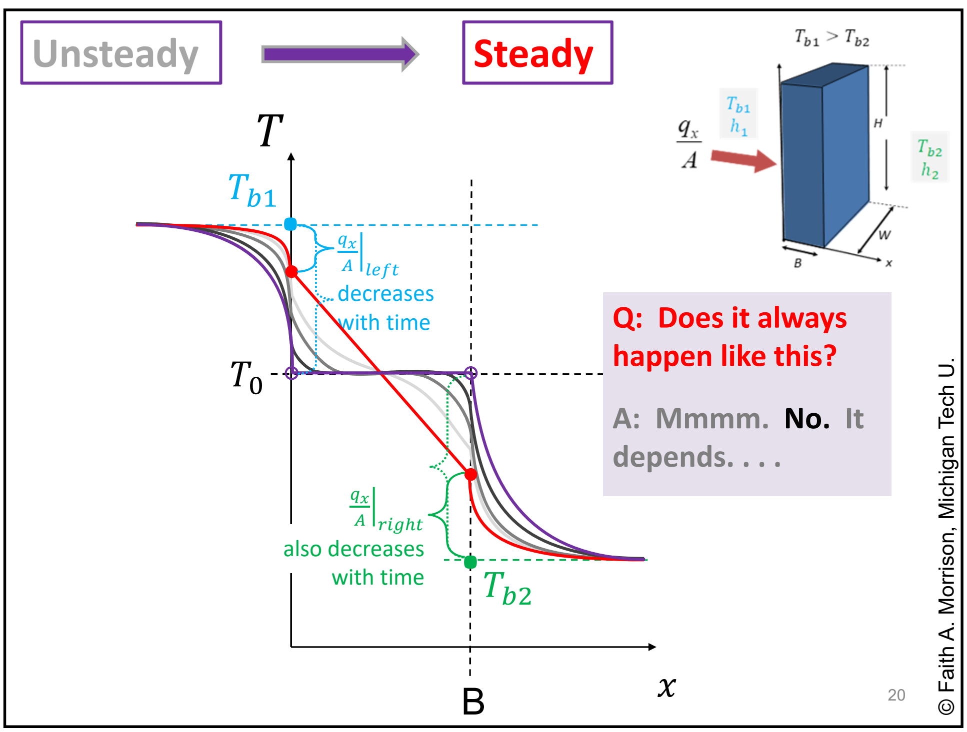
Unsteady
Steady
$T_{b1} > T_{b2}$
$\frac{q_x}{A}$
T_{b1} h_1
T_{b2} h_2
H
W
B
x
T
T_{b1}
$\left.\frac{q_x}{A}\right|_{left}$ decreases with time
T_0
$\left.\frac{q_x}{A}\right|_{right}$ also decreases with time
T_{b2}
B
x
Q: Does it always happen like this?
A: Mmmm. No. It depends. . . .
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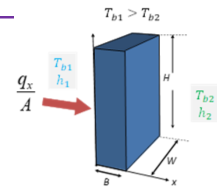
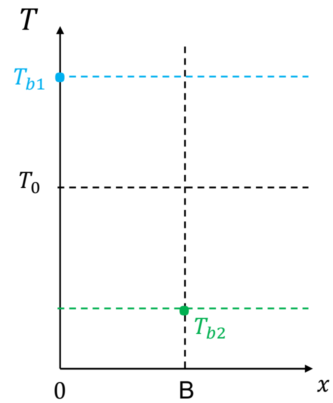
Heat Transfer: Steady vs. Unsteady



What are the various cases that are seen?

- h_i are large

Let's think.



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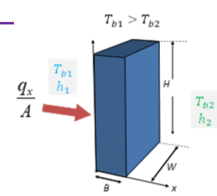
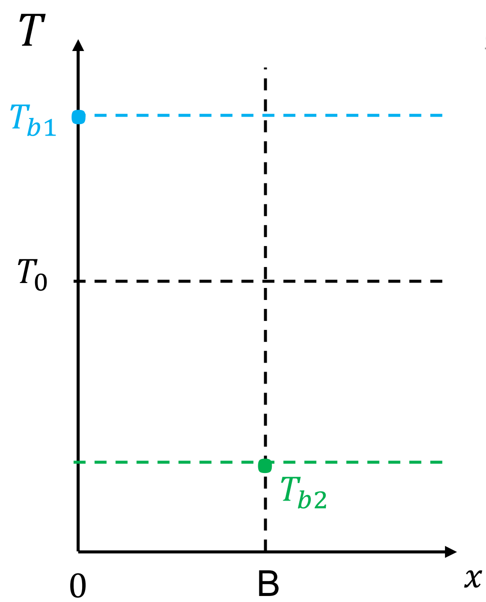
Heat Transfer: Steady vs. Unsteady



- h_i are large



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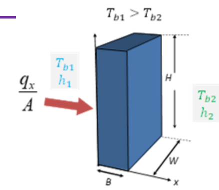
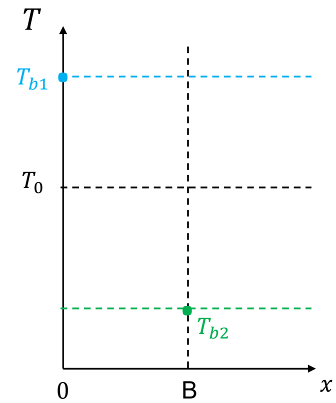
Heat Transfer: Steady vs. Unsteady



What are the various cases that are seen?

