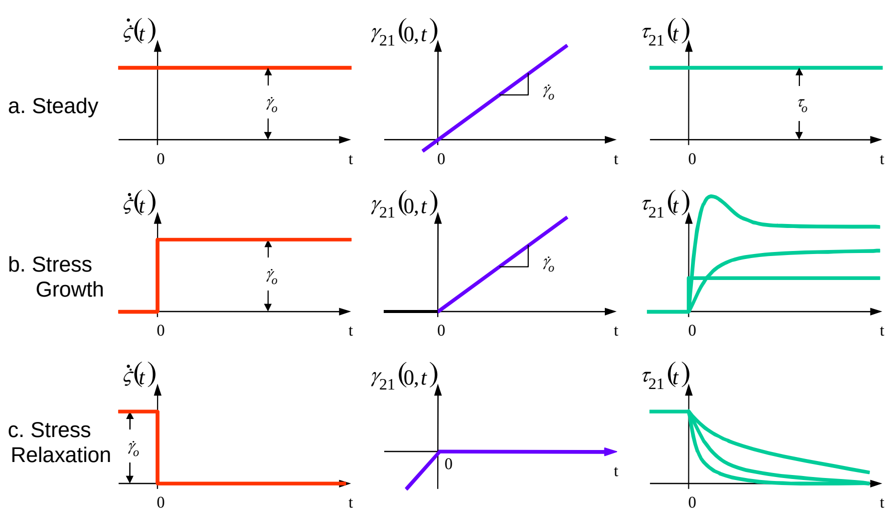
To proceed to better-designed constitutive equations, we need to know more about material behavior, i.e. we need more material functions to predict, and we need measurements of these material functions.

- More non-steady material functions (material functions that tell us about memory)
- Material functions that tell us about nonlinearity (strain)

1

Summary of shear rate kinematics *(part 1)*

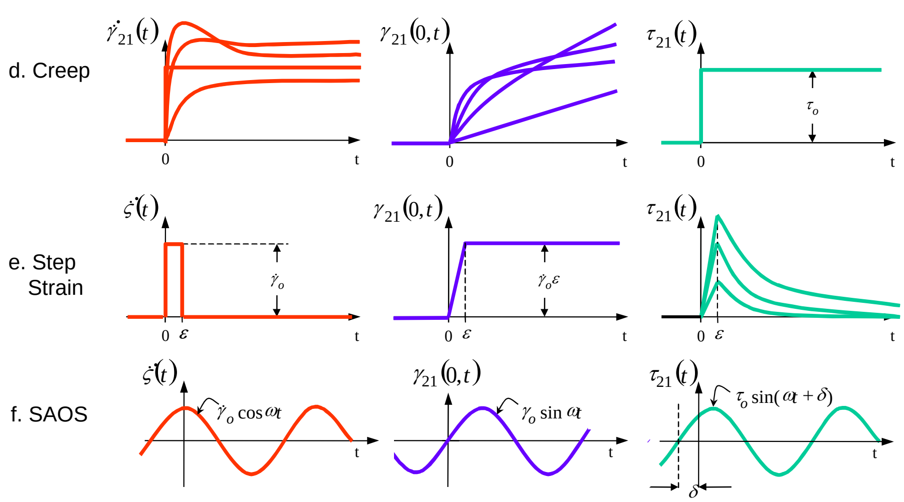


2

The next three families of material functions incorporate the concept of strain.

3

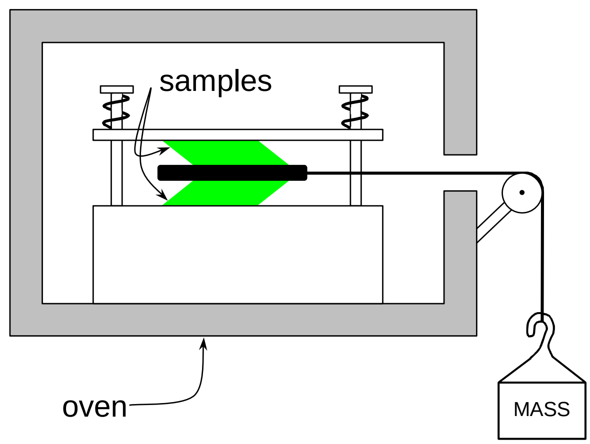
Summary of shear rate kinematics *(part 2)*



4

Shear Creep Flow

Constant shear stress imposed



5

Because shear rate is not prescribed, it becomes something we must measure.

Creep Shear Flow Material Functions

It is <u>unusual</u> to prescribe stress rather than $\dot{\varsigma}(t)$

Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\gamma}_{21}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \tau_{21}(t) = \begin{cases} 0 & t < 0 \\ \tau_0 & t \geq 0 \end{cases}$$

Material Functions:

Since we ***set*** the stress in this experiment (rather than measuring it), the material functions are related to the ***deformation*** of the sample. We need to discuss measurements of deformation before proceeding.

6

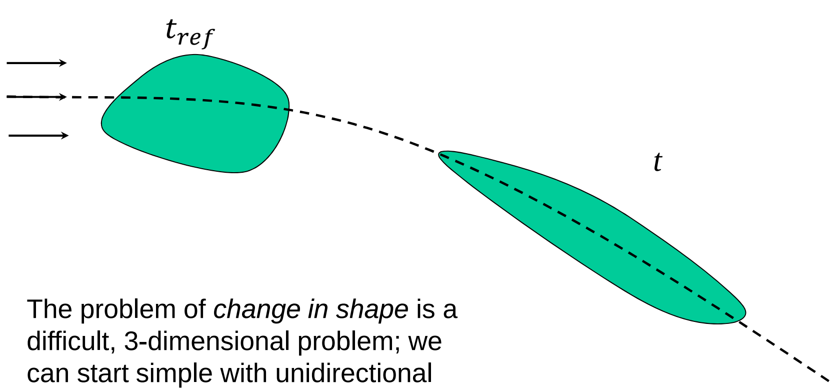
Pause on Material Functions

We need to define and learn to work with *strain*.

