CM3120: Module 3

Diffusion and Mass Transfer I

I. Introduction to diffusion/mass transfer
II. Classic diffusion and mass transfer—Quick Start a): 1D Evaporation
III. Classic diffusion and mass transfer—Quick Start b): 1D Radial droplet
IV. Cycle back: Fick's mass transport law
V. Microscopic species A mass balance
VI. Classic diffusion and mass transfer—c): 1D Mass transfer with chemical reaction

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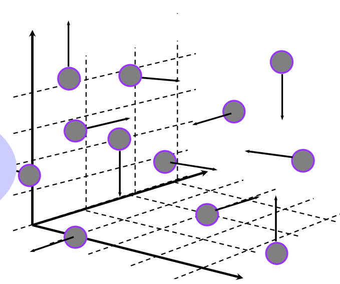
CM3120: Module 3

Module 3 Lecture III

Quick Start 2:

1D Radial Diffusion



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Department of Chemical Engineering
Michigan Technological University

www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

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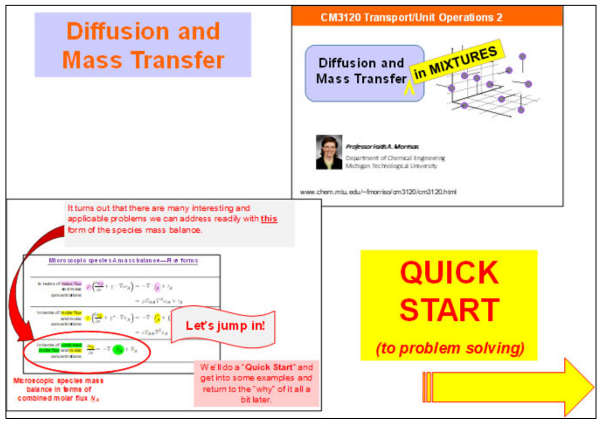
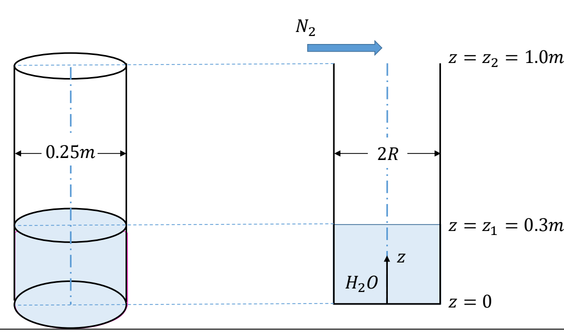


Continuing…

Last time…

QUICK START

Example 1: Water ($40^o C, 1.0\ atm$) slowly and steadily evaporates into nitrogen ($40^o C, 1.0\ atm$) from the bottom of a cylindrical tank as shown in the figure below. A stream of dry nitrogen flows slowly past the open tank. The mole fraction of water in the gas at the top opening of the tank is 0.02. **What is the rate of water evaporation?**



BSL2, p547

Last time…

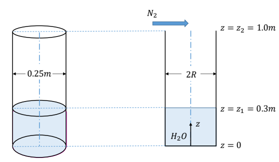
We sought to complete this problem.

The primary goal, however, was (is) **to grow our ability to troubleshoot engineering problems.**

Interrogating the problem:

QUICK START

Example: Water ($40^oC, 1.0\ atm$) slowly and steadily evaporates into nitrogen ($40^oC, 1.0\ atm$) from the bottom of a cylindrical tank as shown in the figure below. A stream of dry nitrogen flows slowly past the open tank. The mole fraction of water in the gas at the top opening of the tank is 0.02. **What is the rate of water evaporation?**



Why does the water evaporate?

What limits the rate of evaporation?

What could be done to accelerate the evaporation?

What could be done to slow down the evaporation?

What is the driving physics?

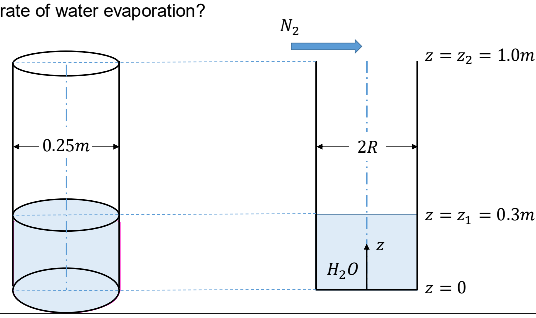
We take note of the questions that were productive in leading us to the solution.

