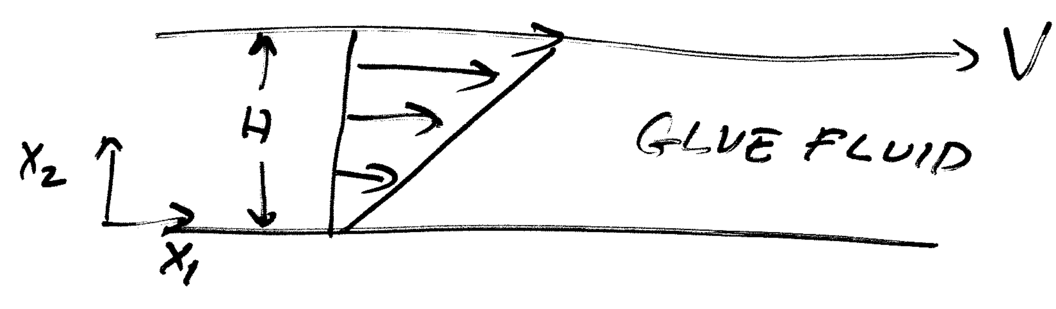
GLUE FLUID CAN ALSO BE USED TO SOLVE FLOW PROBLEMS.

$$\underline{\underline{\tau}}(t) = -\int_{-\infty}^{t} G(t-t')\,\underline{\underline{\dot{\gamma}}}(t')\,dt'$$

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p - \nabla\cdot\underline{\underline{\tau}} + \rho\underline{g}$$

PROBLEM: DRAG FLOW OF AN INCOMPRESSIBLE GLUE FLUID BTWN PARALLEL PLATES



$$\boxed{\begin{array}{lll} \text{BC:} & x_2 = 0 & v_1 = 0 \\ & x_2 = H & v_1 = V \end{array}}$$

continuity: $\nabla \cdot \underline{v} = 0$ $\qquad \underline{v} = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}$

$$0 = \frac{\partial v_1}{\partial x_1} + \cancel{\frac{\partial v_2}{\partial x_2}} + \cancel{\frac{\partial v_3}{\partial x_3}}$$

$$\boxed{\frac{\partial v_1}{\partial x_1} = 0}$$

EOM: $\rho\Big(\underbrace{\cancel{\frac{\partial \underline{v}}{\partial t}}}_{ss} + \underbrace{\cancel{\underline{v} \cdot \nabla \underline{v}}}_{\text{unidir}}\Big) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \underbrace{\cancel{\rho \underline{g}}}_{\text{neglect.}}$

$$0 = -\nabla p - \nabla \cdot \underline{\underline{\tau}}$$

$$\nabla \cdot \underline{\underline{\tau}} = \frac{\partial}{\partial x_i} \underbrace{\hat{e}_i \cdot \tau_{pk} \hat{e}_p}_{\delta_{ip}} \hat{e}_k$$

$$= \frac{\partial}{\partial x_p} \tau_{pk} \hat{e}_k = \begin{pmatrix} \cancel{\frac{\partial \tau_{11}}{\partial x_1}} + \frac{\partial \tau_{21}}{\partial x_2} + \cancel{\frac{\partial \tau_{31}}{\partial x_3}} \\ \frac{\partial \tau_{12}}{\partial x_1} + \cancel{\frac{\partial \tau_{22}}{\partial x_2}} + \cancel{\frac{\partial \tau_{32}}{\partial x_3}} \\ \cancel{\frac{\partial \tau_{13}}{\partial x_1}} + \cancel{\frac{\partial \tau_{23}}{\partial x_2}} + \cancel{\frac{\partial \tau_{33}}{\partial x_3}} \end{pmatrix}_{123}$$

$$-\nabla p = \begin{pmatrix} -\frac{\partial p}{\partial x_1} \\ -\frac{\partial p}{\partial x_2} \\ -\frac{\partial p}{\partial x_3} \end{pmatrix}_{123}$$

④

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} G(t-t')\, \underline{\underline{\dot{\gamma}}}(t')\, dt'$$

$$\underline{\underline{\dot{\gamma}}} = \nabla \underline{v} + (\nabla \underline{v})^T \qquad \nabla \underline{v} = \begin{pmatrix} \cancel{\frac{\partial v_1}{\partial x_1}} & \cancel{\frac{\partial v_2}{\partial x_1}} & \cancel{\frac{\partial v_3}{\partial x_1}} \\ \frac{\partial v_1}{\partial x_2} & \cancel{\frac{\partial v_2}{\partial x_2}} & \cancel{\frac{\partial v_3}{\partial x_2}} \\ \cancel{\frac{\partial v_1}{\partial x_3}} & \cancel{\frac{\partial v_2}{\partial x_3}} & \cancel{\frac{\partial v_3}{\partial x_3}} \end{pmatrix}_{123}$$

$\nabla \cdot \underline{v} = 0$

wide plates $\quad v_2 = v_3 = 0$

$$\underline{\underline{\dot{\gamma}}} = \begin{pmatrix} 0 & \frac{\partial v_1}{\partial x_2} & 0 \\ \frac{\partial v_1}{\partial x_2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

$\underline{\underline{\dot{\gamma}}}$ = constant (steady state)

$$\underline{\underline{\tau}}(t) = -\int_{-\infty}^{t} G(t-t')\,\underline{\underline{\dot{\gamma}}}(t')\,dt'$$

constant ∀ time t'

$$= -\underline{\underline{\dot{\gamma}}} \underbrace{\int_{-\infty}^{t} G(t-t')\,dt'}_{\eta_0 \text{ (see HW6)}}$$

$$\underline{\underline{\tau}}(t) = \begin{pmatrix} 0 & -\frac{\partial v_1}{\partial x_2}\eta_0 & 0 \\ -\frac{\partial v_1}{\partial x_2}\eta_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

EOM

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{\partial p}{\partial x_1} \\ -\frac{\partial p}{\partial x_2} \\ -\frac{\partial p}{\partial x_3} \end{pmatrix}_{123} - \begin{pmatrix} \frac{\partial \tau_{21}}{\partial x_2} \\ \cancel{\frac{\partial \tau_{22}}{\partial x_2}} \\ 0 \end{pmatrix}_{123}$$

$\frac{\partial \tau_{21}}{\partial x_2} = -\frac{\partial v_1}{\partial x_2}\eta_0$

$\frac{\partial \tau_{22}}{\partial x_2} = -\frac{\partial v_1}{\partial x_2}\eta_0$

$v_1 = v_1(x_2)$

x_3-component: $\boxed{\frac{\partial p}{\partial x_3} = 0}$

x_2-component $\boxed{\frac{\partial p}{\partial x_2} = 0}$

x_1-component $\overset{0}{\cancel{\frac{\partial p}{\partial x_1}}} = +\frac{\partial}{\partial x_2}\left(+\frac{\partial v_1}{\partial x_2}\eta_0\right)$

$$\frac{\partial}{\partial x_2}\left(\frac{\partial v_1}{\partial x_2}\right) = 0$$

integrate $\left(\frac{\partial v_1}{\partial x_2}\right) = c_1$

integrate again $\boxed{v_1 = c_1 x_2 + c_2}$

BC: $x_2 = 0 \quad v_1 = 0 \Rightarrow \boxed{c_2 = 0}$

$x_2 = H \quad v_1 = V \Rightarrow c_1 = \frac{V}{H}$

$$\boxed{v_1 = \frac{x_2}{H} V}$$