# CM3120: Module 4

## Diffusion and Mass Transfer II

I. Classic diffusion and mass transfer: d) EMCD
II. Classic diffusion and mass transfer: e) Penetration model
III. Unsteady macroscopic species A mass balances (Intro)
IV. Interphase species A mass transfers—To an interface—$k_x, k_c, k_p$
V. Unsteady macroscopic species A mass balances (Redux)
VI. Interphase species A mass transfers—Across multiple resistances—$K_L, K_G$
VII. Dimensional analysis
VIII. Data correlations

 1

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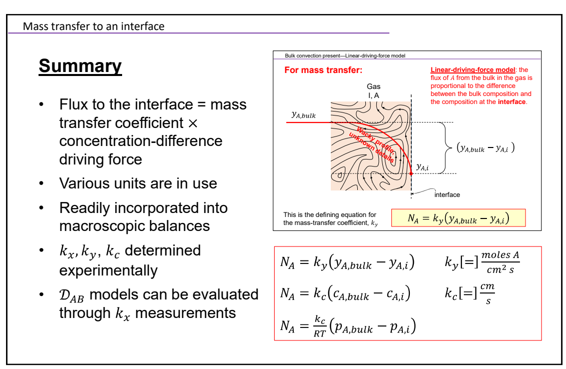
We took a pause to invent the **film mass transfer coefficients $k_y$ (or $k_c, k_p, k_x$)** and the linear transport law for species A in a mixture. These allow us to quantify mass transfer between phases.

$$N_A = k_L(x_A - x_{Ai})$$

We can now return to our attempt to model gas absorber with a macroscopic species A mass balance

**Module 4 Lecture IV:**




 2

Modeling practical devices involving mass transfer

**Example 6:** Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

**Example 6** is presented as a series of **linked examples** that navigate around apparent "dead ends" in modeling mass-transfer units

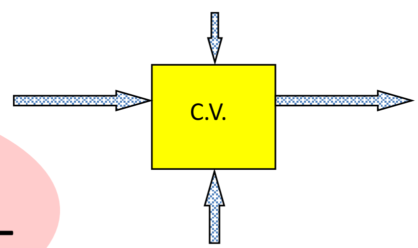
LECTURES

| Identify a question | Invent something | Try to use it |
|---|---|---|
| ✓1. How can we model a large, practical device dependent on mass transfer? **III** | ✓1. Apply the species A mass balance to a macroscopic C.V. **III** | ✓1. Lack a system to account for A going between phases **PAUSE III** |
| ✓2. How an we account for A going between phases? **IV** | ✓2. Invent $k_x$ through linear driving force (LDF) model **IV** | 2. Gets A *to* the boundary, but not ***across*** **PAUSE V** |
| 3. How can we improve LDF model to cross the boundary (bulk-to-bulk transfer)? **VI** | 3. Write LDF in both phases and combine to create overall effect of multiple resistances **VI** | 3. Working, but can we devise a convenient shorthand? **PAUSE VI** |
| 4. Can we model a large, practical device, incorporating $K_L, K_G$ to account for mass xfer between phases? **VI** | 4. Yes **VI** | |

3



# CM3120: Module 4




Module 4 Lecture V

## Unsteady Macroscopic species A mass balances—Redux

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Department of Chemical Engineering
Michigan Technological University

www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

4

Unsteady Macroscopic Species $A$ Mass Balance

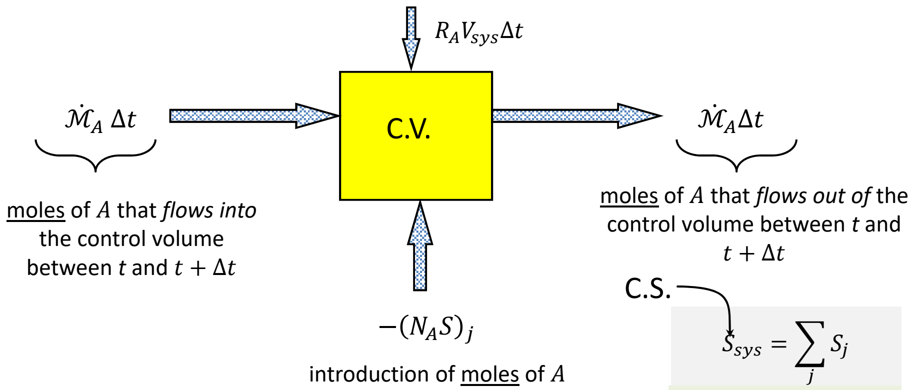
**MOLES**

**Unsteady, Macroscopic, Species A Mass Balance**

balance over time interval $\Delta t$

Macroscopic control volume, C.V.