Last time…

The questions led us to **refine the problem**; to frame it in a way that led to the solution.

QUICK START

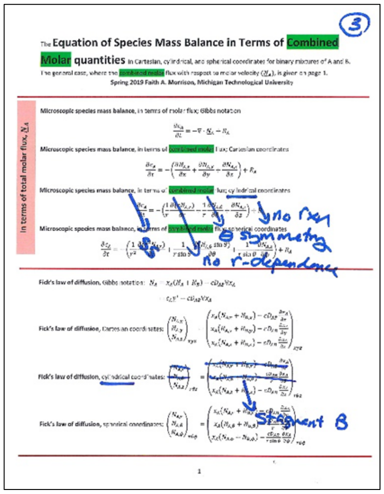
Example 1: Water ($40^oC, 1.0\ atm$) slowly and steadily evaporates into nitrogen ($40^oC, 1.0\ atm$) from the bottom of a cylindrical tank as shown in the figure below. A stream of dry nitrogen flows slowly past the open tank. The mole fraction of water in the gas at the top opening of the tank is 0.02. The geometry is as shown in the figure. What is water mole fraction as a function of vertical position in the tank? You may assume ideal gas properties. What is the rate of water evaporation?

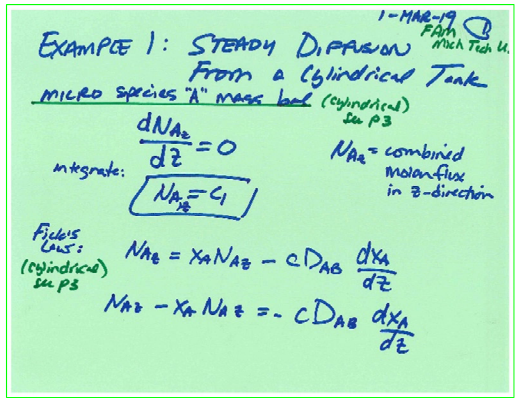


Example 1

Seek concentration distribution

⇒ microscopic species A mass balance.




EXAMPLE 1: STEADY DIFFUSION From a Cylindrical Tank

1-MAR-19 FAM Mich Tech U.

micro species "A" mass bal (cylindrical) see p3

$$\frac{dN_{Az}}{dz} = 0$$

integrate: $N_{Az} = C_1$

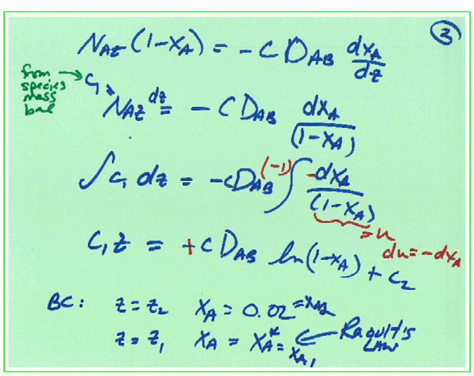
N_{Az} = combined molar flux in z-direction

Fick's Law: (cylindrical) see p3

$$N_{Az} = x_A N_{Az} - cD_{AB}\frac{dx_A}{dz}$$

$$N_{Az} - x_A N_{Az} = -cD_{AB}\frac{dx_A}{dz}$$

Example 1


$$N_{Az}(1-x_A) = -cD_{AB}\frac{dx_A}{dz}$$

from species mass bal → C_1

$$N_{Az}\,dz = -cD_{AB}\frac{dx_A}{(1-x_A)}$$

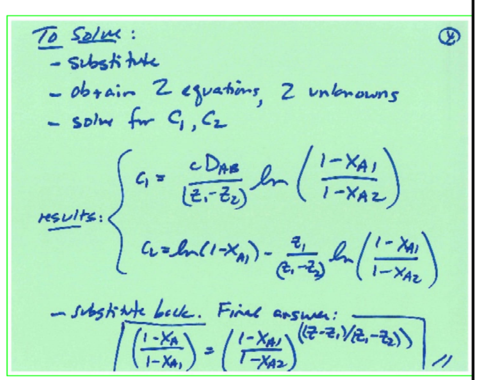
$$\int C_1\,dz = -cD_{AB}(-1)\int \frac{-dx_A}{(1-x_A)}$$

$(1-x_A) = u$, $du = -dx_A$

$$C_1 z = +cD_{AB}\ln(1-x_A) + C_2$$

BC: $z = z_2$ $x_A = 0.02 = x_{A2}$

$z = z_1$ $x_A = x_A^* = x_{A1}$ ← Raoult's Law


To Solve:

