Small-Amplitude Oscillatory Shear Material Functions

Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\varsigma}(t) = \dot{\gamma}_0 \cos \omega t$$

$$\gamma_0 \equiv \frac{\dot{\gamma}_0}{\omega}$$

Material Functions:

$$\frac{-\tau_{21}(t, \gamma_0)}{\gamma_0} = G' \sin \omega t + G'' \cos \omega t$$

$$G'(\omega) \equiv \frac{\tau_0}{\gamma_0} \cos \delta$$

Storage modulus

(δ is the phase difference between stress and strain)

$$G''(\omega) \equiv \frac{\tau_0}{\gamma_0} \sin \delta$$

Loss modulus

What is the strain in this flow?

$$\gamma_{21}(0,t) = \int_0^t \dot{\gamma}_{21}(t') dt'$$

$$= \int_0^t \dot{\gamma}_0 \cos \omega t' \, dt'$$

$$= \frac{\dot{\gamma}_0}{\omega} \sin \omega t$$

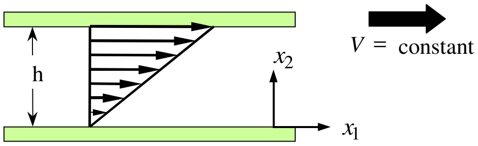
The strain imposed is sinusoidal.

The strain amplitude is $\gamma_0 = \frac{\dot{\gamma}_0}{\omega}$

Generating Small Amplitude Oscillatory Shear (SAOS)

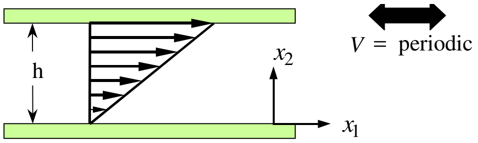
steady shear

$$b(t) = Vt = h\dot{\gamma}_o t$$



small-amplitude
oscillatory shear

$$b(t) = h\gamma_o \sin \omega t$$
$$= \frac{h\dot{\gamma}_o}{\omega} \sin \omega t$$



© Faith A. Morrison, Michigan Tech U.

In SAOS the strain amplitude is small, and a sinusoidal imposed strain induces a sinusoidal measured stress.

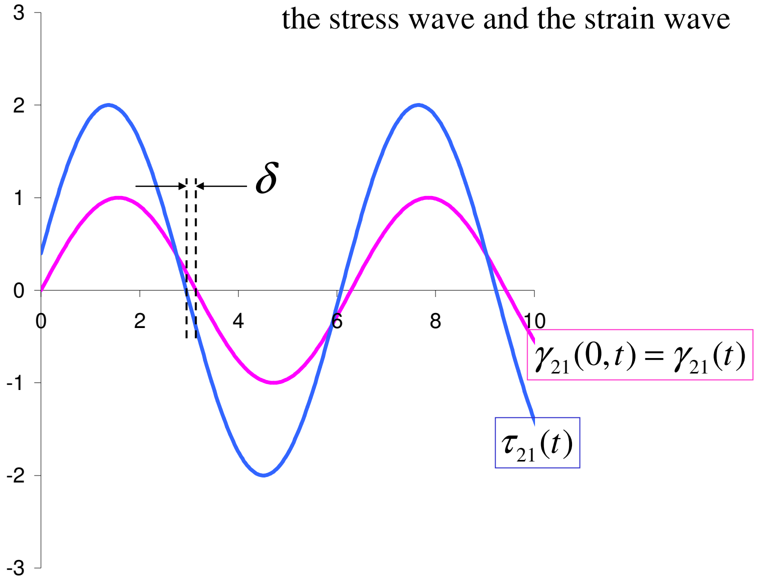
$$\boxed{-\tau_{21}(t) = \tau_0 \sin(\omega t + \delta)}$$

$$\boxed{\begin{aligned} -\tau_{21}(t) &= \tau_0 \sin(\omega t + \delta) \\ &= \tau_0 \sin \omega t \cos \delta + \tau_0 \cos \omega t \sin \delta \\ &= [\tau_0 \cos \delta] \sin \omega t + [\tau_0 \sin \delta] \cos \omega t \end{aligned}}$$

$$\underbrace{\qquad\qquad\qquad}_{\substack{\textit{portion in-phase} \\ \textit{with } \underline{\textit{strain}}}} \qquad \underbrace{\qquad\qquad\qquad}_{\substack{\textit{portion in-phase} \\ \textit{with } \underline{\textit{strain-rate}}}}$$

© Faith A. Morrison, Michigan Tech U.

δ is the phase difference between
the stress wave and the strain wave



SAOS Material Functions

$$\boxed{\frac{-\tau_{21}(t)}{\gamma_0} = \left[\frac{\tau_0 \cos\delta}{\gamma_0}\right] \sin\omega t + \left[\frac{\tau_0 \sin\delta}{\gamma_0}\right] \cos\omega t}$$

portion in-phase with strain

G'

portion in-phase with strain-rate

G''

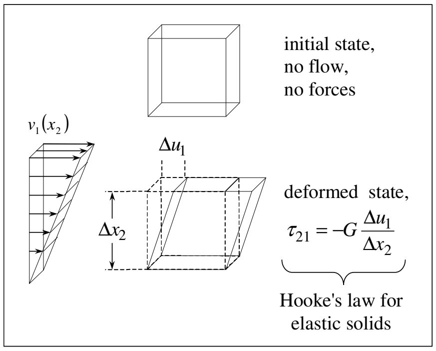
For Newtonian fluids, stress is proportional to strain rate: $\boxed{\tau_{21} = -\mu \dot{\gamma}_{21}}$

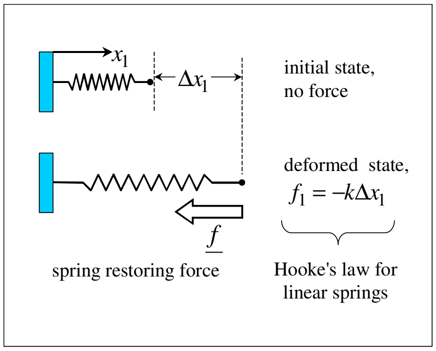
G'' is thus known as the viscous loss modulus. It characterizes the viscous contribution to the stress response.

