# Exam 1 Formulas

Polymer Rheology Prof. Faith Morrison

---

Rate of deformation tensor: $\underline{\underline{\dot{\gamma}}} = \nabla \underline{v} + (\nabla \underline{v})^T$

Rate of deformation: $\dot{\gamma} = \left|\underline{\underline{\dot{\gamma}}}\right|$

Tensor magnitude: $A = \left|\underline{\underline{A}}\right| = +\sqrt{\frac{\underline{\underline{A}}:\underline{\underline{A}}}{2}}$

Total stress tensor: $\underline{\underline{\Pi}} = p\underline{\underline{I}} + \underline{\underline{\tau}}$
(Bird sign convention on stress)

---

Navier-Stokes Equation: $\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$

Cauchy Momentum Equation: $\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$
(Bird sign convention on stress)

Continuity Equation: $\frac{\partial \rho}{\partial t} = -\nabla \cdot \left(\rho \underline{v}\right)$

Newtonian, incompressible constitutive equation: $\underline{\underline{\tau}} = -\mu(\nabla \underline{v} + (\nabla \underline{v})^T)$
(Bird sign convention on stress)

Fluid force $\underline{F}_{on}$ on a surface S:
(Bird sign convention on stress)

$$\underline{F}_{on} = \iint_S \left[\hat{n} \cdot -\underline{\underline{\Pi}}\right]\Big|_{surface} dS$$

Flow rate $Q$ through a surface S:

$$Q = \iint_S \left[\hat{n} \cdot \underline{v}\right]_{surface} dS$$

Fluid torque $\underline{T}_{on}$ on a surface S: ($\underline{\tilde{R}}$ is the lever arm vector from the axis of rotation to the point of application of the force)
(Bird sign convention on stress)

$$\underline{T}_{on} = \iint_S \left[\underline{\tilde{R}} \times (\hat{n} \cdot -\underline{\underline{\Pi}})\right]_{surface} dS$$

---

## Table of Integrals

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \text{ , } n \text{ is a constant}$$

$$\int \frac{1}{y} du = \ln u + C$$

$$\int \ln u \, du = u \ln u - u + C$$

$$\int_a^b u \, dv \; = uv\Big|_a^b - \int_a^b v \, du$$

---

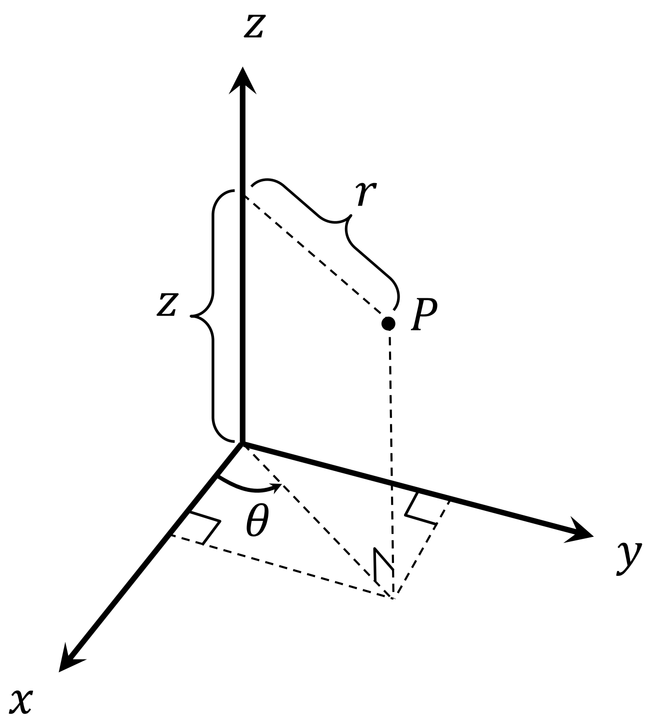
## Miscellaneous

$$\underline{\underline{A}} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}_{123}$$