- substitute
- obtain 2 equations, 2 unknowns
- solve for C_1, C_2

results:

$$C_1 = \frac{cD_{AB}}{(z_1-z_2)}\ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)$$

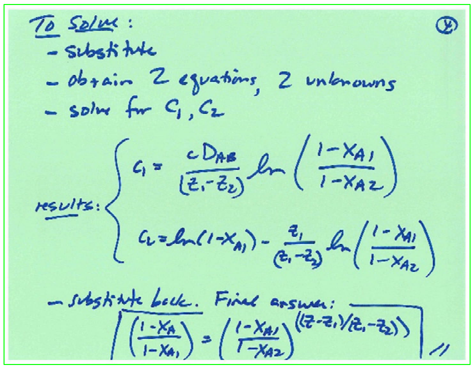
$$C_2 = \ln(1-x_{A1}) - \frac{z_1}{(z_1-z_2)}\ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)$$

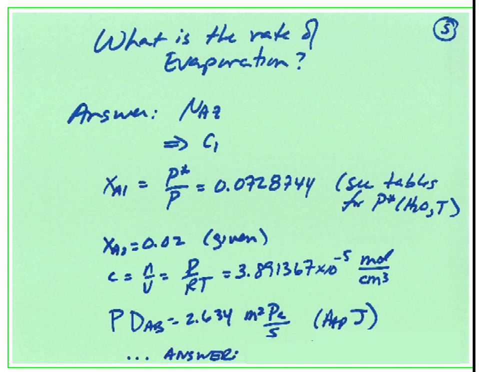
- substitute back. Final answer:

$$\left(\frac{1-x_A}{1-x_{A1}}\right) = \left(\frac{1-x_{A1}}{1-x_{A2}}\right)^{(z-z_1)/(z_1-z_2)}$$

First, we obtained the flux $N_{A,z}$ and the concentration distribution $x_A(z)$.

Example 1





Then, we answered the question: $N_{A,z} = C_1 = \frac{cD_{AB}}{(z_1 - z_2)} \ln\left(\frac{1 - x_{A1}}{1 - x_{A2}}\right)$

$A_{xs} = \frac{\pi D_{tank}^2}{4}$

The rate of evaporation
$A_{xs}N_{A,z} = 3.9 \times 10^{-6}$ mol/s.

BSL2, p547

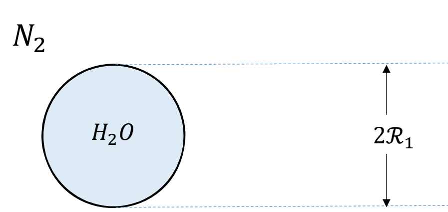
The primary goals are

QUICK START

- **to grow our ability to troubleshoot engineering problems**
- **To explore "classic" mass transfer circumstances and add them to our tool belt**

Let's do another problem

Example 2: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). What is the rate of evaporation and how does the water concentration vary in the gas?



Ver 1

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QUICK START

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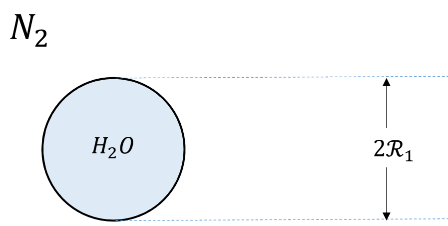

Let's Interrogate the problem.

Ver 1

BSL2, p550

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QUICK START

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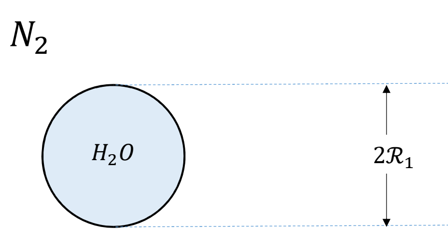

Why does the water evaporate?

What limits the rate of evaporation?

What could be done to accelerate the evaporation?

What could be done to slow down the evaporation?

What is the driving physics?

Can we use any ideas from previous experience?

Ver 1

BSL2, p550

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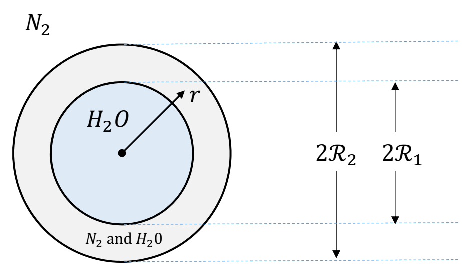
QUICK START

Example 2: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. What is the rate of evaporation and how does the water concentration vary in the gas as a function distance from the droplet? You may assume ideal gas properties for air.



