

CM3120: Module 4

Diffusion and Mass Transfer II

- I. Classic diffusion and mass transfer: d) EMCD
- II. Classic diffusion and mass transfer: e) Penetration model
- III. Unsteady macroscopic species A mass balances (Intro)
- IV. Interphase species A mass transfers—To an interface— k_x, k_c, k_p
- V. **Unsteady macroscopic species A mass balances (Redux)**
- VI. Interphase species A mass transfers—Across multiple resistances— K_L, K_G
- VII. Dimensional analysis
- VIII. Data correlations

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We took a pause to invent the **film mass transfer coefficients k_y (or k_c, k_p, k_x)** and the linear transport law for species A in a mixture. These allow us to quantify mass transfer between phases.

$$N_A = k_L(x_A - x_{Ai})$$

We can now return to our attempt to model gas absorber with a macroscopic species A mass balance

Module 4
Lecture IV:

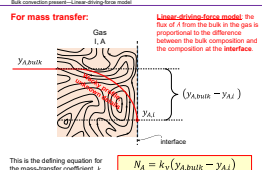
Mass transfer to an interface

Summary

- Flux to the interface = mass transfer coefficient × concentration-difference driving force
- Various units are in use
- Readily incorporated into macroscopic balances
- k_x, k_y, k_c determined experimentally
- D_{AB} models can be evaluated through k_x measurements

Bulk convective flow—Linear driving force model

For mass transfer:



This is the defining equation for the mass-transfer coefficient, k_y .

Linear-driving force model: the flux of A from the bulk in the gas is proportional to the difference between the bulk composition and the composition at the interface.

$$N_A = k_y(y_{A,bulk} - y_{Ai})$$

$$N_A = k_y(y_{A,bulk} - y_{Ai}) \quad k_y [=] \frac{\text{moles A}}{\text{cm}^2 \text{ s}}$$

$$N_A = k_c(c_{A,bulk} - c_{Ai}) \quad k_c [=] \frac{\text{cm}}{\text{s}}$$

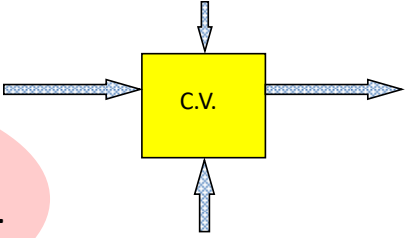
$$N_A = \frac{k_c}{RT}(p_{A,bulk} - p_{Ai})$$

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Modeling practical devices involving mass transfer		<p>Example 6: Height of a packed bed absorber</p> <p>How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?</p>
<p>Example 6 is presented as a series of linked examples that navigate around apparent “dead ends” in modeling mass-transfer units</p>		
LECTURES		
<p>Identify a question</p> <ol style="list-style-type: none"> ✓ How can we model a large, practical device dependent on mass transfer? III ✓ How can we account for A going between phases? IV 3. How can we improve LDF model to cross the boundary (bulk-to-bulk transfer)? VI 4. Can we model a large, practical device, incorporating K_L, K_G to account for mass xfer between phases? VI 	<p>Invent something</p> <ol style="list-style-type: none"> ✓ Apply the species A mass balance to a macroscopic C.V. III ✓ Invent k_x through linear driving force (LDF) model IV 3. Write LDF in both phases and combine to create overall effect of multiple resistances VI 4. Yes VI 	<p>Try to use it</p> <ol style="list-style-type: none"> ✓ Lack a system to account for A going between phases PAUSE III 2. Gets A <u>to</u> the boundary, but not <u>across</u> PAUSE V 3. Working, but can we devise a convenient shorthand? PAUSE VI
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CM3120: Module 4



Module 4 Lecture V

Unsteady Macroscopic species A mass balances— Redux



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Unsteady Macroscopic Species A Mass Balance

MOLES

Unsteady, Macroscopic, Species A Mass Balance

balance over time interval Δt

R_A = net rate of production of moles of A in the C.V. by reaction, per unit volume

$R_A V_{sys} \Delta t$

C.V.

$-(N_A S)_j$

introduction of moles of A into the C.V. by mass transfer across the j^{th} bounding control surface S_j (C.S.)

C.S. = control surface
C.V. = control volume

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Unsteady Macroscopic Species A Mass Balance

MOLES

accumulation = net flow in + production + introduction

$$\frac{d}{dt} (\mathcal{M}_{A,sys}) = -\Delta \dot{\mathcal{M}}_A + R_A V_{sys} - \sum_j (N_A S)_j$$

R_A = net rate of production of moles of A in the C.V. by reaction, per unit volume

$R_A V_{sys} \Delta t$

C.V.

$-(N_A S)_j$

introduction of moles of A into the C.V. by mass transfer across the j^{th} bounding control surface S_j (C.S.)

$S_{sys} = \sum_j S_j$
 Δ is "out" - "in"
C.S. = control surface
C.V. = control volume

$\mathcal{M}_{A,sys} = c_A V_{sys}$ = total moles of A in the C.V.

$\Delta \dot{\mathcal{M}}_A = \sum_{j,outs} \dot{\mathcal{M}}_{A,j} - \sum_{j,ins} \dot{\mathcal{M}}_{A,j}$ = bulk out

R_A = net rate of production of moles of A in the C.V. by reaction, per unit volume

V_{sys} = system volume

$N_{A_j} = K \Delta c_{df}$ = molar flux of A out through the j^{th} C.S.

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Unsteady Macroscopic Species A Mass Balance

MOLES

accumulation = net flow in + production + introduction

$$\frac{d}{dt}(\mathcal{M}_{A,sys}) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum_j (N_A S)_j$$

The choice of the “system,” i.e. of the control volume, is an important first step.