$R_A$ = net rate of production of moles of $A$ in the C.V. by reaction, per unit volume




introduction of moles of $A$ into the C.V. by mass transfer across the $j^{th}$ bounding control surface $S_j$ (C.S.)

C.S. = control surface
C.V. = control volume

BSL2, p727

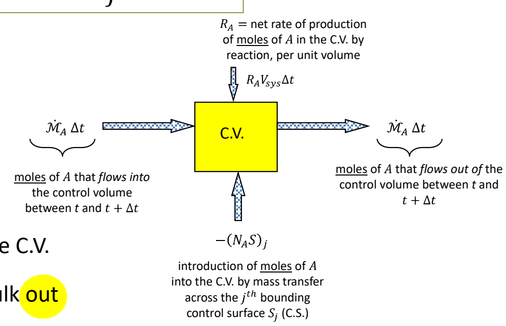
 5

Unsteady Macroscopic Species $A$ Mass Balance

**MOLES**

*accumulation = net flow in + production + introduction*

$$\frac{d}{dt}\left(\mathcal{M}_{A,sys}\right) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum_j (N_A S)_j$$




$\mathcal{M}_{A,sys} = c_A V_{sys}$ = total moles of $A$ in the C.V.

$\Delta\dot{\mathcal{M}}_A = \sum_{j,outs} \dot{\mathcal{M}}_{A,j} - \sum_{j,ins} \dot{\mathcal{M}}_{A,j}$ = bulk out

$R_A$ = net rate of production of moles of $A$ in the C.V. by reaction, per unit volume

$V_{sys}$ = system volume

$N_{A_j} = K\Delta c_{df}$ =molar flux of $A$ out through the $j^{th}$ C.S.

$S_{sys} = \sum_j S_j$
$\Delta$ is "out"- "in"
C.S. = control surface
C.V. = control volume

 6

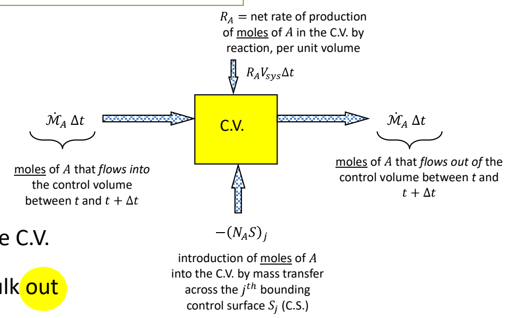
Unsteady Macroscopic Species $A$ Mass Balance

**MOLES**

*accumulation = net flow in + production + introduction*

$$\frac{d}{dt}\left(\mathcal{M}_{A,sys}\right) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum_j (N_A S)_j$$

**The choice of the "system," i.e. of the *control volume,* is an important first step.**

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**(think "source" and "sink")**

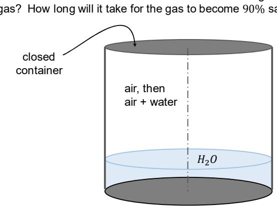
 7

Unsteady Macroscopic Species $A$ Mass Balance

Review: Macroscopic Species A Mass Balance

Unsteady Macroscopic Species $A$ Mass Balance

Example: Bone dry air and liquid water (water volume = 0.80 $liters$) are introduced into a closed container (cross sectional area = 150 $cm^2$; total volume = 19.2 $liters$). Both air and water are at 25$^oC$ throughout this scenario. Three minutes after the air and water are placed in the closed container, the vapor is found to be 5.0% saturated with water vapor. What is the mass transfer coefficient for the water transferring from the liquid to the gas? How long will it take for the gas to become 90% saturated with water?