7

Deformation (strain)

We need a way to quantify "change in shape"



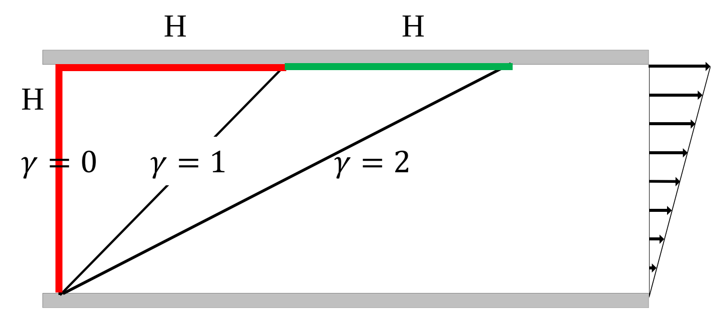
The problem of *change in shape* is a difficult, 3-dimensional problem; we can start simple with unidirectional flow (shear).

8

Strain in Shear

$$\gamma = \frac{\Delta u_1}{\Delta x_2}$$

Relative change in displacement



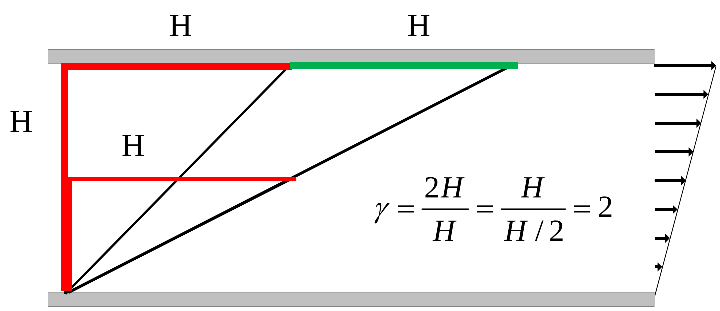
The strain is related to the *change of shape* of the deformed particle.

9

Strain in Shear

$$\gamma = \frac{\Delta u_1}{\Delta x_2}$$

Relative change in displacement



The strain is related to the *change of shape* of the deformed particle.

There is no unique way to measure "change of shape."

10

Deformation (strain)

$$\underline{r}(t_{ref}) = \begin{pmatrix} x_1(t_{ref}) \\ x_2(t_{ref}) \\ x_3(t_{ref}) \end{pmatrix}_{123}$$

$$\underline{r}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}_{123}$$

This vector keeps track of the location of a fluid particle as a function of time.

$$\underline{u}(t_{ref}, t) \equiv \underline{r}(t) - \underline{r}(t_{ref})$$

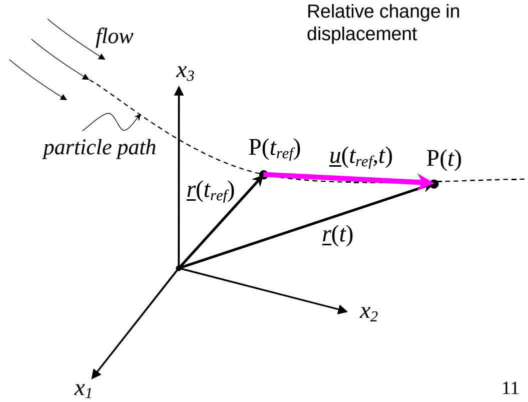
Displacement function

Current position compared to reference position

$$\gamma_{21}(t_{ref}, t) \equiv \frac{\partial u_1}{\partial x_2}$$

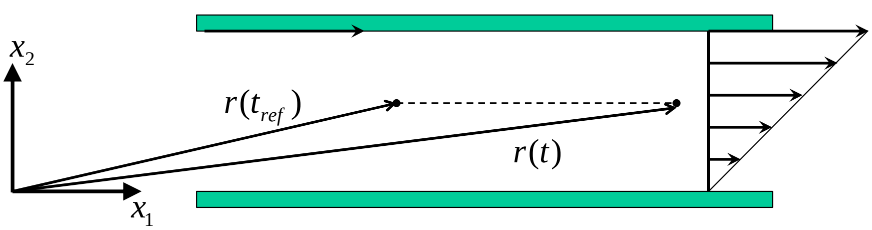
Shear strain

Relative change in displacement



11

What is the strain in the standard flow *steady shear*?



$$\underline{v} = \begin{pmatrix} \dot{\gamma}_0 x_2 \\ 0 \\ 0 \end{pmatrix}_{123} = \begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{pmatrix}_{123}$$

We can integrate this differential equation because $\dot{\gamma}_0$ is a constant. We obtain $x_1(t)$.

Deformation in shear flow (strain)

steady

$$\underline{r}(t_{ref}) = \begin{pmatrix} x_1(t_{ref}) \\ x_2(t_{ref}) \\ x_3(t_{ref}) \end{pmatrix}_{123}$$

$$\underline{r}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}_{123} = \begin{pmatrix} x_1(t_{ref}) + (t - t_{ref})\dot{\gamma}_0 x_2 \\ x_2(t_{ref}) \\ x_3(t_{ref}) \end{pmatrix}_{123}$$

$$\underline{u}(t_{ref}, t) \equiv \underline{r}(t) - \underline{r}(t_{ref}) = \begin{pmatrix} (t - t_{ref})\dot{\gamma}_0 x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

Displacement function

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Deformation in shear flow (strain)

steady

$$\underline{u}(t_{ref}, t) \equiv \underline{r}(t) - \underline{r}(t_{ref}) = \begin{pmatrix} (t - t_{ref})\dot{\gamma}_0 x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

Displacement function

Our choice for measuring change in shape:

$$\gamma_{21}(t_{ref}, t) \equiv \frac{\partial u_1}{\partial x_2} = \frac{du_1}{dx_2}$$

Shear strain

$$\gamma_{21}(t_{ref}, t) = (t - t_{ref})\dot{\gamma}_0$$

(for steady shear or in unsteady shear for short time intervals)

14

For **unsteady** shear, $\dot{\gamma}$ is a function of time:

$$\underline{v} = \begin{pmatrix} \dot{\gamma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123} = \begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{pmatrix}_{123}$$

This integration is less straightforward.