We are developing a *model* to address the questions of interest

Ver 2

BSL2, p550

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QUICK START

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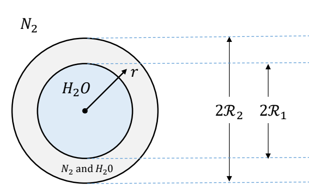

Ver 2

SOLVE

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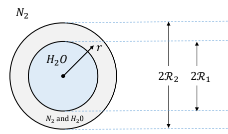
QUICK START

SOLUTION:

Example 2: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. What is the rate of evaporation and how does the water concentration vary as a function distance from the droplet? You may assume ideal gas properties for air.



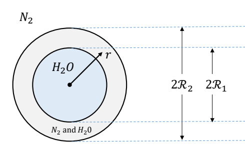
Ver 2

See hand slides

QUICK START

Ver 2

Example 2: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. What is the rate of evaporation and how does the water concentration vary as a function distance from the droplet? You may assume ideal gas properties for air.



Ver 2

Assumptions:
- Steady diffusion; stagnant air
- Uniform film surrounds droplet
- Ideal gas
- Constant temperature and pressure
- Constant $c, \mathcal{D}_{AB}$

Solution:

$x_A(r)$

$$\frac{1-x_A}{1-x_{A1}} = \left(\frac{1-x_{A2}}{1-x_{A1}}\right)^{\left(\frac{\frac{1}{\mathcal{R}_1}-\frac{1}{r}}{\frac{1}{\mathcal{R}_1}-\frac{1}{\mathcal{R}_2}}\right)}$$

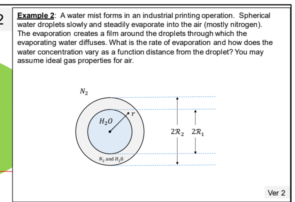
Homework problem 3.14 do algebra; problem 3.8 plot this solution

QUICK START

We can now explore these assumptions, and modify, if needed, for more complex problems.

Ver 2

Assumptions:
- Uniform film surrounds droplet
- Ideal gas
- Constant temperature and pressure



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Let's Interrogate the problem.

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Film model of mass transfer

QUICK START

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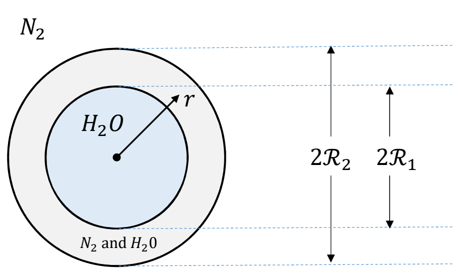

Assumptions:
- Uniform film
- Ideal gas
- Constant pressure
- Constant temperature

How good are these assumptions?

Are there energy issues?

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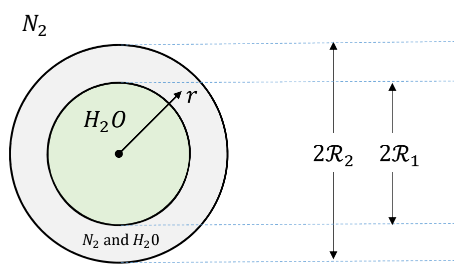
Film model of mass transfer (more complex)

QUICK START

Example 2: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. We can model the diffusion process as shown in the figure. The temperature in the film is not constant but varies as $T(r)/T(\mathcal{R}_1) = (r/\mathcal{R}_1)^n$. What is the rate of evaporation and how does the water concentration vary as a function distance from the droplet?

Note: *not* isothermal



$\Delta\hat{H}_{vap} \neq 0$

Ver 3

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Film model of mass transfer (more complex)

QUICK START

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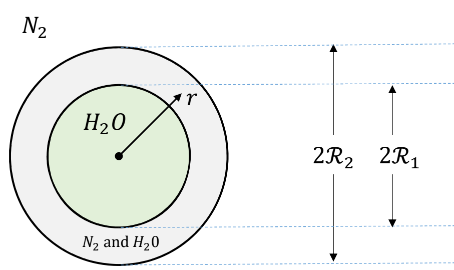

$\Delta\hat{H}_{vap} \neq 0$

What does changing temperature impact?

How do we modify the model?

