

## CM3120: Module 4

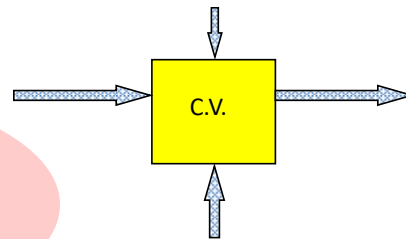
### Diffusion and Mass Transfer II

- I. Classic diffusion and mass transfer: d) EMCD
- II. Classic diffusion and mass transfer: e) Penetration model
- III. **Unsteady macroscopic species A mass balances (Intro)**
- IV. Interphase species A mass transfers—To an interface— $k_x, k_c, k_p$
- V. Unsteady macroscopic species A mass balances (Redux)
- VI. Interphase species A mass transfers—Across multiple resistances— $K_L, K_G$
- VII. Dimensional analysis
- VIII. Data correlations

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## CM3120: Module 4

Module 4 Lecture III  
**Unsteady Macroscopic  
 species A mass balances**  
*(Introduction)*



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[www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html](http://www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html)

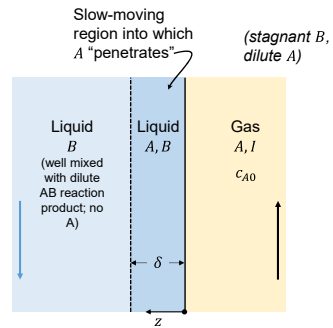
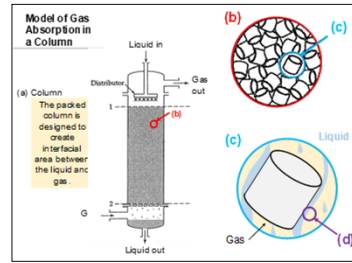
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Unsteady Macroscopic Species A Mass Balance—Intro

In lecture II we developed a picture of how the mass moves around in an absorber (**penetration model**).

*How can we design a packed bed gas absorber to achieve a desired separation?*

In momentum and heat transfer we often sorted out “devices” with a macroscopic balance. Shall we try this?



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Unsteady Macroscopic Species A Mass Balance

**Example 6**

A practical problem

**Example 6:** Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

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**Model of Gas Absorption in a Column**

**From earlier lecture:**

(a) Column  
 The packed column is designed to create interfacial area between the liquid and gas.

(b)

(c)

(d)

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1D Steady Diffusion—Gas Absorption with Chemical Solvent

**From earlier lecture:**

**The Mass-Transfer Model**

We focus on the **liquid-gas interface** where the mass transfer takes place.

**Idealize** the entire device as comprising only the appropriate amount of this interface, with mass transfer taking place there.

**Retain the role of the column**, by forcing the appropriate concentrations of species A to enter and exit the column.

$$a \equiv \frac{\text{interfacial area}}{\text{volume}}$$

A property of the column and packing, in operation

(d)

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**Example 6**

**Unsteady Macroscopic Species A Mass Balance—Intro**

### What is our model for the entire column??

(our assumptions)

1. Control volume =
- 2.

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### Model of Gas Absorption in a Column

top of column

Liquid B attracts A; the concentration of A increases as the liquid passes through the column