We can obtain the unsteady result for strain by applying the steady result over short time intervals (where $\dot{\gamma}$ may be approximated as a constant) and add up the strains.

short time interval between t_p and t_{p+1}:

$$\gamma_{21}(t_p, t_{p+1}) = \frac{\partial u_1}{\partial x_2} = \dot{\gamma}_{21}(t_{p+1})\Delta t$$

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For **unsteady** shear:

$$\gamma_{21}(t_p, t_{p+1}) = \frac{\partial u_1}{\partial x_2} = \dot{\gamma}_{21}(t_{p+1})\Delta t$$

(short time interval)

For a long time interval, we add up the strains over short time intervals.

short time interval: $\gamma_{21}(t_p, t_{p+1}) = \dot{\gamma}_{21}(t_{p+1})\Delta t$

long time interval: $\gamma_{21}(t_1, t_2) = \sum_{p=0}^{N-1} \gamma_{21}(t_p, t_{p+1}) = \sum_{p=0}^{N-1} \Delta t \dot{\gamma}_{21}(t_{p+1})$

Taking the limit as $\Delta t \rightarrow 0$,

$$\gamma_{21}(t_1, t_2) = \lim_{\Delta t \rightarrow 0} \left[\sum_{p=0}^{N-1} \Delta t \dot{\gamma}_{21}(t_{p+1}) \right] = \int_{t_1}^{t_2} \dot{\gamma}_{21}(t') dt'$$

Strain at t_2 with respect to fluid configuration at t_1 in **unsteady** shear flow.

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Change of Shape

For shear flow (steady or unsteady):

$$\gamma_{21}(t_1,t_2) = \int_{t_1}^{t_2} \dot{\gamma}_{21}(t')dt'$$

Strain at t_2 with respect to fluid configuration at t_1 in shear flow (steady or unsteady).

Note also, by Leibnitz rule:

$$\frac{d\gamma_{21}}{dt} = \frac{d}{dt}\int_{t_{ref}}^{t} \dot{\gamma}_{21}(t')dt'$$

$$= \int_{t_{ref}}^{t} \tfrac{\partial}{\partial t}\left(\dot{\gamma}_{21}(t')\right)dt' + \dot{\gamma}_{21}(t)\tfrac{d(t)}{dt} - \dot{\gamma}_{21}(t_{ref})\tfrac{d(t_{ref})}{dt}$$

$$\frac{d\gamma_{21}}{dt} = \dot{\gamma}_{21}(t)$$

Deformation rate

Now we can continue with material functions based on strain.

17

Because shear rate is not prescribed, it becomes something we must measure.

Creep Shear Flow Material Functions

Kinematics:

It is unusual to prescribe stress rather than $\dot{\varsigma}(t)$

$$\underline{v} \equiv \begin{pmatrix} \dot{\gamma}_{21}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \tau_{21}(t) = \begin{cases} 0 & t < 0 \\ \tau_0 & t \geq 0 \end{cases}$$

Material Functions:

Since we ***set*** the stress in this experiment (rather than measuring it), the material functions are related to the ***deformation*** of the sample..

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Creep Shear Flow Material Functions

Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\gamma}_{21}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \tau_{21}(t) = \begin{cases} 0 & t < 0 \\ \tau_0 & 0 \le t \le t_2 \\ 0 & t > t_2 \end{cases}$$

Material Functions:

$$\underset{(t_2 \to \infty)}{J(t, \tau_0)} \equiv \frac{\gamma_{21}(0,t)}{-\tau_0}$$

Shear creep compliance

$$\underset{(\tilde{t} = t - t_2,\, t \ge t_2)}{J_r(\tilde{t}, \tau_0)} = R(\tilde{t}, \tau_0) \equiv \frac{\gamma_r(\tilde{t})}{-\tau_0}$$

$$\gamma_r(\tilde{t}) = \gamma_{21}(0, t_2) - \gamma_{21}(0, t)$$

Recoverable creep compliance

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Creep Recovery

-After creep, stop pulling forward and allow the flow to reverse

-In linear-viscoelastic materials, we can calculate the recovery material function from creep measurements

$$\gamma_r(\tilde{t}) = \gamma_{21}(0, t_2) - \gamma_{21}(0, t)$$

Recoverable strain Recoil strain	Strain at the end of the forward motion	Strain at the end of the recovery

$$J_r(\tilde{t}, \tau_0) \equiv \frac{\gamma_r(\tilde{t})}{-\tau_0}$$

Recoverable creep compliance

20

Material functions predicted for *creep* of a Newtonian fluid

Newtonian: $\underline{\underline{\tau}}(t) = -\mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$

Shear creep compliance $\underset{(t_2 \to \infty)}{J(t,\tau_0)} = ?$

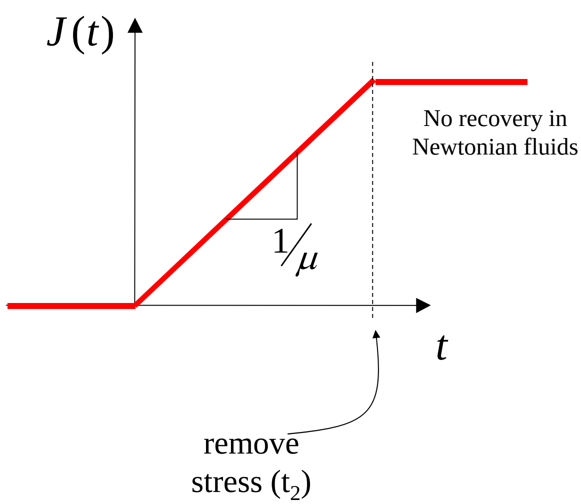
Recoverable creep compliance $J_r(\tilde{t},\tau_0) = ?$

21

Material functions predicted for *creep* of a Newtonian fluid

$J(t) = \left(\frac{1}{\mu}\right)t$

$J_r(\tilde{t}) = 0$



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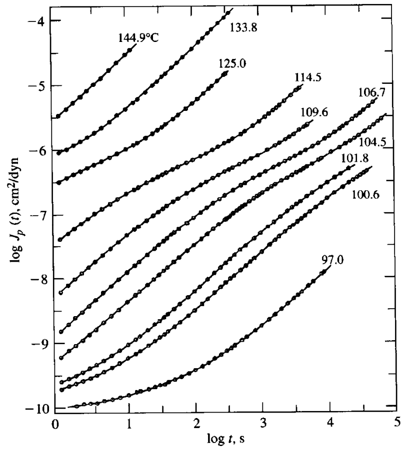
Shear Creep of a Viscoelastic liquid

$$\underline{v} \equiv \begin{pmatrix} \dot{\gamma}_{21}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\tau_{21}(t) = \begin{cases} 0 & t < 0 \\ \tau_0 & t \geq 0 \end{cases}$$

$$J_p = \frac{J(T) T \rho}{T_{ref} \rho_{ref}}$$

Data have been corrected for vertical shift.



