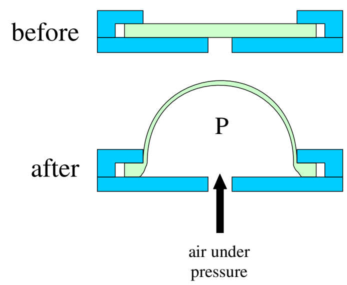
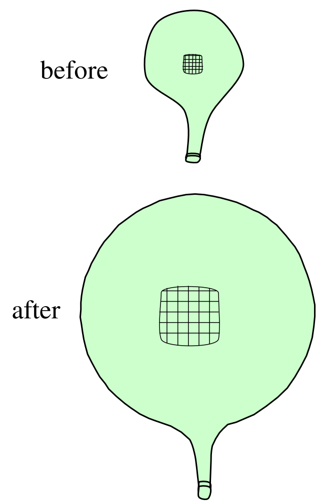
A second type of shear-free flow: **Biaxial Stretching**

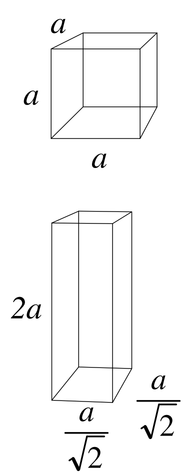


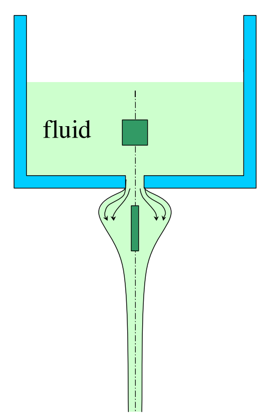
$$\underline{v} \equiv \begin{pmatrix} -\frac{\dot{\varepsilon}(t)}{2} x_1 \\ -\frac{\dot{\varepsilon}(t)}{2} x_2 \\ \dot{\varepsilon}(t) x_3 \end{pmatrix}_{123} \qquad \dot{\varepsilon}(t) < 0$$



How do uniaxial and biaxial deformations differ?

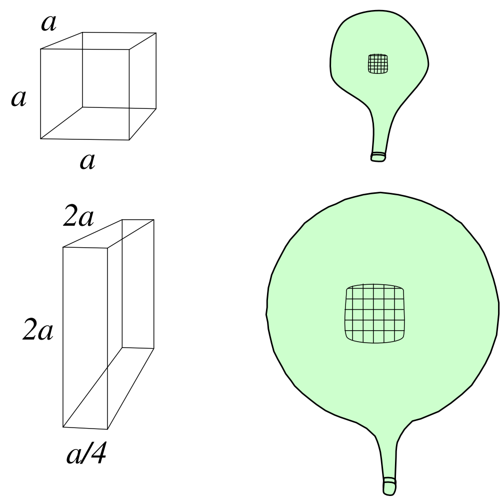
Consider a uniaxial flow in which a particle is doubled in length in the flow direction.





How do uniaxial and biaxial deformations differ?

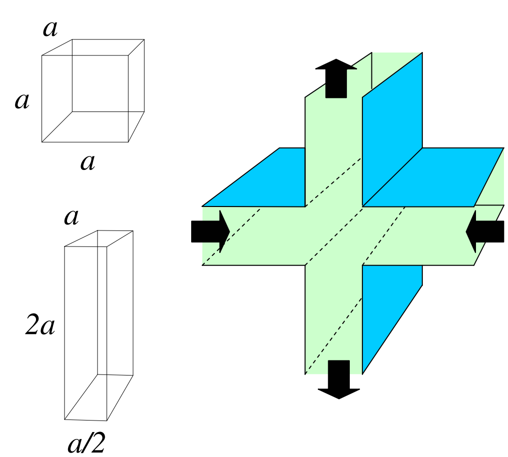
Consider a biaxial flow in which a particle is doubled in length in the flow direction.



A third type of shear-free flow:

Planar Elongational Flow

$$\underline{v} \equiv \begin{pmatrix} -\dot{\varepsilon}(t)x_1 \\ 0 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123} \qquad \dot{\varepsilon}(t) > 0$$



All three shear-free flows can be written together as:

$$\underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)(1+b)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)(1-b)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

Elongational flow: b=0, $\dot{\varepsilon}(t) > 0$

Biaxial stretching: b=0, $\dot{\varepsilon}(t) < 0$

Planar elongation: b=1, $\dot{\varepsilon}(t) > 0$

Why have we chosen these flows?

ANSWER: Because these simple flows have **symmetry**.

And symmetry allows us to draw conclusions about the stress tensor that is associated with these flows ***for any fluid*** subjected to that flow.

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In general:

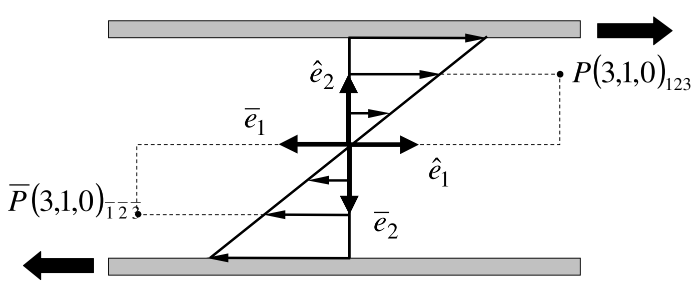
$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}_{123}$$

But the stress tensor is symmetric – leaving 6 independent stress components.

Can we choose a flow to use in which there are fewer than 6 independent stress components?

