One final comment on measuring stresses. . .

What is measured is the total stress, $\underline{\underline{\Pi}}$:

$$\underline{\underline{\Pi}} = \begin{pmatrix} p+\tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & p+\tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & p+\tau_{33} \end{pmatrix}_{123}$$

For the normal stresses we are faced with the difficulty of separating p from τ_{ii}.

Compressible fluids:

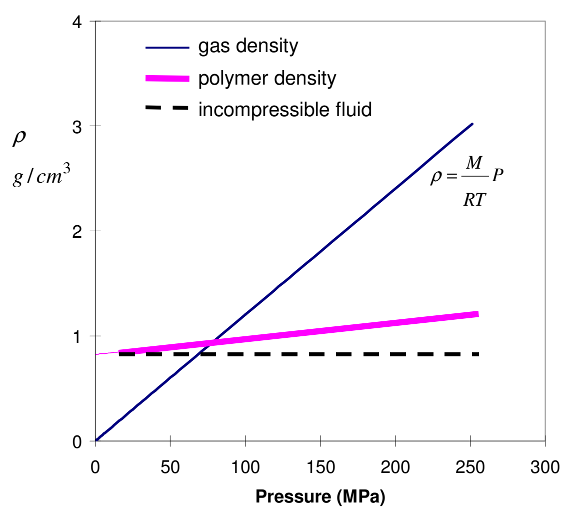
$$p = \frac{nRT}{V}$$

Get p from measurements of T and V.

Incompressible fluids:

?

Density does not vary (much) with pressure for polymeric fluids.



For incompressible fluids it is not possible to separate *p* from τ_{ii}.

Luckily, this is not a problem since we only need $\nabla \cdot \underline{\underline{\Pi}} = \nabla p + \nabla \cdot \underline{\underline{\tau}}$

Equation of motion

$$\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} = -\nabla \underline{\underline{\Pi}} + \rho \underline{g}$$
$$= -\nabla P - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$

We do not need τ_{ii} directly to solve for velocities

Solution? *Normal stress differences*

Normal Stress Differences

First normal stress difference $N_1 \equiv \Pi_{11} - \Pi_{22} = \tau_{11} - \tau_{22}$

Second normal stress difference $N_2 \equiv \Pi_{22} - \Pi_{33} = \tau_{22} - \tau_{33}$

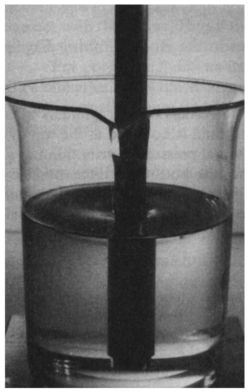
In shear flow, three stress quantities are measured τ_{21}, N_1, N_2

In elongational flow, two stress quantities are measured $\tau_{33} - \tau_{11}, \tau_{22} - \tau_{11}$

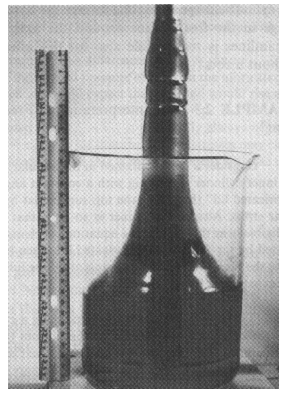
First normal stress effects: rod climbing

$$\tau_{11} - \tau_{22} < 0$$

Extra tension in the 1-direction pulls azimuthally and upward (see DPL p65).



Newtonian - glycerin



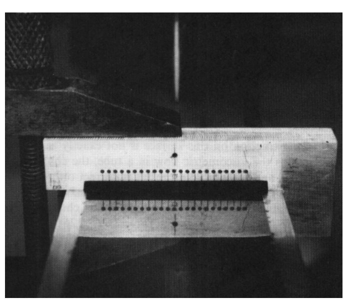
Viscoelastic - solution of polyacrylamide in glycerin

Bird, et al., Dynamics of Polymeric Fluids, vol. 1, Wiley, 1987, Figure 2.3-1 page 63. (DPL)

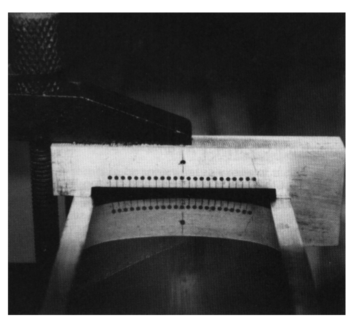
Second normal stress effects: inclined open-channel flow

$$\tau_{22} - \tau_{33} > 0$$

Extra tension in the 2-direction pulls down the free surface where dv_1/dx_2 is greatest (see DPL p65).



Newtonian - glycerin

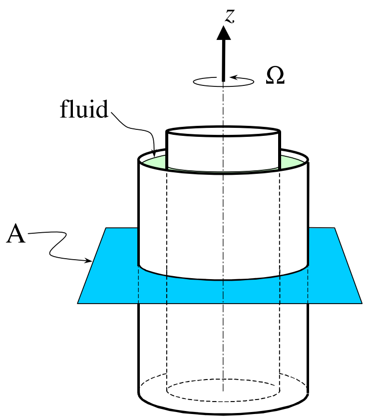


Viscoelastic - 1% soln of polyethylene oxide in water

$N_2 \simeq -N_1/10$

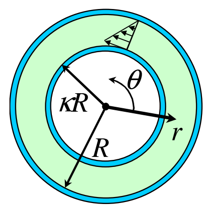
R. I. Tanner, Engineering Rheology, Oxford 1985, Figure 3.6 page 104

Example: Can the equation of motion predict rod climbing for typical values of N_1, N_2?



$$\underline{v} = \begin{pmatrix} 0 \\ v_\theta \\ 0 \end{pmatrix}_{r\theta z}$$

cross-section A:



What is $\frac{d\Pi_{zz}}{dr}$?

Bird et al. p64

What's next?

Shear	Shear-free (elongational, extensional)
$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$	$\underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)(1+b)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)(1-b)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$

Even with just these 2 (or 4) standard flows, we can still generate an *infinite* number of flows by varying $\dot{\varsigma}(t)$ and $\dot{\varepsilon}(t)$.

Elongational flow: b=0, $\dot{\varepsilon}(t) > 0$
Biaxial stretching: b=0, $\dot{\varepsilon}(t) < 0$
Planar elongation: b=1, $\dot{\varepsilon}(t) > 0$

We seek to quantify the behavior of non-Newtonian fluids

Procedure:

1. Choose a flow type (shear or a type of elongation).
2. Specify $\dot{\varsigma}(t)$ or $\dot{\varepsilon}(t)$ as appropriate.
3. Impose the flow on a fluid of interest.
4. Measure stresses.

shear	τ_{21}, N_1, N_2
elongation	$\tau_{33} - \tau_{11}, \tau_{22} - \tau_{11}$

5. Report stresses in terms of material functions.

6a. Compare measured material functions with predictions of these material functions (from proposed constitutive equations).

7a. Choose the most appropriate constitutive equation for use in numerical modeling.

6b. Compare measured material functions with those measured on other materials.

7a. Draw conclusions on the likely properties of the unknown material based on the comparison.