$$J(t, \tau_0) \equiv \frac{\gamma_{21}(0,t)}{-\tau_0}$$

Figure 6.53, p. 210
Plazek; PS melt

23

Characteristics of a Creep Curve

•At long times the creep compliance $J(t, \tau_0)$ becomes a straight line (steady flow).

$$\left.\frac{dJ}{dt}\right|_{\substack{steady\\state}} = \frac{d\gamma_{21}}{dt}\left(\frac{1}{-\tau_0}\right)$$

$$= \frac{\dot{\gamma}_{t\to\infty}}{-\tau_0}$$

$$= \frac{1}{\eta(\dot{\gamma}_{t\to\infty})}$$

The slope at steady state is the inverse of the steady state viscosity

•We can define a steady-state compliance

$$\Rightarrow J(t)\Big|_{\substack{steady\\state}} = \frac{1}{\eta(\dot{\gamma}_{t\to\infty})} t + C$$

$J_s(\tau_0)$

Steady-state compliance

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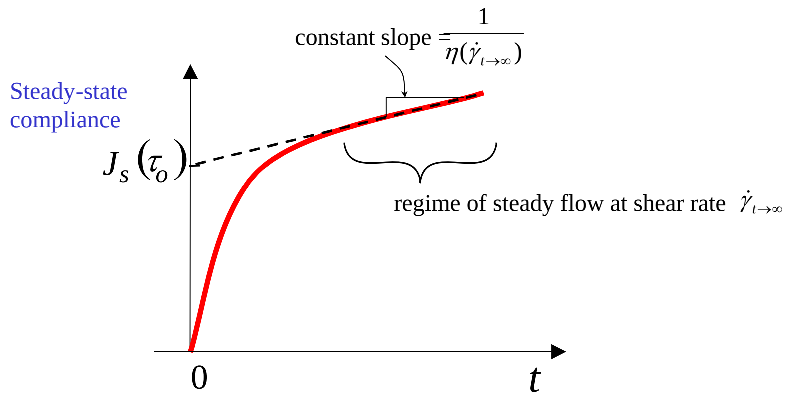
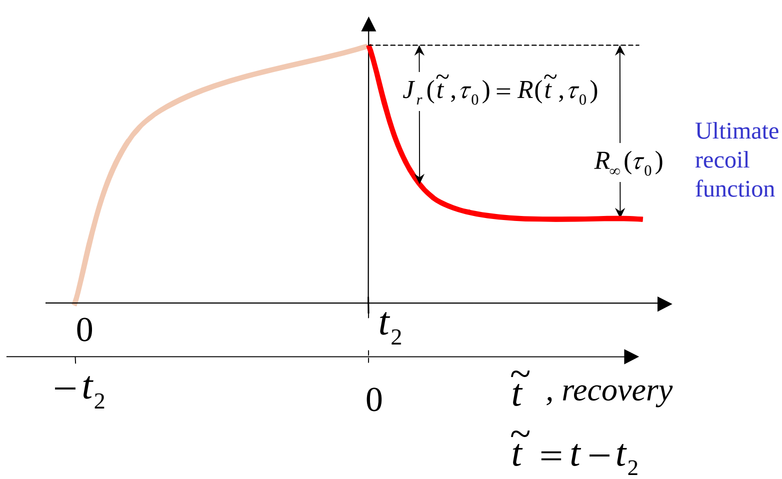
Shear creep material functions

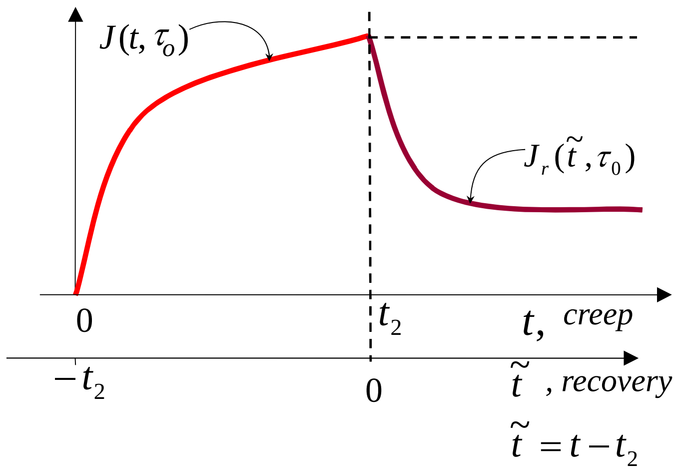


Characteristics of a Creep Recovery Curve

•At long recoil times we can define an ultimate recoil function $R_\infty(\tau_0)$



In the Linear Viscoelastic (LVE) Limit, it is easy to relate the two shear creep material functions



27

Linear Viscoelastic Creep (no dependence on τ_0)

total strain = recoverable strain + non-recoverable strain

$$\gamma(t) = \gamma_r(t) + t\dot{\gamma}_{t\to\infty}$$

$$\frac{\gamma(t)}{-\tau_0} = \frac{\gamma_r(t)}{-\tau_0} + t\left(\frac{\dot{\gamma}_0}{-\tau_0}\right)$$