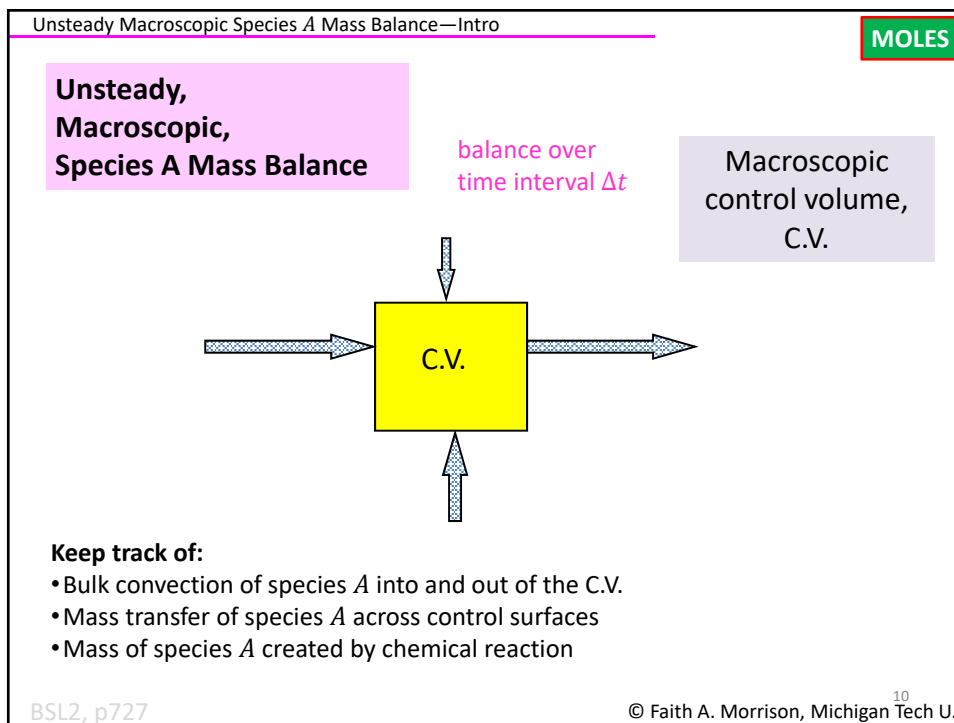
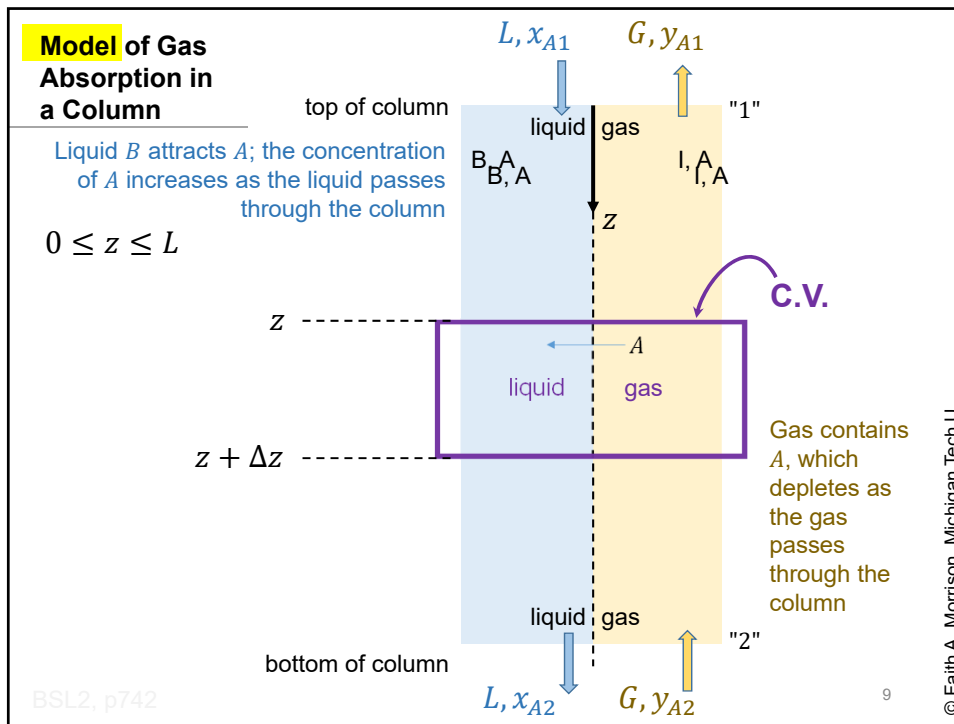
The preference of species A for liquid B rather than inert gas I is due to a difference in chemical potential

