Exam 2 Formulas

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Rate of deformation tensor: $\underline{\underline{\dot{\boldsymbol{\gamma}}}} = \nabla \underline{\boldsymbol{v}} + (\nabla \underline{\boldsymbol{v}})^T$

Rate of deformation: $\dot{\boldsymbol{\gamma}} = \left|\underline{\underline{\dot{\boldsymbol{\gamma}}}}\right|$

Tensor magnitude: $\boldsymbol{A} = \left|\underline{\underline{\boldsymbol{A}}}\right| = +\sqrt{\frac{\underline{\underline{\boldsymbol{A}}}:\underline{\underline{\boldsymbol{A}}}}{2}}$

Shear strain: $\boldsymbol{\gamma_{21}(t_a, t_b) = \int_{t_a}^{t_b} \dot{\gamma}_{21}(t'')dt''}$

Navier-Stokes Equation: $\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$

Cauchy Momentum Equation: $\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$

Continuity Equation: $\frac{\partial \rho}{\partial t} = -\nabla \cdot \left(\rho \underline{v}\right)$

Fluid force $\underline{\boldsymbol{F}}$ on a surface S:

$$\underline{\boldsymbol{F}} = \iint_S \left[\hat{\boldsymbol{n}} \cdot -\underline{\underline{\boldsymbol{\Pi}}}\right]\Big|_{surface} \boldsymbol{dA}$$

Flow rate $\boldsymbol{Q}$ through a surface S:

$$\boldsymbol{Q} = \iint_S \left[\hat{\boldsymbol{n}} \cdot \underline{\boldsymbol{v}}\right]_{surface} \boldsymbol{dA}$$

Fluid torque $\underline{\boldsymbol{T}}$ on a surface S: ($\underline{\boldsymbol{R}}$ is the vector from the axis of rotation to the point of application of the force)

$$\underline{\boldsymbol{T}} = \iint_S \left[\underline{\boldsymbol{R}} \times (\hat{\boldsymbol{n}} \cdot -\underline{\underline{\boldsymbol{\Pi}}})\right]_{surface} \boldsymbol{dA}$$

Newtonian, incompressible fluid: $\underline{\underline{\tau}} = -\mu(\nabla \underline{v} + (\nabla \underline{v})^T)$

Generalized Newtonian fluid (GNF): $\underline{\underline{\tau}} = -\eta(\dot{\gamma})\underline{\underline{\dot{\gamma}}}$

Power-law GNF model: $\eta(\dot{\gamma}) = m\dot{\gamma}^{n-1}$

(Note that m and n are parameters of the model and are constants)

Carreau-Yasuda GNF model: $\eta(\dot{\gamma}) = \eta_\infty + (\eta_0 - \eta_\infty)[1 + (\dot{\gamma}\lambda)^a]^{\frac{n-1}{a}}$

(Note that a, λ and n, η_0, and η_∞ are parameters of the model and are constants)

Elongational flow (uniaxial, biaxial): $\underline{v} = \begin{pmatrix} -\frac{\dot{\varepsilon}(t)}{2} x_1 \\ -\frac{\dot{\varepsilon}(t)}{2} x_2 \\ \dot{\varepsilon}(t) x_3 \end{pmatrix}_{123}$

Shear flow: $\underline{v} = \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$

Steady shearing kinematics: $\dot{\varsigma}(t) = \dot{\gamma}_0$ for all values of time t

Start-up of steady shearing kinematics: $\dot{\varsigma}(t) = \begin{cases} 0 & t < 0 \\ \dot{\gamma}_0 & t \geq 0 \end{cases}$

Cessation of steady shearing kinematics: $\dot{\varsigma}(t) = \begin{cases} \dot{\gamma}_0 & t < 0 \\ 0 & t \geq 0 \end{cases}$