What types of materials generate stress in proportion to the strain imposed? Answer: elastic solids

Hooke's Law for elastic solids

$$\boxed{\tau_{21} = -G\gamma_{21}}$$





Similar to the linear spring law

SAOS Material Functions

$$\boxed{\frac{-\tau_{21}(t)}{\gamma_0} = \underbrace{\left[\frac{\tau_0\cos\delta}{\gamma_0}\right]}_{G'}\sin\omega t + \underbrace{\left[\frac{\tau_0\sin\delta}{\gamma_0}\right]}_{G''}\cos\omega t}$$

portion in-phase with strain — G'

portion in-phase with strain-rate — G''

For Hookean solids, stress is proportional to strain : $\boxed{\tau_{21} = -G\gamma_{21}}$

G' is thus known as the elastic storage modulus. It characterizes the elastic contribution to the stress response.

(note: SAOS material functions may also be expressed in complex notation. See pp. 156-159 of Morrison, 2001)

Steady Elongational Flow Material Functions

Kinematics:

$$\underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)(1+b)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)(1-b)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

$$\dot{\varepsilon}(t) = \dot{\varepsilon}_0 = \text{constant}$$

Elongational flow: b=0, $\dot{\varepsilon}(t) > 0$
Biaxial stretching: b=0, $\dot{\varepsilon}(t) < 0$
Planar elongation: b=1, $\dot{\varepsilon}(t) > 0$

Material Functions:

$$\bar{\eta} \text{ or } \bar{\eta}_B \text{ or } \bar{\eta}_{P_1} \equiv \frac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$$

Uniaxial or Biaxial or First Planar Elongational Viscosity

$$\bar{\eta}_{P_2} \equiv \frac{-(\tau_{22} - \tau_{11})}{\dot{\varepsilon}_0}$$

Second Planar Elongational Viscosity

What is the strain in this flow? *Hencky strain*

$$\varepsilon(t_{ref}, t) = \int_{t_{ref}}^{t} \dot{\varepsilon}(t')dt'$$

(choose $t_{ref}=0$)

$$= \dot{\varepsilon}_0 t$$

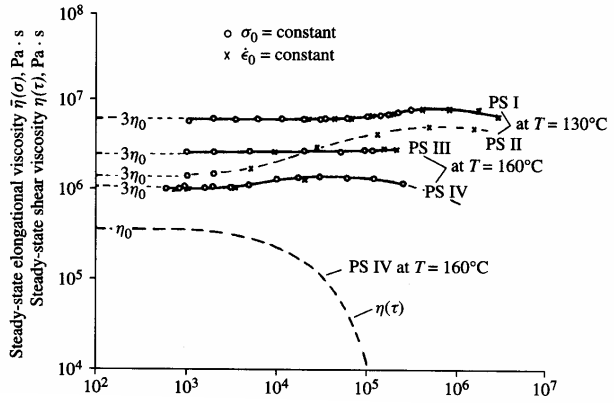
The strain imposed is proportional to time.

$$= \ln \frac{l}{l_0}$$

The ratio of current length to initial length is exponential in time.

Steady State Elongation Viscosity

Both tension thinning and thickening are observed.



$$\text{Trouton ratio: } Tr \equiv \frac{\bar{\eta}}{\eta_0}$$

Figure 6.60, p. 215
Munstedt.; PS melt

The other elongational experiments are analogous to shear experiments *(see text)*

Elongational stress growth

Elongational stress cessation *(nearly impossible)*

Elongational creep

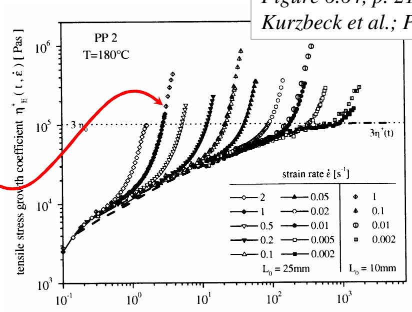
Step elongational strain

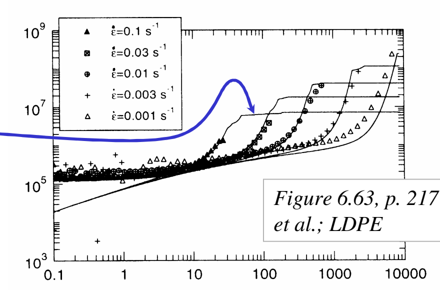
Small-amplitude Oscillatory Elongation (SAOE)

Start-up of Steady Elongation

Figure 6.64, p. 218 Kurzbeck et al.; PP

Strain-hardening





Fit to an advanced constitutive equation (12 mode pom-pom model)

Figure 6.63, p. 217 Inkson et al.; LDPE