- k is large

Let's think.



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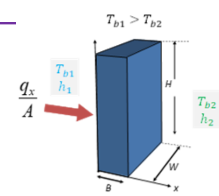
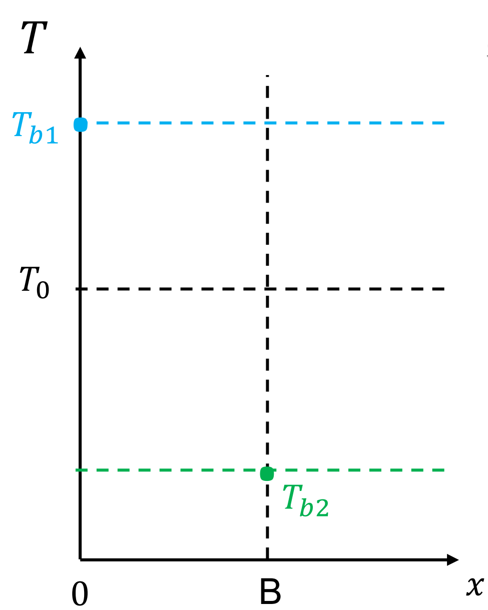
Heat Transfer: Steady vs. Unsteady



- k is large



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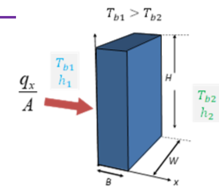
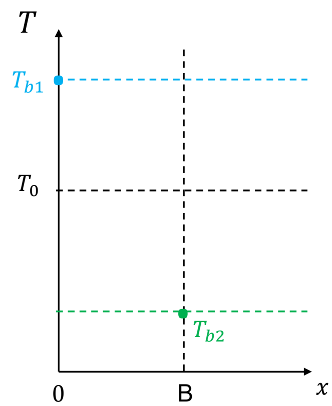
Heat Transfer: Steady vs. Unsteady

What are the various cases that are seen?



- Neither slab conduction nor fluid convection dominates

Let's think.



Heat Transfer: Steady vs. Unsteady

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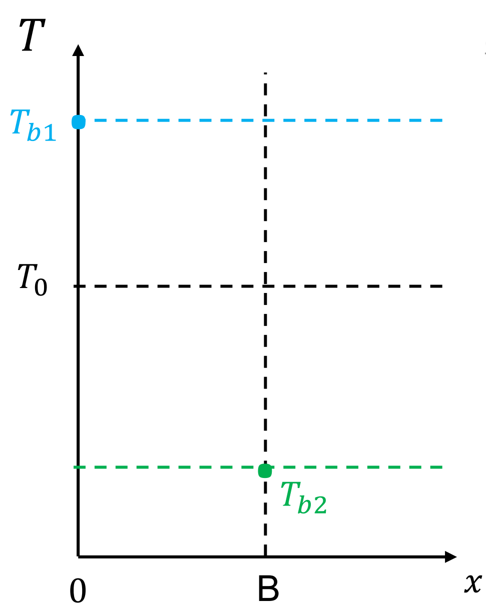
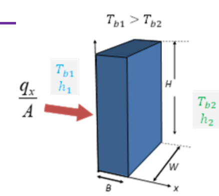
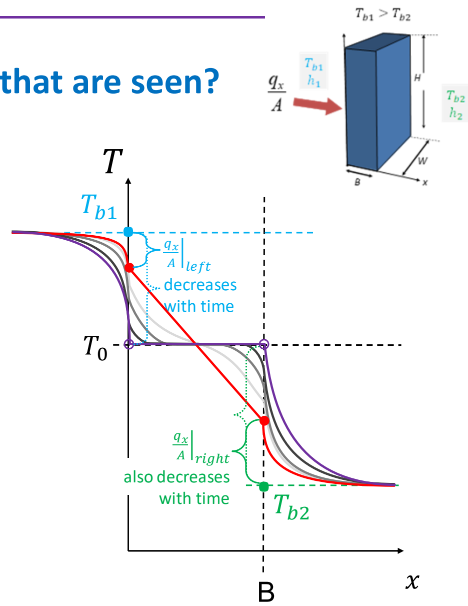




Heat Transfer: Steady vs. Unsteady

What are the various cases that are seen?

- If h_i is large, the wall temp is just the bulk temperature (fast convection)
- If k is large, the temp profile is always straight (quasi-steady state in the slab) and the convection works to keep up (heat transfer is limited by h_i; fast conduction in slab)
- If neither mechanism dominates, it's complicated!
- If the boundary conditions vary with $t, \underline{x}$, it's complicated!



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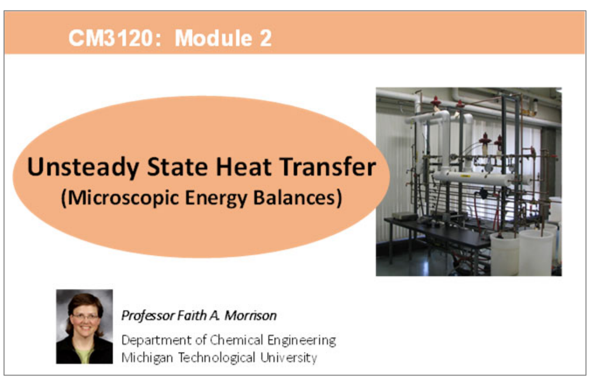

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