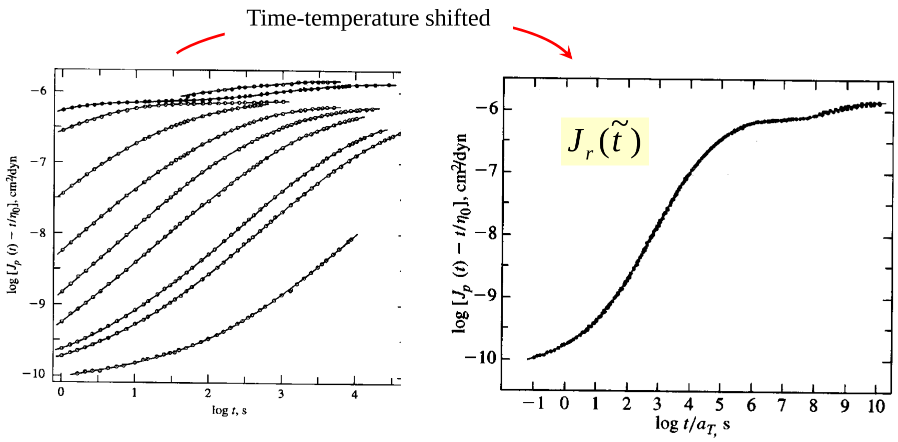
$$J(t) = J_r(t) + \frac{t}{\eta_0}$$

$$J_r(t) = J(t) - \frac{t}{\eta_0}$$

For LVE materials, we can obtain R(t) without a *recovery* experiment

28

Shear Creep - Recoverable Compliance

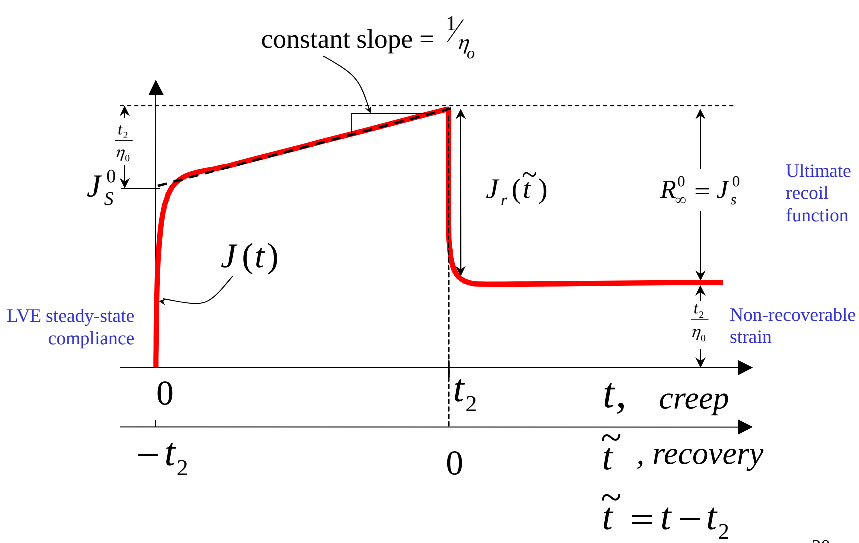


Figures 6.54, 6.55, p. 211
Plazek; PS melt

29

Shear creep material functions

Linear-viscoelastic (LVE) limit



30

Step Shear Strain Material Functions

Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\varsigma}(t) = \lim_{\varepsilon \to 0} \begin{cases} 0 & t < 0 \\ \dot{\gamma}_0 & 0 \le t < \varepsilon \\ 0 & t \ge \varepsilon \end{cases}$$

$$\dot{\gamma}_0 \varepsilon = \text{constant} = \gamma_0$$

Material Functions:

$$G(t,\gamma_0) \equiv \frac{-\tau_{21}(t,\gamma_0)}{\gamma_0}$$

Relaxation modulus

First normal-stress relaxation modulus

$$G_{\Psi_1} \equiv \frac{-(\tau_{11} - \tau_{22})}{\gamma_0^2}$$

Second normal-stress relaxation modulus

$$G_{\Psi_2} \equiv \frac{-(\tau_{22} - \tau_{33})}{\gamma_0^2}$$

31

What is the strain in this flow?

$$\gamma_{21}(-\infty, t) = \int_{-\infty}^{t} \dot{\gamma}_{21}(t')\, dt'$$

$$= \int_{-\infty}^{t} \lim_{\varepsilon \to 0} \begin{cases} 0 & t' < 0 \\ \frac{\gamma_0}{\varepsilon} & 0 \le t' < \varepsilon \\ 0 & t \ge \varepsilon \end{cases} dt'$$

$$= \lim_{\varepsilon \to 0} \int_0^{\varepsilon} \frac{\gamma_0}{\varepsilon}\, dt'$$

$$= \gamma_0$$

The strain imposed is a constant

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Step shear strain - *strain dependence*

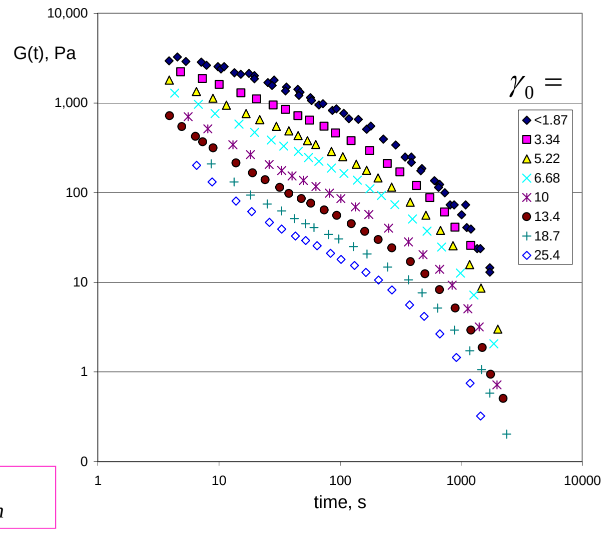

Figure 6.57, p. 212
Einaga et al.; PS soln

33

Linear viscoelastic limit

$$\lim_{\gamma_0 \to 0} G(t, \gamma_0) = G(t)$$

At small strains the relaxation modulus is independent of strain.

The polystyrene solutions on the previous slide show time-strain independence, i.e. the curves have the same shape at different strains.

Damping function, *h*

$$h(\gamma_0) \equiv \frac{G(t, \gamma_0)}{G(t)}$$

The damping function summarizes the non-linear effects as a function of strain amplitude.

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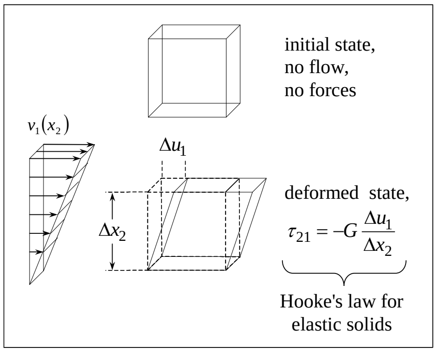
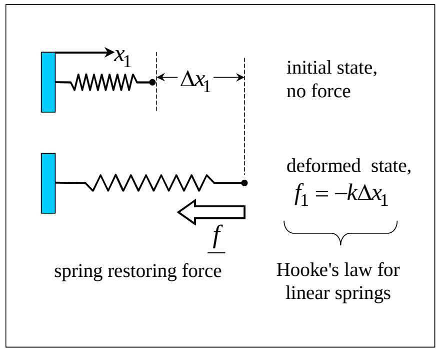
What types of materials generate stress in proportion to the strain imposed? Answer: elastic solids

