Intro to strain (continued)

For steady shear, with t_{ref}=0: $\gamma_{21}(0,t) \equiv \frac{\partial u_1}{\partial x_2} = t\dot{\gamma}_0$

(short time interval)

For a long time interval, we add up the strains over short time intervals.

$$\gamma_{21}(t_p, t_{p+1}) = \dot{\gamma}_0 \Delta t$$

$$\gamma_{21}(0,t) = \sum_{p=0}^{N-1} \gamma_{21}(t_p, t_{p+1}) = (N\Delta t)\dot{\gamma}_0 = t\dot{\gamma}_0$$

Same, because flow is steady.

For **unsteady** shear:

$$\gamma_{21}(t_p, t_{p+1}) = \frac{\partial u_1}{\partial x_2} = \dot{\gamma}_{21}(t_{p+1})\Delta t$$ *(short time interval)*

For a long time interval, we add up the strains over short time intervals.

$$\gamma_{21}(t_p, t_{p+1}) = \dot{\gamma}_{21}(t_{p+1})\Delta t$$

$$\gamma_{21}(t_1, t_2) = \sum_{p=0}^{N-1} \gamma_{21}(t_p, t_{p+1}) = \sum_{p=0}^{N-1} \Delta t \dot{\gamma}_{21}(t_{p+1})$$

Taking the limit as Δt goes to zero,

$$\gamma_{21}(t_1, t_2) = \lim_{\Delta t \to 0} \left[\sum_{p=0}^{N-1} \Delta t \dot{\gamma}_{21}(t_{p+1}) \right] = \int_{t_1}^{t_2} \dot{\gamma}_{21}(t')dt'$$

Strain at t_2 with respect to fluid configuration at t_1 in **unsteady** shear flow.

Creep Shear Flow Material Functions

Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\gamma}_{21}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \tau_{21}(t) = \begin{cases} 0 & t < 0 \\ \tau_0 & t \geq 0 \end{cases}$$

Material Functions:

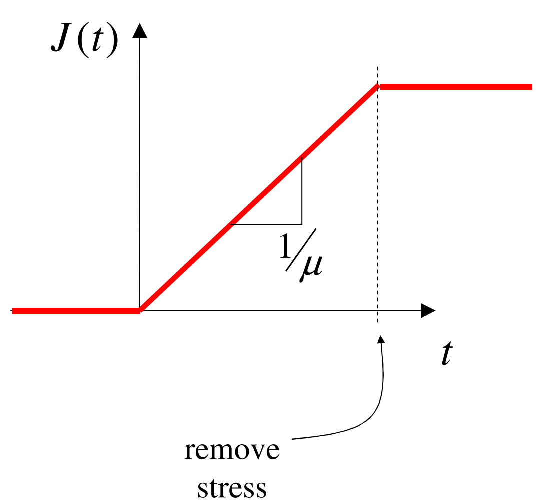
$$J(t, \tau_0) \equiv \frac{\gamma_{21}(0,t)}{-\tau_0}$$

Shear creep compliance

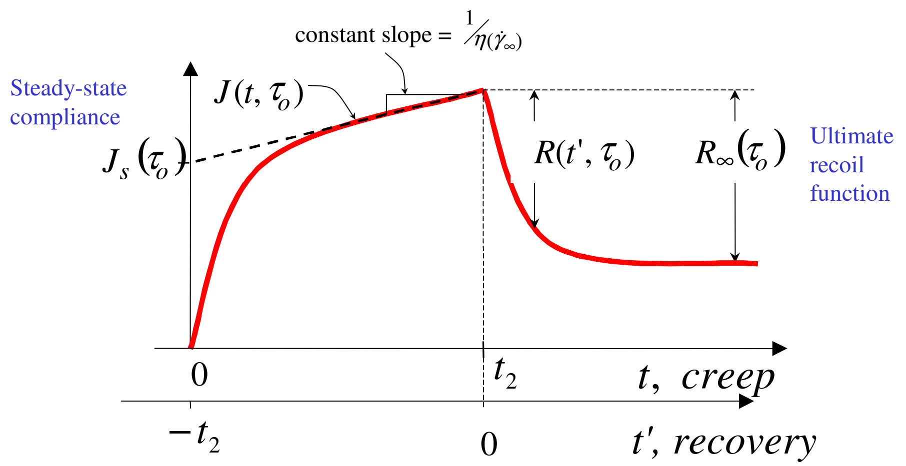
$$J_r(t', \tau_0) \equiv \frac{\gamma_r(t')}{-\tau_0}$$

