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Predict G', G'' for GMM

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} \left\{ \sum_{k=1}^{N} \frac{\eta_k}{\lambda_k} e^{-(t-t')/\lambda_k} \right\} \underline{\underline{\dot\gamma}}(t')\, dt'$$

$$\underline{\underline{\dot\gamma}} = \nabla \underline{v} + (\nabla \underline{v})^T = \begin{pmatrix} 0 & \frac{\partial v_1}{\partial x_2} & 0 \\ \frac{\partial v_1}{\partial x_2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

$$= \begin{pmatrix} 0 & \dot\gamma_0 \cos\omega t & 0 \\ \dot\gamma_0 \cos\omega t & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

$$\tau_{21}(t) = \tau_{12}(t) = -\int_{-\infty}^{t} \sum_{k=1}^{N} \frac{\eta_k}{\lambda_k} e^{-\frac{(t-t')}{\lambda_k}} \dot\gamma_0 \cos\omega t'\, dt'$$

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$$-\tau_{21}(t) = \sum_{k=1}^{N} \frac{\eta_k \dot\gamma_0}{\lambda_k} \int_{-\infty}^{t} e^{-\frac{(t-t')}{\lambda_k}} \cos\omega t'\, dt'$$

$$I = \int_{-\infty}^{t} e^{-\frac{(t-t')}{\lambda}} \cos\omega t'\, dt'$$

$$\int u\,dv = uv - \int v\,du$$

$$u = e^{-\frac{(t-t')}{\lambda_k}} \qquad dv = \tfrac{1}{\omega}\cos\omega t' (dt'\omega)$$

$$du = +\frac{1}{\lambda_k} e^{-\frac{(t-t')}{\lambda_k}} dt' \qquad v = \frac{1}{\omega}\sin\omega t'$$

$$\frac{1}{\omega} e^{-(t-t')/\lambda_k} \sin\omega t' \Big|_{-\infty}^{t} - \frac{1}{\lambda_k}\int_{-\infty}^{t} \frac{1}{\omega}\sin\omega t'\, e^{-(t-t')/\lambda_k}$$

$$= \frac{1}{\omega}\sin\omega t - 0 - \frac{1}{\omega\lambda_k}\int \ldots\, dt'$$

What is this ?

$$II = \int_{-\infty}^{t} \sin \omega t' \, e^{-(t-t')/\lambda_k} \, dt'$$

$$u = e^{-(t-t')/\lambda_k} \qquad dv = \frac{1}{\omega} \sin \omega t' (dt' \omega)$$

$$du = \frac{1}{\lambda_k} e^{-\frac{(t-t')}{\lambda_k}} \qquad v = -\frac{1}{\omega} \cos \omega t'$$

$$= e^{-\frac{(t-t')}{\lambda_k}} \left(-\frac{1}{\omega}\right) \cos \omega t' \Big|_{-\infty}^{t} + \frac{1}{\lambda_k \omega} \int_{-\infty}^{t} \cos \omega t' \, e^{-\frac{(t-t')}{\lambda_k}} dt'$$

$$= \left(-\frac{1}{\omega}\right) \cos \omega t + 0 + \frac{1}{\lambda_k \omega} I$$

SUBSTITUTE BACK + SOLVE FOR I

$$I = \frac{1}{\omega} \sin \omega t - \frac{1}{\lambda_k \omega} II$$

$$= \frac{1}{\omega} \sin \omega t - \frac{1}{\lambda_k \omega} \left[-\frac{1}{\omega} \cos \omega t + \frac{1}{\lambda_k \omega} I \right]$$

$\boxed{\text{SOLVE FOR } I}$

$$I + \frac{1}{\lambda_k^2 \omega^2} I = \frac{1}{\omega} \sin \omega t + \frac{1}{\lambda_k \omega^2} \cos \omega t$$

$$\left(1 + \frac{1}{\lambda_k^2 \omega^2}\right) I = \frac{1}{\omega} \sin \omega t + \frac{1}{\lambda_k \omega^2} \cos \omega t$$

$$I = \frac{\frac{1}{\omega} (\lambda_k^2 \omega^2)}{\lambda_k^2 \omega^2 + 1} \sin \omega t + \frac{\frac{1}{\lambda_k \omega^2} (\lambda_k^2 \omega^2)}{\lambda_k^2 \omega^2 + 1} \cos \omega t$$

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$$-\tau_{21}(t) = \sum_{k=1}^{N} \frac{\eta_k \dot{\gamma}_0}{\lambda_k} \quad I$$

$$= \sum_{k=1}^{N} \frac{\eta_k \dot{\gamma}_0}{\lambda_k} \left[\frac{\lambda_k^2 \omega}{1+\omega^2\lambda_k^2} \sin\omega t + \frac{\lambda_k}{1+\lambda_k^2\omega^2} \cos\omega t \right]$$

$$-\frac{\tau_{21}(t)}{\gamma_0} = \frac{\omega}{\cancel{\dot{\gamma}_0}} \sum_{k=1}^{N} \frac{\eta_k \cancel{\dot{\gamma}_0}}{\cancel{\lambda_k}} \frac{\lambda_k^{\cancel{2}} \omega}{1+\omega^2\lambda_k^2} \sin\omega t$$

$$+ \frac{\omega}{\cancel{\dot{\gamma}_0}} \sum_{k=1}^{N} \frac{\eta_k \cancel{\dot{\gamma}_0}}{\cancel{\lambda_k}} \frac{\cancel{\lambda_k}}{1+\lambda_k^2\omega^2} \cos\omega t$$

$$\boxed{G' = \sum_{k=1}^{N} \eta_k \frac{\omega^2 \lambda_k}{1+\omega^2\lambda_k^2} \qquad G'' = \sum_{k=1}^{N} \frac{\omega \eta_k}{1+\lambda_k^2\omega^2}}$$