Hooke's Law for elastic solids $\tau_{21} = -G\gamma_{21}$





Similar to the linear spring law

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Small-Amplitude Oscillatory Shear Material Functions

Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\varsigma}(t) = \dot{\gamma}_0 \cos \omega t$$

$$\gamma_0 \equiv \frac{\dot{\gamma}_0}{\omega}$$

Material Functions:

$$\frac{-\tau_{21}(t,\gamma_0)}{\gamma_0} = G' \sin \omega t + G'' \cos \omega t$$

$$G'(\omega) \equiv \frac{\tau_0}{\gamma_0}\cos\delta$$

Storage modulus

(δ is the phase difference between stress and strain waves)

$$G''(\omega) \equiv \frac{\tau_0}{\gamma_0}\sin\delta$$

Loss modulus

36

What is the strain in this flow?

$$\gamma_{21}(0,t) = \int_0^t \dot{\gamma}_{21}(t')dt'$$

$$= \int_0^t \dot{\gamma}_0 \cos \omega t' \, dt'$$

$$= \frac{\dot{\gamma}_0}{\omega} \sin \omega t$$

The strain imposed is sinusoidal.

The strain amplitude is $\gamma_0 = \frac{\dot{\gamma}_0}{\omega}$

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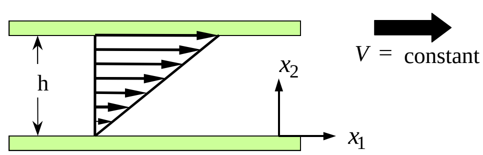
Generating Small Amplitude Oscillatory Shear (SAOS)

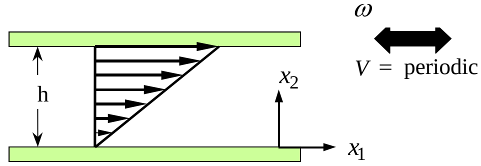
steady shear

$b(t) = Vt = h\dot{\gamma}_0 t$



small-amplitude oscillatory shear

$$b(t) = h\gamma_0 \sin \omega t = \frac{h\dot{\gamma}_0}{\omega} \sin \omega t$$



38

In SAOS the strain amplitude is small, and a sinusoidal imposed strain induces a sinusoidal measured stress.

$$-\tau_{21}(t) = \tau_0 \sin(\omega t + \delta)$$

$$\begin{aligned} -\tau_{21}(t) &= \tau_0 \sin(\omega t + \delta) \\ &= \tau_0 \sin \omega t \cos \delta + \tau_0 \cos \omega t \sin \delta \\ &= \underbrace{[\tau_0 \cos \delta]}_{\text{portion in-phase with strain}} \sin \omega t + \underbrace{[\tau_0 \sin \delta]}_{\text{portion in-phase with strain-rate}} \cos \omega t \end{aligned}$$

portion in-phase with strain

portion in-phase with strain-rate

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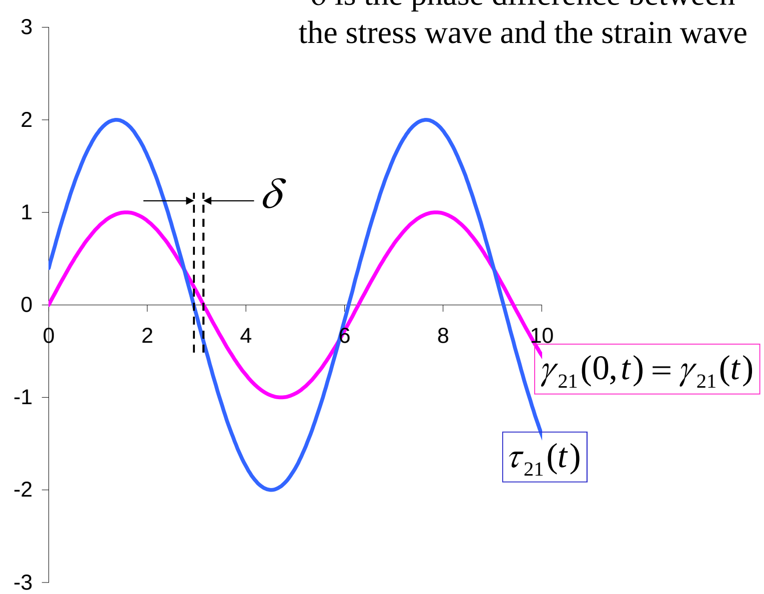
δ is the phase difference between the stress wave and the strain wave



40

SAOS Material Functions

$$\frac{-\tau_{21}(t)}{\gamma_0} = \left[\frac{\tau_0 \cos\delta}{\gamma_0}\right]\sin\omega t + \left[\frac{\tau_0 \sin\delta}{\gamma_0}\right]\cos\omega t$$

portion in-phase with strain — G'

portion in-phase with strain-rate — G''

For Newtonian fluids, stress is proportional to strain rate: $\tau_{21} = -\mu\dot{\gamma}_{21}$

G'' is thus known as the viscous loss modulus. It characterizes the viscous contribution to the stress response.

41

SAOS Material Functions

$$\frac{-\tau_{21}(t)}{\gamma_0} = \left[\frac{\tau_0 \cos\delta}{\gamma_0}\right]\sin\omega t + \left[\frac{\tau_0 \sin\delta}{\gamma_0}\right]\cos\omega t$$

portion in-phase with strain — G'

portion in-phase with strain-rate — G''

For Hookean solids, stress is proportional to strain : $\tau_{21} = -G\gamma_{21}$

G' is thus known as the elastic storage modulus. It characterizes the elastic contribution to the stress response.