Recoverable creep compliance

Material functions predicted for *creep* of a Newtonian fluid



Shear creep material functions



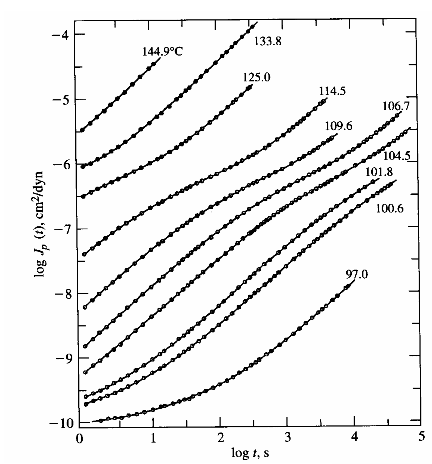
Shear Creep

$$\underline{v} \equiv \begin{pmatrix} \dot{\gamma}_{21}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\tau_{21}(t) = \begin{cases} 0 & t < 0 \\ \tau_0 & t \geq 0 \end{cases}$$

$$J_p = \frac{J(T)T\rho}{T_{ref}\,\rho_{ref}}$$

Data have been corrected for vertical shift.



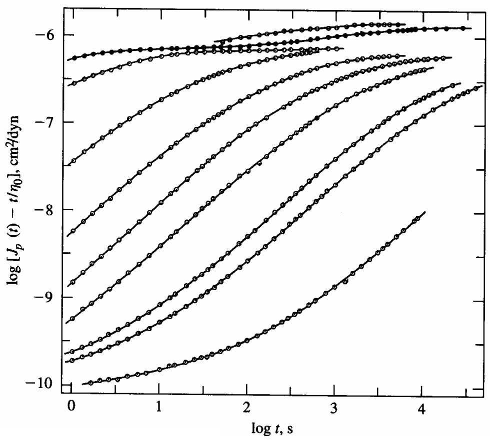
$$J(t,\tau_0) \equiv \frac{\gamma_{21}(0,t)}{-\tau_0}$$

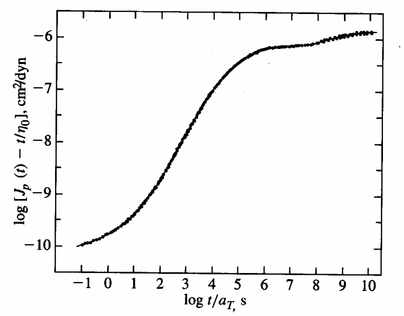
Figure 6.53, p. 210
Plazek; PS melt

Shear Creep - Recoverable Compliance

$$J(t) = R(t) + \frac{t}{\eta_0}$$

$$\underbrace{\gamma(t)}_{\text{total strain}} = \overbrace{\gamma_r(t)}^{\text{recoverable strain}} + \underbrace{t\dot{\gamma}_\infty}_{\text{non-recoverable strain}}$$





Figures 6.54, 6.55, p. 211
Plazek; PS melt

At long times the creep compliance $J(t, \tau_0)$ becomes a straight line.

$$\left.\frac{dJ}{dt}\right|_{\substack{steady\\state}} = \frac{d\gamma_{21}}{dt}\left(\frac{1}{-\tau_0}\right)$$

$$= \frac{\dot{\gamma}_\infty}{-\tau_0}$$

$$= \frac{1}{\eta(\dot{\gamma}_\infty)}$$

the slope at steady state is the inverse of the steady viscosity

$$\left.\frac{dJ}{dt}\right|_{\substack{steady\\state}} = \frac{1}{\eta(\dot{\gamma}_\infty)} \Rightarrow \left.J(t)\right|_{\substack{steady\\state}} = \frac{1}{\eta(\dot{\gamma}_\infty)}t + C$$