**Solution:**

(mass transfer coefficient for evaporating water)

$$\frac{c_A}{c_A^*} = 1 - e^{-\left(\frac{k_c S}{V_{gas}}\right)t}$$

$$t = 2.3\ hours$$

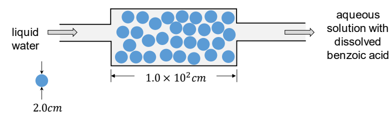
 8

Unsteady Macroscopic Species $A$ Mass Balance

**Review: Macroscopic Species A Mass Balance**

Unsteady Macroscopic Species $A$ Mass Balance

**<u>Example</u>:** Flow through a packed bed of soluble spherical pellets.




Two centimeter diameter spheres of benzoic acid (soluble in water) are packed into a bed as shown. The spheres have $23\ cm^2$ of surface area per $cm^3$ volume of bed. What is the mass transfer coefficient when pure water flowing in ("superficial velocity"= $5.0\ cm/s$) exits $62\%$ saturated with benzoic acid?

## Solution:

(dissolving benzoic acid in packed bed, pseudo steady state)

$$k_c = \frac{v^0}{aL}\left(-\ln\left(1 - \frac{c_{AL}}{c_A^*}\right)\right)$$

$$k_c = 2.1 \times 10^{-3} cm/s$$

9



Unsteady Macroscopic Species $A$ Mass Balance

The **macroscopic species A mass balance** worked well for the these two problems.

Both problems involved transfer from the **interface conditions** to the **bulk**.

Let's now try a problem in which **the mass transfer is from one bulk condition to a second bulk condition:**

**Example 6—Revisited**

**<u>Example</u> 6:** Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

(started in Module 4, Lecture III)

10

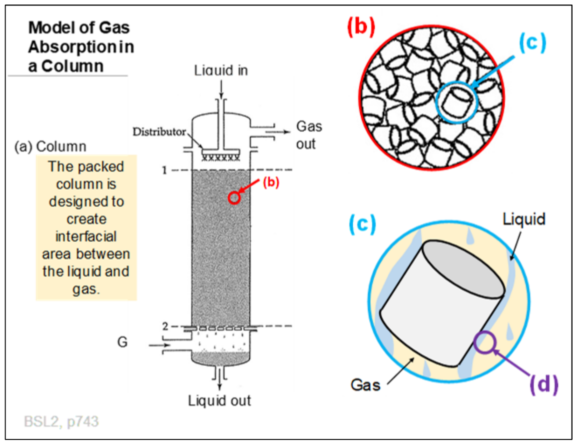
Unsteady Macroscopic Species $A$ Mass Balance

**Example 6—Revisited**

**Example 6:** Height of a packed bed absorber

How can we use the linear driving force model for mass transfer to design a packed bed gas absorber to achieve a desired separation?

**From earlier lecture:**

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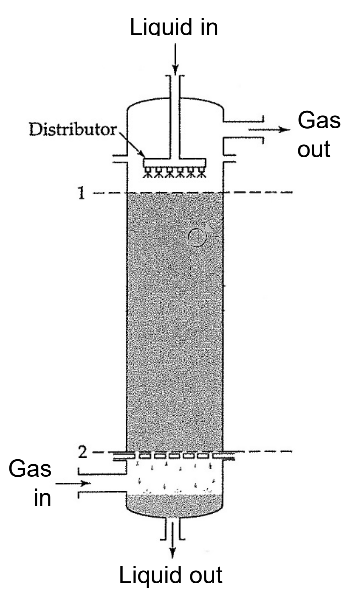
 11

Unsteady Macroscopic Species $A$ Mass Balance—Gas Absorption




Unsteady Macroscopic Species $A$ Mass Balance

**MOLES**

*accumulation = net flow in + production + introduction*

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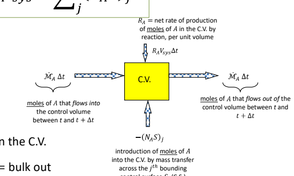