(think “source” and “sink”)

$\mathcal{M}_{A,sys} = c_A V_{sys}$ = total moles of A in the C.V.

$\Delta\dot{\mathcal{M}}_A = \sum_{j,out} \dot{\mathcal{M}}_{A,j} - \sum_{j,ins} \dot{\mathcal{M}}_{A,j}$ = bulk out

R_A = net rate of production of moles of A in the C.V. by reaction, per unit volume

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Unsteady Macroscopic Species A Mass Balance

Review: Macroscopic Species A Mass Balance

Unsteady Macroscopic Species A Mass Balance

Example: Bone dry air and liquid water (water volume = 0.80 liters) are introduced into a closed container (cross sectional area = 150 cm²; total volume = 19.2 liters). Both air and water are at 25°C throughout this scenario. Three minutes after the air and water are placed in the closed container, the vapor is found to be 5.0% saturated with water vapor. What is the mass transfer coefficient for the water transferring from the liquid to the gas? How long will it take for the gas to become 90% saturated with water?

Solution:

(mass transfer coefficient for evaporating water)

$$\frac{C_A}{C_A^*} = 1 - e^{-\left(\frac{k_c S}{V_{gas}}\right)t}$$

$t = 2.3 \text{ hours}$

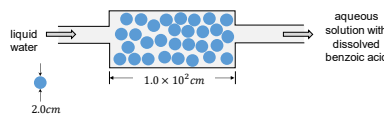
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Unsteady Macroscopic Species A Mass Balance

Review: Macroscopic Species A Mass Balance

Unsteady Macroscopic Species A Mass Balance

Example: Flow through a packed bed of soluble spherical pellets.



Two centimeter diameter spheres of benzoic acid (soluble in water) are packed into a bed as shown. The spheres have 23 cm^2 of surface area per cm^3 volume of bed. What is the mass transfer coefficient when pure water flowing in ("superficial velocity" = 5.0 cm/s) exits 62% saturated with benzoic acid?

Solution:

(dissolving benzoic acid in packed bed, pseudo steady state)

$$k_c = \frac{v^0}{aL} \left(-\ln \left(1 - \frac{c_{AL}}{c_A^*} \right) \right)$$

$$k_c = 2.1 \times 10^{-3} \text{ cm/s}$$

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Unsteady Macroscopic Species A Mass Balance

The **macroscopic species A mass balance** worked well for the these two problems.

Both problems involved transfer from the **interface conditions** to the **bulk**.

Let's now try a problem in which **the mass transfer is from one bulk condition to a second bulk condition**:

Example 6—Revisited**Example 6:** Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

(started in Module 4, Lecture III)

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Unsteady Macroscopic Species A Mass Balance

Example 6—Revisited

Example 6: Height of a packed bed absorber

How can we use the linear driving force model for mass transfer to design a packed bed gas absorber to achieve a desired separation?

From earlier lecture:

Model of Gas Absorption in a Column

(a) Column: The packed column is designed to create interfacial area between the liquid and gas.

(b) and (c) show detailed views of the liquid-gas interface with labels (b) and (d).

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Unsteady Macroscopic Species A Mass Balance—Gas Absorption

MOLES

accumulation = net flow in + production + introduction

$$\frac{d}{dt}(\mathcal{M}_{A,sys}) = -\Delta \dot{\mathcal{M}}_A + R_A V_{sys} - \sum_j (N_A S)_j$$

R_A = net rate of production of moles of A in the C.V. by reaction, per unit volume
 $\dot{\mathcal{M}}_A \Delta t$ = moles of A that flow into the control volume between t and t + Δt
 $\dot{\mathcal{M}}_A \Delta t$ = moles of A that flow out of the control volume between t and t + Δt
 $(N_A S)_j$ = introduction of moles of A into the C.V. by mass transfer across the jth bounding control surface S_j (C.S.)

$\mathcal{M}_{A,sys} = c_A V_{sys}$ = total moles of A in the C.V.
 $\Delta \dot{\mathcal{M}}_A = \sum_{j,out} \dot{\mathcal{M}}_{A,j} - \sum_{j,in} \dot{\mathcal{M}}_{A,j}$ = bulk out
 R_A = net rate of production of moles of A in the C.V. by reaction, per unit volume
 V_{sys} = system volume
 $N_{A,j} = K(\Delta c_{A,j})$ = molar flux of A out through the jth C.S.

$S_{sys} = \sum_j S_j$
 Δ is "out" - "in"
 C.S. = control surface
 C.V. = control volume

? You try.

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Unsteady Macroscopic Species A Mass Balance—Gas Absorption

Example 6—Revisited

Use film coefficients to obtain N_A ?

$$a \equiv \frac{\text{interfacial area}}{\text{volume}}$$
 A property of the column and packing, in operation

$aA_{xs}\Delta z = \text{area for mass transfer, } S$

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Unsteady Macroscopic Species A Mass Balance—Gas Absorption

Example 6—Revisited

Difficulty:
The film coefficients do not allow us to include the status of the bulk liquid phase in the driving force for this problem.

Gas phase film LDF model:

$$N_A = k_p(p_A - p_{Ai})$$

$$a \equiv \frac{\text{interfacial area}}{\text{volume}}$$
 A property of the column and packing, in operation

$aA_{xs}\Delta z = \text{area for mass transfer, } S$

We need a technique that uses a **bulk-to-bulk** driving force

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Unsteady Macroscopic Species A Mass Balance—Gas Absorption

Next? Develop the macroscopic mass-transfer expression we need for bulk-to-bulk transfer $N_A S$.

Needed:

$$N_A = (?) = (\text{expression})(\Delta c_{df})$$

Bulk-to-bulk, like $\dot{Q} = UA\Delta T_{lm}$