Steady-state compliance $J_s(\tau_0)$

Creep Recovery - after creep, stop pulling forward and allow the flow to reverse

$$\gamma_r(t) = \gamma_{21}(0,t_2) - \gamma_{21}(0,t)$$

Recoverable strain
Recoil strain

Strain at the end of the forward motion

Strain at the end of the recovery

$$J_r(t',\tau_0) \equiv \frac{\gamma_r(t')}{-\tau_0}$$

Recoverable creep compliance

Linear Viscoelastic Creep

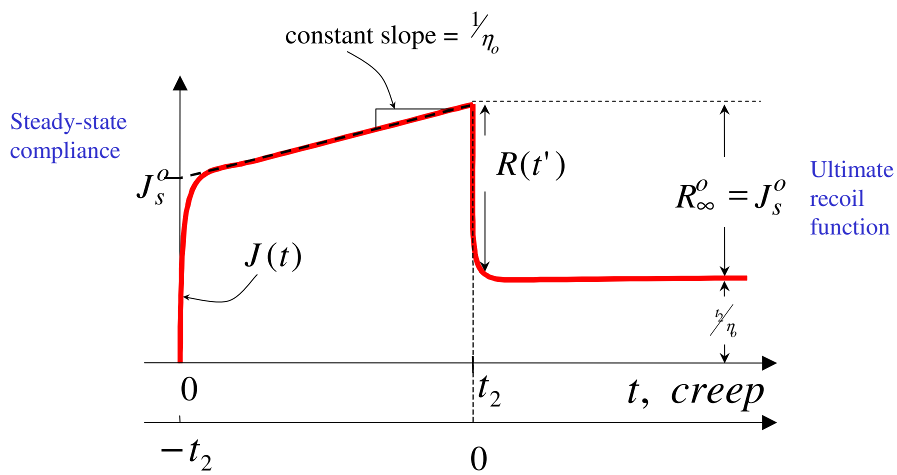
$$\underbrace{\gamma(t)}_{\text{total strain}} = \overbrace{\gamma_r(t)}^{\text{recoverable strain}} + \underbrace{t\dot{\gamma}_\infty}_{\text{non-recoverable strain}}$$

$$J(t) = R(t) + \frac{t}{\eta_0}$$

This is a way to get R(t) without measuring it

Shear creep material functions

Linear-viscoelastic limit



Step Shear Strain Material Functions

Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\varsigma}(t) = \lim_{\varepsilon \to 0} \begin{cases} 0 & t < 0 \\ \dot{\gamma} & 0 \le t < \varepsilon \\ 0 & t \ge \varepsilon \end{cases}$$

$$\dot{\gamma}\varepsilon = \text{constant} = \gamma_0$$

Material Functions:

$$G(t, \gamma_0) \equiv \frac{-\tau_{21}(t, \gamma_0)}{\gamma_0}$$

Relaxation modulus

First normal-stress relaxation modulus

$$G_{\Psi_1} \equiv \frac{-(\tau_{11} - \tau_{22})}{\gamma_0^2}$$

Second normal-stress relaxation modulus

$$G_{\Psi_2} \equiv \frac{-(\tau_{22} - \tau_{33})}{\gamma_0^2}$$

What is the strain in this flow?

$$\gamma_{21}(-\infty,t) = \int_{-\infty}^{t} \dot{\gamma}_{21}(t')dt'$$

$$= \int_{-\infty}^{t} \lim_{\varepsilon\to 0} \begin{cases} 0 & t'<0 \\ \frac{\gamma_0}{\varepsilon} & 0\le t'<\varepsilon \\ 0 & t\ge\varepsilon \end{cases} dt'$$

$$= \lim_{\varepsilon\to 0} \int_0^{\varepsilon} \frac{\gamma_0}{\varepsilon}\,dt'$$

$$= \gamma_0$$

The strain imposed is a constant

Step shear strain - *strain dependence*

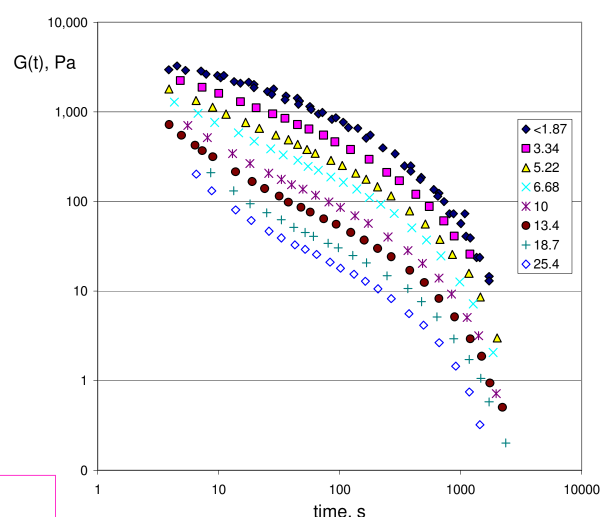


Figure 6.57, p. 212
Einaga et al.; PS soln

Linear viscoelastic limit

$$\lim_{\gamma_0 \to 0} G(t, \gamma_0) = G(t)$$

At small strains the relaxation modulus is independent of strain.

Damping function, *h*

$$h(\gamma_0) \equiv \frac{G(t, \gamma_0)}{G(t)}$$

The damping function summarizes the non-linear effects as a function of strain amplitude.