Ver 3

BSL2, p550

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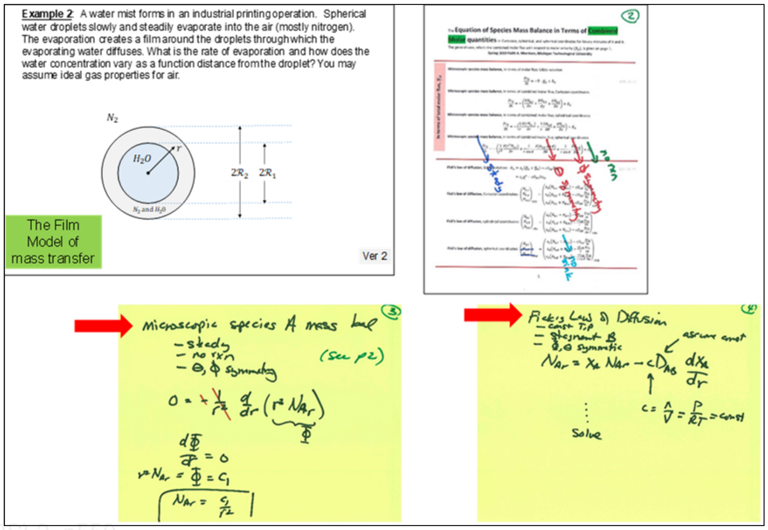
Film model of mass transfer (more complex)

Where did we assume "isothermal" in our previous solution?



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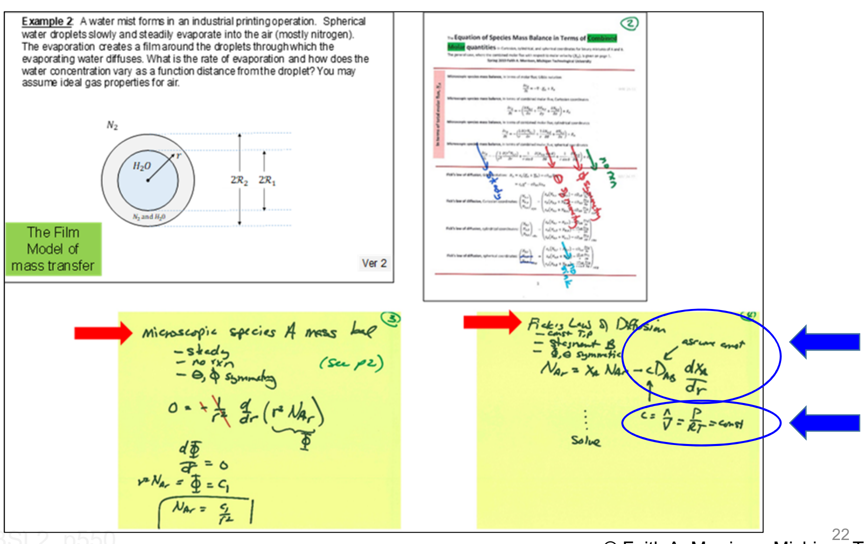
Film model of mass transfer (more complex)

Where did we assume "isothermal" in our previous solution?

When we integrated to obtain $x_A(r)$.



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Film model of mass transfer (more complex)

QUICK START

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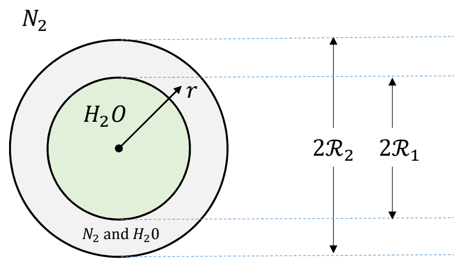
$$D_{AB}(T)/D_{AB,1} = (T/T_1)^{3/2}$$



Note: *not* isothermal

HW 3.7 **(stretch)**

Ver 4

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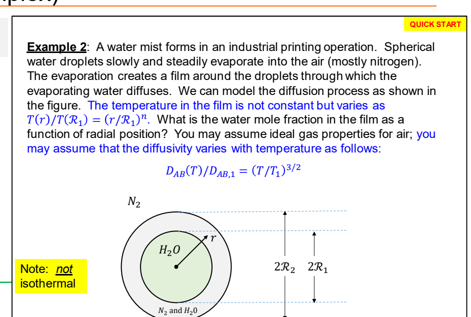
Film model of mass transfer (more complex)

Ver 4

Assumptions:
- Uniform film surrounds droplet
- Ideal gas
- Temperature follows power law
- Diffusivity follows power law
- Pressure is constant