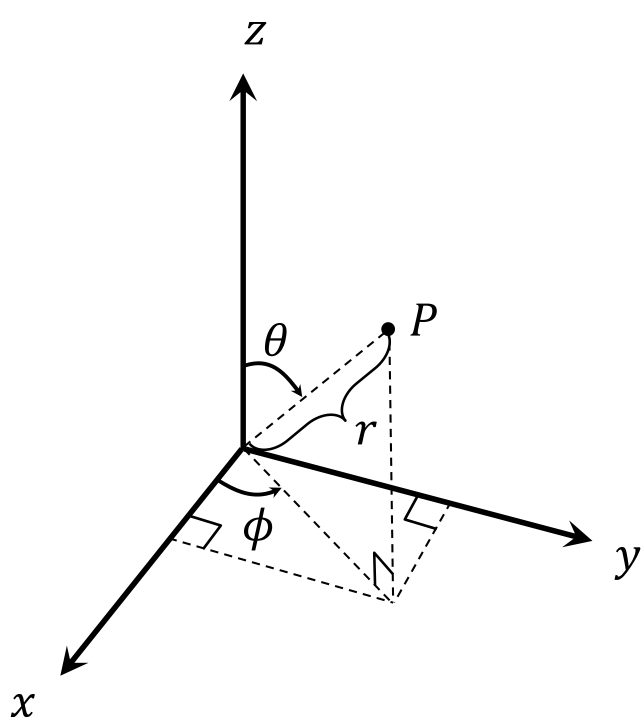
$$\frac{d}{ds}(uw) = u\frac{dw}{ds} + w\frac{du}{ds}$$

$$\underline{u} \times \underline{w} = \det \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \begin{pmatrix} u_2 w_3 - u_3 w_2 \\ -(u_1 w_3 - u_3 w_1) \\ u_1 w_2 - u_2 w_1 \end{pmatrix}_{123}$$

**Cylindrical Coordinate System:** Note that the $\theta$-coordinate swings around the $z$-axis and the $r$-coordinate is perpendicular to the z-axis.




**Spherical Coordinate System:** Note that the $\theta$-coordinate swings down from the $z$-axis and the $r$-coordinate emits radially from the origin; these are different from their definitions in the cylindrical system above.



| System | Coordinates | Basis vectors |
|---|---|---|
| Spherical | $x = r\sin\theta\cos\phi$ | $\hat{e}_r = (\sin\theta\cos\phi)\hat{e}_x + (\sin\theta\sin\phi)\hat{e}_y + \cos\theta\hat{e}_z$ |
| Spherical | $y = r\sin\theta\sin\phi$ | $\hat{e}_\theta = (\cos\theta\cos\phi)\hat{e}_x + (\cos\theta\sin\phi)\hat{e}_y + (-\sin\theta)\hat{e}_z$ |
| Spherical | $z = r\cos\theta$ | $\hat{e}_\phi = (-\sin\phi)\hat{e}_x + \cos\phi\,\hat{e}_y$ |
| Cylindrical | $x = r\cos\theta$ | $\hat{e}_r = \cos\theta\,\hat{e}_x + \sin\theta\,\hat{e}_y$ |
| Cylindrical | $y = r\sin\theta$ | $\hat{e}_\theta = (-\sin\theta)\hat{e}_x + \cos\theta\,\hat{e}_y$ |
| Cylindrical | $z = z$ | $\hat{e}_z = \hat{e}_z$ |

| Coordinate system | surface differential $dS$ |
|---|---|
| Cartesian (top, $\hat{n} = \hat{e}_z$) | $dS = dxdy$ |
| Cartesian (side a, $\hat{n} = \hat{e}_y$) | $dS = dxdz$ |
| Cartesian (side b, $\hat{n} = \hat{e}_x$) | $dS = dydz$ |
| cylindrical (top, $\hat{n} = \hat{e}_z$) | $dS = rdrd\theta$ |
| cylindrical (side, $\hat{n} = \hat{e}_r$) | $dS = Rd\theta dz$ |
| spherical, ($\hat{n} = \hat{e}_r$) | $dS = R^2\sin\theta d\theta d\phi$ |

| Coordinate system | volume differential $dV$ |
|---|---|
| Cartesian | $dV = dxdydz$ |
| cylindrical | $dV = rdrd\theta dz$ |
| spherical | $dV = r^2\sin\theta drd\theta d\phi$ |