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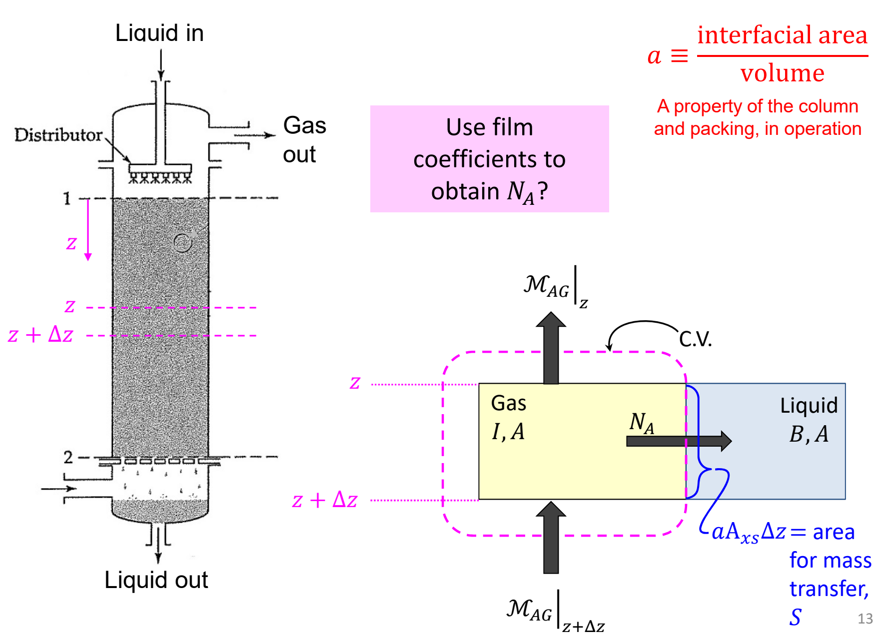
?

You try.

 12

Unsteady Macroscopic Species $A$ Mass Balance—Gas Absorption

**Example 6—Revisited**



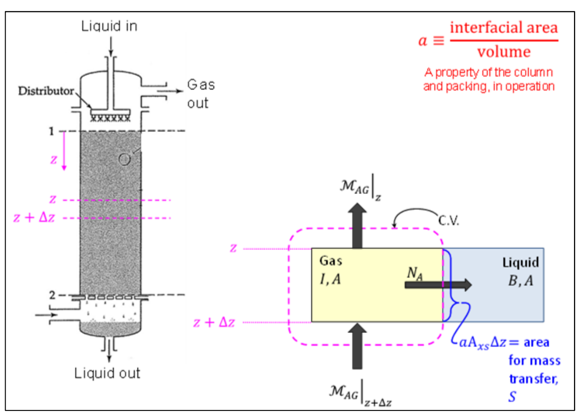

13



Unsteady Macroscopic Species $A$ Mass Balance—Gas Absorption

**Example 6—Revisited**

**Difficulty:**
The film coefficients do not allow us to include the status of the bulk liquid phase in the driving force for this problem.




**Gas phase film LDF model:**

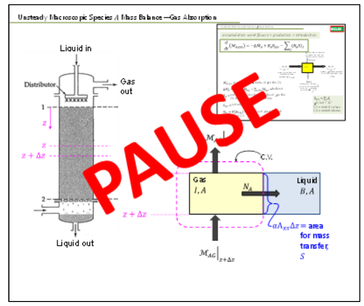
$$N_A = k_p(p_A - p_{Ai})$$

We need a technique that uses a **bulk-to-bulk** driving force

14

## Unsteady Macroscopic Species $A$ Mass Balance—Gas Absorption

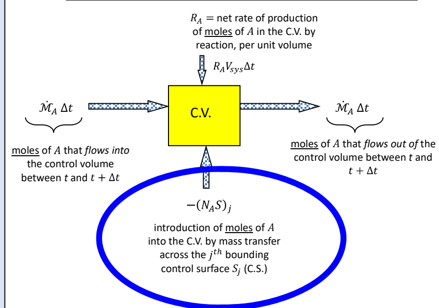
**Next?** Develop the macroscopic mass-transfer expression we need for bulk-to-bulk transfer $N_A S$.




**Needed:**

$$N_A = (?)$$
$$= (\text{expression})(\Delta c_{df})$$

Bulk-to-bulk, like
$\dot{Q} = UA\Delta T_{lm}$




15