QUICK START

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$$D_{AB}(T)/D_{AB,1} = (T/T_1)^{3/2}$$

N_2, H_2O, r, N_2 and H_2O, $2\mathcal{R}_2$, $2\mathcal{R}_1$

Note: *not* isothermal

Solution:

$x_A(r)$

$$\frac{1-x_A}{1-x_{A1}} = \left(\frac{1-x_{A2}}{1-x_{A1}}\right)^{\left(\frac{\frac{1}{\mathcal{R}_1^{1+n/2}}-\frac{1}{r^{1+n/2}}}{\frac{1}{\mathcal{R}_1^{1+n/2}}-\frac{1}{\mathcal{R}_2^{1+n/2}}}\right)}$$

HW 3.7

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Introduction to Diffusion and Mass Transfer in Mixtures QUICK START

Recurring Modeling Assumptions in Diffusion ("Classics")

- Near a liquid-gas interface, the region in the gas near the liquid is a film where slow diffusion takes place
- The vapor near the liquid-gas interface is often ***saturated*** (Raoult's law, $x_A = p_A^*/p$)
- If component A has no sink, flux $\underline{N}_A = 0$.
- If A diffuses through stagnant B, $\underline{N}_B = 0$.
- If A is dilute in B, we can neglect the convection term ($N_{Az} = J_{Az}^*$)
- Because diffusion is slow, we can make a quasi-steady-state assumption
-
-
-
-
-
-

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We have been performing a "Quick Start,"

And have found the **combined molar flux** formulation useful.

It turns out that there are many interesting and applicable problems we can address readily with **this** form of the species mass balance.

Microscopic species A mass balance—Five forms

In terms of mass flux and mass concentrations: $\rho\left(\frac{\partial \omega_A}{\partial t} + \underline{v}\cdot\nabla\omega_A\right) = -\nabla\cdot\underline{j}_A + r_A$
$= \rho D_{AB}\nabla^2\omega_A + r_A$

In terms of molar flux and molar concentrations: $c\left(\frac{\partial x_A}{\partial t} + \underline{v}^*\cdot\nabla x_A\right) = -\nabla\cdot\underline{J}_A^* + ($
$= cD_{AB}\nabla^2 x_A$

In terms of combined molar flux and molar concentrations: $\frac{\partial c_A}{\partial t} = -\nabla\cdot\underline{N}_A + R_A$

Let's jump in!

Microscopic species mass balance in terms of combined molar flux $\underline{N}_A$

We'll do a "**Quick Start**" and get into some examples and return to the "why" of it all a bit later.

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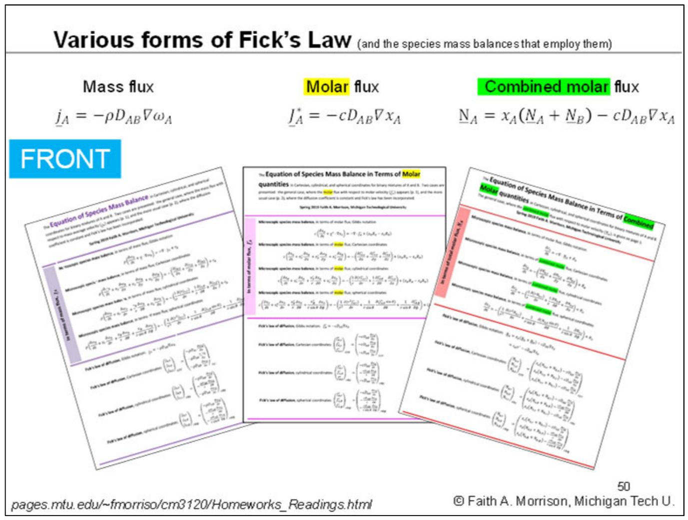
There are times it is not useful. We need to go back and discuss *how/why/when* this all works.



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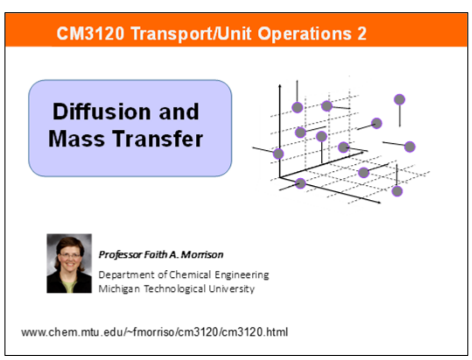
Now, Cycling Back:

Diffusion and Mass Transfer



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