Gas contains A, which depletes as the gas passes through the column

bottom of column

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Unsteady Macroscopic Species A Mass Balance—Gas Absorption Example 6

$a \equiv \frac{\text{interfacial area}}{\text{volume}}$  A property of the column and packing, in operation

Mass flux of species A from gas to liquid  $\equiv N_{A,z}$

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Unsteady Macroscopic Species A Mass Balance—Intro MOLES

**Unsteady, Macroscopic, Species A Mass Balance**

balance over time interval  $\Delta t$

$R_A$  = net rate of production of moles of A in the C.V. by reaction, per unit volume

Macroscopic control volume, C.V.

$\dot{\mathcal{M}}_A \Delta t$  → **C.V.** →  $\dot{\mathcal{M}}_A \Delta t$

moles of A that flows into the control volume between  $t$  and  $t + \Delta t$       moles of A that flows out of the control volume between  $t$  and  $t + \Delta t$

Mass flux of species A from gas to liquid  $\equiv N_{A,z}$

$-(N_A S)_j$

introduction of moles of A into the C.V. by mass transfer across the  $j^{th}$  bounding control surface  $S_j$  (C.S.)

C.S. = control surface  
C.V. = control volume

$\dot{S}_{sys} = \sum_j S_j$

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Unsteady Macroscopic Species A Mass Balance—Intro

**MOLES**

*accumulation = net flow in + production + introduction*

$$\frac{d}{dt}(\mathcal{M}_{A,sys}) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum_j (N_A S)_j$$

$\mathcal{M}_{A,sys} = c_A V_{sys}$  = total moles of A in the C.V.

$\Delta\dot{\mathcal{M}}_A = \sum_{j,out} \dot{\mathcal{M}}_{A,j} - \sum_{j,in} \dot{\mathcal{M}}_{A,j}$  = bulk out

$R_A$  = net rate of production of moles of A in the C.V. by reaction, per unit volume

$V_{sys}$  = system volume

$N_{A_j} = K\Delta c_{df}$  = molar flux of A out through the  $j^{th}$  C.S.

$S_{sys} = \sum_j S_j$

$\Delta$  is "out" - "in"

C.S. = control surface

C.V. = control volume

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Unsteady Macroscopic Species A Mass Balance--Intro

**MOLES**

*accumulation = net flow in + production + introduction*

$$\frac{d}{dt}(\mathcal{M}_{A,sys}) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum_i (N_A S)_i$$

**Need:**

Mass flux of species A from gas to liquid  $\equiv N_{A,z}$

$\mathcal{M}_{A,sys} = c_A V_{sys}$  = total moles of A in the C.V.

$\Delta\dot{\mathcal{M}}_A = \sum_{j,out} \dot{\mathcal{M}}_{A,j} - \sum_{j,in} \dot{\mathcal{M}}_{A,j}$  = bulk out

$R_A$  = net rate of production of moles of A in the C.V. by reaction, per unit volume

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Unsteady Macroscopic Species A Mass Balance
**Example 6**

**Example 6:** Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

Unsteady Macroscopic Species A Mass Balance--Intro

MOLES

accumulation = net flow in + production + introduction

$$\frac{d}{dt}(M_{A,sys}) = -\Delta \dot{M}_A + R_A V_{sys} - \sum_i (N_A S)_i$$

$c_A$  = molar concentration of A in the C.V. by unit volume

$M_{A,sys} = c_A V_{sys}$  = moles of A in the C.V.

$\Delta \dot{M}_A = \sum_{j,outs} \dot{M}_j - \sum_{j,ins} \dot{M}_j$  = bulk out

$R_A$  = net rate of production of moles of A in the C.V. by reaction, per unit volume

$V_{sys}$  = system volume

$N_A = K \Delta c_{df}$  = molar flux of A out through the  $j^{th}$  C.S.