(note: SAOS material functions may also be expressed in complex notation. See pp. 156-159 of Morrison, 2001)

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SAOS Moduli of a Polymer Melt

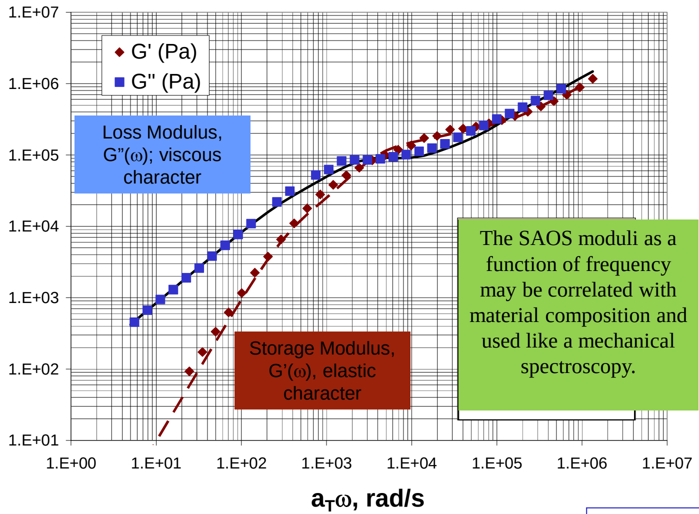
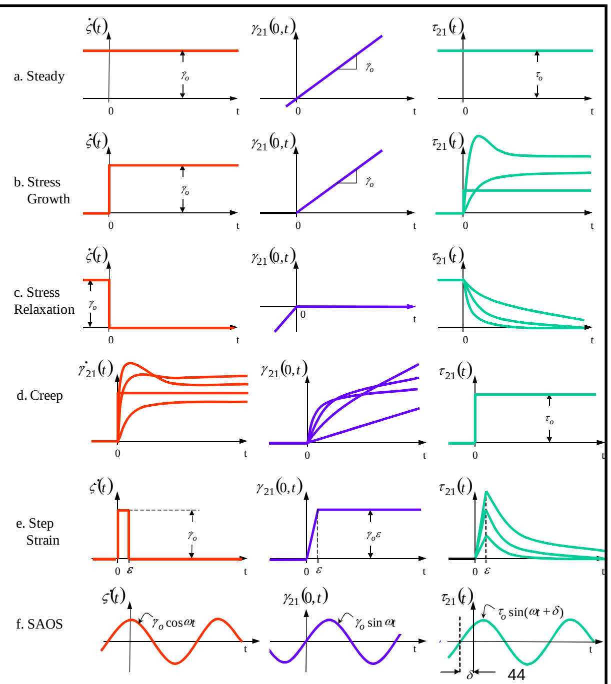


Figure 8.8, p. 284 data from Vinogradov, PS melt

We have discussed six shear material functions;

Now, the equivalent elongational material functions



Steady Elongational Flow Material Functions

Kinematics:

$$\underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)(1+b)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)(1-b)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

$$\dot{\varepsilon}(t) = \dot{\varepsilon}_0 = \text{constant}$$

Elongational flow: b=0, $\dot{\varepsilon}(t) > 0$

Biaxial stretching: b=0, $\dot{\varepsilon}(t) < 0$

Planar elongation: b=1, $\dot{\varepsilon}(t) > 0$

Material Functions:

$$\bar{\eta} \text{ or } \bar{\eta}_B \text{ or } \bar{\eta}_{P_1} \equiv \frac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$$

Uniaxial or Biaxial or First Planar Elongational Viscosity

$$\bar{\eta}_{P_2} \equiv \frac{-(\tau_{22} - \tau_{11})}{\dot{\varepsilon}_0}$$

Second Planar Elongational Viscosity

45

What is the strain in this flow?

(to answer, review how strain was developed/defined for previous flows. . .)

How did we do that before . . ?

Recall, for shear . . .

Deformation (strain)

$$\underline{r}(t_{ref}) = \begin{pmatrix} x_1(t_{ref}) \\ x_2(t_{ref}) \\ x_3(t_{ref}) \end{pmatrix}_{123}$$

$$\underline{r}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}_{123}$$

This vector keeps track of the location of a fluid particle as a function of time.

$$\underline{u}(t_{ref},t) \equiv \underline{r}(t) - \underline{r}(t_{ref})$$

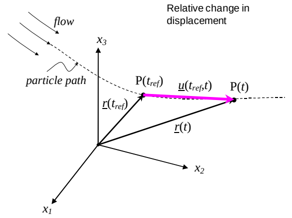
Displacement function

Current position compared to reference position

$$\gamma_{21}(t_{ref},t) \equiv \frac{\partial u_1}{\partial x_2}$$

Shear strain

Relative change in displacement



$$\underline{r}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}_{123} = \begin{pmatrix} x_1(t_{ref}) + (t - t_{ref})\dot{\gamma}_0 x_2 \\ x_2(t_{ref}) \\ x_3(t_{ref}) \end{pmatrix}_{123}$$

$$\underline{u}(t_{ref},t) \equiv \underline{r}(t) - \underline{r}(t_{ref}) = \begin{pmatrix} (t - t_{ref})\dot{\gamma}_0 x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

Displacement function

Our choice for measuring change in shape:

$$\gamma_{21}(t_{ref},t) \equiv \frac{\partial u_1}{\partial x_2}$$

Shear strain

$$\gamma_{21}(t_{ref},t) = (t - t_{ref})\dot{\gamma}_0$$

(for steady shear or in unsteady shear for short time intervals)

Path to strain for shear:

$$\underline{r}(t_{ref}), \underline{r}(t) \rightarrow \quad \underline{u} \rightarrow \quad \nabla \underline{u} \rightarrow \quad \underline{\underline{\gamma}}(t_{ref}, t)$$

Try to follow for elongation.

$$\underline{r}(t_{ref}) = \begin{pmatrix} x_1(t_{ref}) \\ x_2(t_{ref}) \\ x_3(t_{ref}) \end{pmatrix}_{123} \qquad \underline{r}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}_{123} \quad = ?$$

$$\underline{u}(t_{ref},t) \equiv \underline{r}(t) - \underline{r}(t_{ref}) = ?$$

$$\underline{\underline{\gamma}} = \nabla \underline{u} + (\nabla \underline{u})^T = ?$$

Shear	Elongation	
$v_1 = \dot{\gamma}_0 x_2$	$v_3 = \dot{\varepsilon}_0 x_3$	$\frac{\partial v}{\partial x} = constant$
$\frac{dx_1}{dt} = \dot{\gamma}_0 x_2$	$\frac{dx_3}{dt} = \dot{\varepsilon}_0 x_3$	
$dx_1 = \dot{\gamma}_0 x_2 dt$	$\frac{dx_3}{x_3} = \dot{\varepsilon}_0 dt$	
$x_1 = x_{1,0} + \dot{\gamma}_0 \Delta t \; x_2$		
$\frac{\partial(x_1 - x_{1,0})}{\partial x_2} = \dot{\gamma}_0 \Delta t$	$\ln \frac{x_3}{x_{3,0}} = \dot{\varepsilon}_0 \Delta t$	Piece of deformation over time interval Δt

Notes:

•The way we quantified deformation for shear, du_1/dx_2, is not so appropriate for elongation.