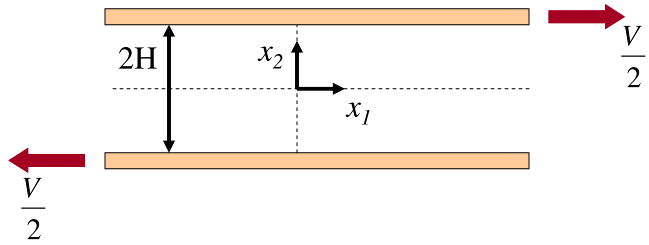
Yes we can – **symmetric flows**

How does the stress tensor simplify for shear (*and later, elongational*) flow?



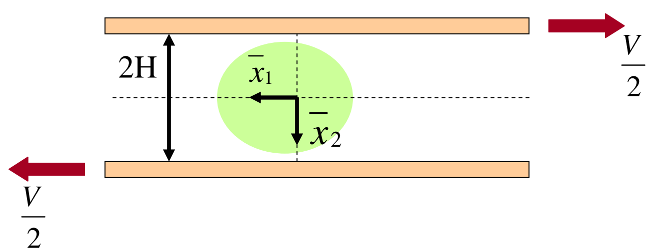
What would the velocity function be for a Newtonian fluid in this coordinate system?

$$\underline{v} = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}_{123}$$



What would the velocity function be for a Newtonian fluid in **this** coordinate system?

$$\underline{v} = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}_{123}$$



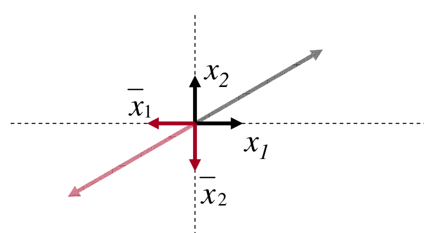
Vectors are independent of coordinate system, but in general the coefficients will be different when the same vector is written in two different coordinate systems:

$$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{123} = \begin{pmatrix} \bar{v}_1 \\ \bar{v}_2 \\ \bar{v}_3 \end{pmatrix}_{\bar{1}\bar{2}\bar{3}}$$

For shear flow and the two particular coordinate systems we have just examined,

$$\underline{v} = \begin{pmatrix} \frac{V}{2H} x_2 \\ 0 \\ 0 \end{pmatrix}_{123} = \begin{pmatrix} \frac{V}{2H} \bar{x}_2 \\ 0 \\ 0 \end{pmatrix}_{\bar{1}\bar{2}\bar{3}}$$

$$\underline{v} = \begin{pmatrix} \frac{V}{2H} x_2 \\ 0 \\ 0 \end{pmatrix}_{123} = \begin{pmatrix} \frac{V}{2H} \bar{x}_2 \\ 0 \\ 0 \end{pmatrix}_{\bar{1}\bar{2}\bar{3}}$$



If we plug in the **same number** in for x_2 and $\bar{x}_2$, we will NOT be asking about the same point in space, but we WILL get the same exact velocity vector.

Since stress is calculated from the velocity field, we will get the **same exact stress tensor** when we calculate it from either vector representation also.

$$v_p = \bar{v}_p$$
$$\tau_{pk} = \bar{\tau}_{pk}$$

This is an unusual circumstance only true for the particular coordinate systems chosen.

What do we learn if we formally transform $\underline{v}$ from one coordinate system to the other?

$$\hat{e}_1 = -\bar{e}_1 \qquad x_1 = -\bar{x}_1$$
$$\hat{e}_2 = -\bar{e}_2 \qquad x_2 = -\bar{x}_2$$
$$\hat{e}_3 = \bar{e}_3 \qquad x_3 = \bar{x}_3$$

Because of symmetry, there are only 5 nonzero components of the extra stress tensor in shear flow.

SHEAR:

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & 0 \\ \tau_{21} & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}_{123}$$

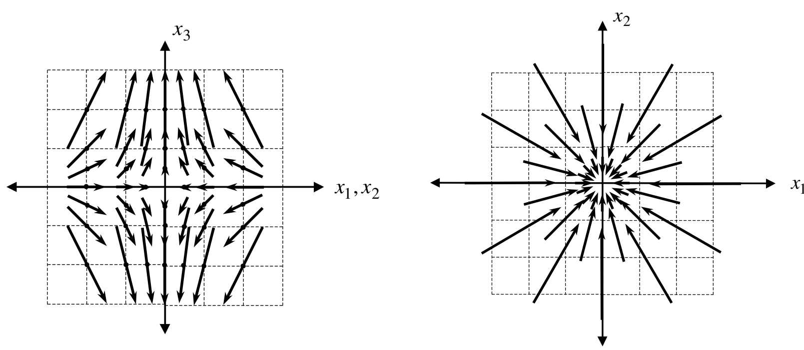
This greatly simplifies the experimentalists tasks as only four stress components ($\tau_{21}=\tau_{12}$) must be measured instead of 6.

Conclusion:

We have found a coordinate system (the shear coordinate system) in which there are only 5 non-zero coefficients of the stress tensor. In addition, $\tau_{21} = \tau_{12}$.

This leaves only four stress components to be measured for this flow, expressed in this coordinate system.

How does the stress tensor simplify for elongational flow?



There is 180° of symmetry around all three coordinate axes.

Because of symmetry, there are only 3 nonzero components of the extra stress tensor in elongational flows.

ELONGATION:

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & 0 & 0 \\ 0 & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}_{123}$$

This greatly simplifies the experimentalists tasks as only three stress components must be measured instead of 6.