Pause: We need a way to account for **mass transfer from one phase to another** in a macroscopic control volume before this can become an effective tool.

**We will return to this problem in Lecture IV.**

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Unsteady Macroscopic Species A Mass Balance--Intro

### Macroscopic Species A Balances Summary (so far)

- To model entire devices, we need **macroscopic** control volumes
- The diffusion coefficient is **not a convenient** way to model mass transfer into a macroscopic control volume.
- In **heat transfer** we used Newton's law of cooling in the macroscopic balance as the source of convective heat flow:
 
$$\dot{q}_{in} = hA(T_b - T_w)$$
- Maybe someone has invented a "**mass transfer coefficient**"  $K$  analogous to the heat transfer coefficient  $h$ ?
 
$$N_A = K \Delta c_{df}$$

Take a **PAUSE** to figure this out

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|---|---|---|
| Modeling practical devices involving mass transfer  |   | <p><b>Example 6:</b> Height of a packed bed absorber</p> <p>How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?</p>   |
| <p><b>Example 6</b> is presented as a series of <b>linked examples</b> that navigate around apparent “dead ends” in modeling mass-transfer units</p>  |   |   |
| <p><b>Identify a question</b></p> <ol style="list-style-type: none"> <li>How can we model a large, practical device dependent on mass transfer?</li> <li>How can we account for A going between phases?</li> <li>How can we improve LDF model to cross the boundary (bulk-to-bulk transfer)?</li> <li>Can we model a large, practical device, incorporating <math>K_L, K_G</math> to account for mass xfer between phases?</li> </ol> | <p><b>Invent something</b></p> <ol style="list-style-type: none"> <li>Apply the species A mass balance to a macroscopic C.V.</li> <li>Invent <math>k_x</math> through linear driving force (LDF) model</li> <li>Write LDF in both phases and combine to create overall effect of multiple resistances</li> <li>Yes</li> </ol> | <p><b>Try to use it</b></p> <ol style="list-style-type: none"> <li>Lack a system to account for A going between phases</li> <li>Gets A <u>to</u> the boundary, but not <u>across</u></li> <li>Working, but can we devise a convenient shorthand?</li> </ol> |
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|--|--|---|
| Modeling practical devices involving mass transfer   |  | <p><b>Example 6:</b> Height of a packed bed absorber</p> <p>How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?</p>   |
| <p><b>Example 6</b> is presented as a series of <b>linked examples</b> that navigate around apparent “dead ends” in modeling mass-transfer units</p>   |  |   |
| <b>LECTURES</b>  |  |   |
| <p><b>Identify a question</b></p> <ol style="list-style-type: none"> <li>✓ How can we model a large, practical device dependent on mass transfer? <b>III</b></li> <li>How can we account for A going between phases? <b>IV</b></li> <li>How can we improve LDF model to cross the boundary (bulk-to-bulk transfer)? <b>VI</b></li> <li>Can we model a large, practical device, incorporating <math>K_L, K_G</math> to account for mass xfer between phases? <b>VI</b></li> </ol> | <p><b>Invent something</b></p> <ol style="list-style-type: none"> <li>✓ Apply the species A mass balance to a macroscopic C.V. <b>III</b></li> <li>Invent <math>k_x</math> through linear driving force (LDF) model <b>IV</b></li> <li>Write LDF in both phases and combine to create overall effect of multiple resistances <b>VI</b></li> <li>Yes <b>VI</b></li> </ol> | <p><b>Try to use it</b></p> <ol style="list-style-type: none"> <li>✓ Lack a system to account for A going between phases <b>PAUSE III</b></li> <li>Gets A <u>to</u> the boundary, but not <u>across</u> <b>PAUSE V</b></li> <li>Working, but can we devise a convenient shorthand? <b>PAUSE VI</b></li> </ol> |
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