•Velocity gradient constant in both flows (but not the same gradient)

•(Velocity gradient) (Δt) is a measure of deformation that accumulates linearly with flow

Press on:

$$\text{strain} = \int \begin{matrix} velocity \\ gradient \end{matrix} \; dt$$

homogeneous flows
(velocity gradient the same everywhere in the flow)

Shear: $$\gamma_{21}(t_1, t_2) = \int_{t_1}^{t_2} \dot{\gamma}_{21}(t')dt'$$

Elongation: $$\varepsilon(t_1, t_2) = \int_{t_1}^{t_2} \dot{\varepsilon}(t')dt'$$

Note:

Need a better definition of strain for the general case

Hencky strain

$$\varepsilon(t_{ref},t) = \int_{t_{ref}}^{t} \dot{\varepsilon}(t')dt'$$

(choose $t_{ref}=0$)

$$= \dot{\varepsilon}_0 t$$

The strain imposed is proportional to time.

$$= \ln \frac{l}{l_0}$$

The ratio of current length to initial length is exponential in time.

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What does the **Newtonian** Fluid model predict in uniaxial steady elongational flow?

$$\underline{\underline{\tau}} = -\mu \dot{\underline{\underline{\gamma}}} = -\mu\left[\nabla \underline{v} + \left(\nabla \underline{v}\right)^T\right]$$

Again, since we know $\underline{\boldsymbol{v}}$, we can just plug it in to the constitutive equation and calculate the stresses.

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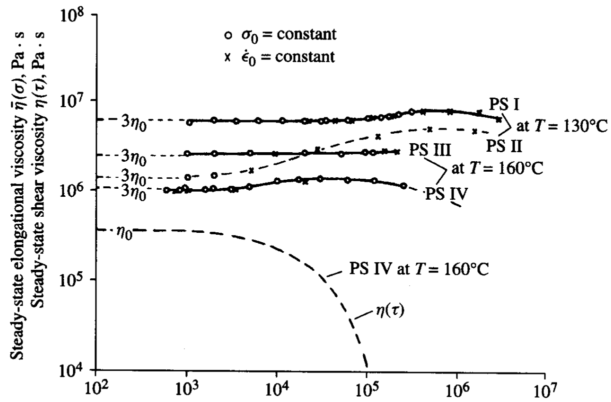
Steady State Elongation Viscosity



Both tension thinning and thickening are observed.

Figure 6.60, p. 215
Munstedt.; PS melt

Trouton ratio: $Tr \equiv \dfrac{\bar{\eta}}{\eta_0}$

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What does the model we guessed at predict for steady uniaxial elongational flow?

$$\underline{\underline{\tau}} = -M\left(\dot{\gamma}_0\right)\left[\nabla \underline{v} + \left(\nabla \underline{v}\right)^T\right]$$

$$M\left(\dot{\gamma}_0\right) = \begin{cases} M_0 & \dot{\gamma}_0 < \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 \geq \dot{\gamma}_c \end{cases}$$

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What if we make the following replacement?

$$\dot{\gamma}_0 \rightarrow \frac{\partial v_1}{\partial x_2}$$

This at least can be written for any flow and it is equal to the shear rate in shear flow.

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$$\underline{\underline{\tau}} = -M(\dot{\gamma}_0)\left[\nabla \underline{v} + (\nabla \underline{v})^T\right]$$

Observations

$$M(\dot{\gamma}_0) = \begin{cases} M_0 & \dot{\gamma}_0 < \dot{\gamma}_c \\ m\dot{\gamma}_0^{n-1} & \dot{\gamma}_0 \ge \dot{\gamma}_c \end{cases}$$

•The model contains parameters that are specific to shear flow – makes it impossible to adapt for elongational or mixed flows

•Also, the model should only contain quantities that are independent of coordinate system (i.e. invariant)

We will try to salvage the model by replacing the flow-specific kinetic parameter with something that is frame-invariant and not flow-specific.

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We will take out the shear rate and replace with the magnitude of the rate-of-deformation tensor (which is related to the second invariant of that tensor).

$$\underline{\underline{\tau}} = -M\left(\left|\dot{\underline{\underline{\gamma}}}\right|\right)\left[\nabla \underline{v} + (\nabla \underline{v})^T\right]$$

$$M\left(\left|\dot{\underline{\underline{\gamma}}}\right|\right) = \begin{cases} M_0 & \left|\dot{\underline{\underline{\gamma}}}\right| < \dot{\gamma}_c \\ m\left|\dot{\underline{\underline{\gamma}}}\right|^{n-1} & \left|\dot{\underline{\underline{\gamma}}}\right| \geq \dot{\gamma}_c \end{cases}$$

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The other elongational experiments are analogous to shear experiments *(see text)*

Elongational stress growth

Elongational stress cessation *(nearly impossible)*

Elongational creep

Step elongational strain

Small-amplitude Oscillatory Elongation (SAOE)

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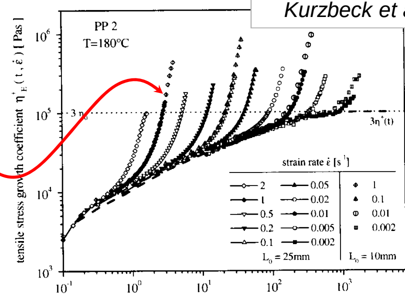
Start-up of Steady Elongation

Figure 6.64, p. 218
Kurzbeck et al.; PP



Strain-hardening

Fit to an advanced constitutive equation (12 mode pom-pom model)

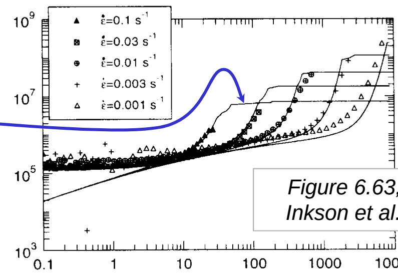


Figure 6.63, p. 217
Inkson et al.; LDPE

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What's next?

✓Underlying physics (mass, momentum balances, stress tensor)

✓Standard flows

✓Material functions

?

- Constitutive equations
- Model flows/solve engineering problems

We want to design constitutive equations based on the material behavior of real non-Newtonian fluids.

What is the behavior of non-Newtonian fluids?

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