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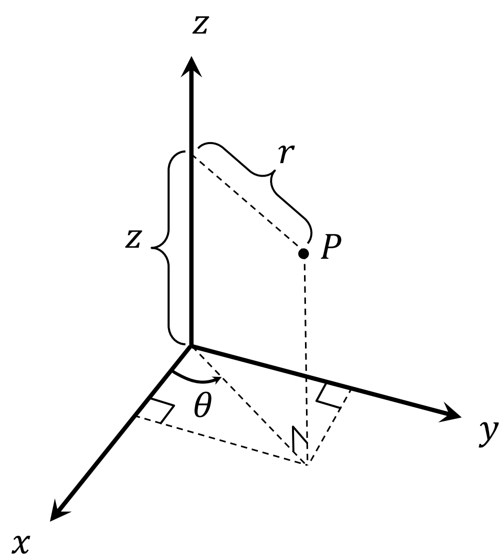
Shear viscosity: $\eta = \frac{-(\tau_{21})}{\dot{\gamma}_0}$

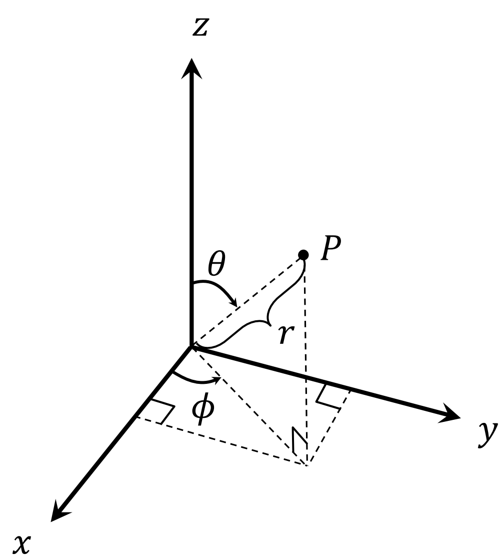
Shear normal stress coefficients: $\Psi_1 = \frac{-(\tau_{11} - \tau_{22})}{\dot{\gamma}_0^2}, \Psi_2 = \frac{-(\tau_{22} - \tau_{33})}{\dot{\gamma}_0}$

Elongational viscosity: $\bar{\eta} = \frac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$

Cylindrical Coordinate System: Note that the $\boldsymbol{\theta}$-coordinate swings around the $\boldsymbol{z}$-axis and the $\boldsymbol{r}$-coordinate is perpendicular to the z-axis.



Spherical Coordinate System: Note that the $\boldsymbol{\theta}$-coordinate swings down from the $\boldsymbol{z}$-axis and the $\boldsymbol{r}$-coordinate emits radially from the origin; these are different from their definitions in the cylindrical system above.



System	Coordinates	Basis vectors
Spherical	$\boldsymbol{x = r \sin\theta \cos\phi}$	$\boldsymbol{\hat{e}_r = (\sin\theta\cos\phi)\hat{e}_x + (\sin\theta\sin\phi)\hat{e}_y + \cos\theta\,\hat{e}_z}$
Spherical	$\boldsymbol{y = r \sin\theta \sin\phi}$	$\boldsymbol{\hat{e}_\theta = (\cos\theta\cos\phi)\hat{e}_x + (\cos\theta\sin\phi)\hat{e}_y + (-\sin\theta)\hat{e}_z}$
Spherical	$\boldsymbol{z = r\cos\theta}$	$\boldsymbol{\hat{e}_\phi = (-\sin\phi)\hat{e}_x + \cos\phi\,\hat{e}_y}$
Cylindrical	$\boldsymbol{x = r\cos\theta}$	$\boldsymbol{\hat{e}_r = \cos\theta\,\hat{e}_x + \sin\theta\,\hat{e}_y}$
Cylindrical	$\boldsymbol{y = r\sin\theta}$	$\boldsymbol{\hat{e}_\theta = (-\sin\theta)\hat{e}_x + \cos\theta\,\hat{e}_y}$
Cylindrical	$\boldsymbol{z = z}$	$\boldsymbol{\hat{e}_z = \hat{e}_z}$