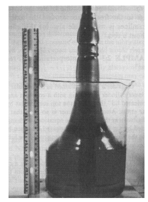
The steady shear viscosity function η can be fit to experimental data to an arbitrarily high precision.

Does this mean that *Generalized Newtonian Fluid* models are okay to use in all situations?

Not necessarily. A constitutive model needs to be able to predict *all stresses* in *all flows,* not just shear stresses in steady shearing. We need to check predictions.

For example, does the GNF predict the shear normal stresses?

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & 0 \\ \tau_{21} & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}_{123}$$



Generalized Newtonian Fluid (GNF) constitutive equation

$$\underline{\underline{\tau}} = -\eta(\dot{\gamma}) \begin{pmatrix} 2\frac{\partial v_1}{\partial x_1} & \frac{\partial v_1}{\partial x_2}+\frac{\partial v_2}{\partial x_1} & \frac{\partial v_1}{\partial x_3}+\frac{\partial v_3}{\partial x_1} \\ \frac{\partial v_1}{\partial x_2}+\frac{\partial v_2}{\partial x_1} & 2\frac{\partial v_2}{\partial x_2} & \frac{\partial v_3}{\partial x_2}+\frac{\partial v_2}{\partial x_3} \\ \frac{\partial v_1}{\partial x_3}+\frac{\partial v_3}{\partial x_1} & \frac{\partial v_3}{\partial x_2}+\frac{\partial v_2}{\partial x_3} & 2\frac{\partial v_3}{\partial x_3} \end{pmatrix}_{123}$$

In Shear Flow:

$$\underline{v} = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \dot{\gamma} \equiv \left|\frac{\partial v_1}{\partial x_2}\right|$$

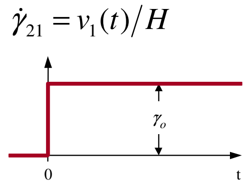
$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}_{123} = \begin{pmatrix} 0 & -\eta(\dot{\gamma})\frac{\partial v_1}{\partial x_2} & 0 \\ -\eta(\dot{\gamma})\frac{\partial v_1}{\partial x_2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

No matter what we pick for the function $\eta(\dot{\gamma})$, **we cannot predict** shear normal stresses with a Generalized Newtonian Fluid.

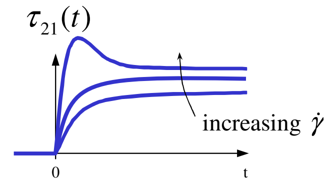
What does the GNF predict for start-up shear stresses?

shear stress response

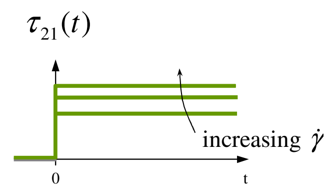
imposed shear rate



What the data show:

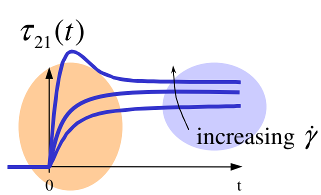


What the GNF models predict:



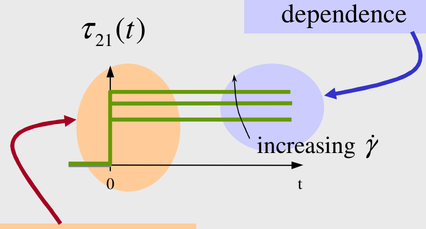
Start-up shear stresses

What the data show:



What the **GNF** models predict:

Correctly captures rate dependence

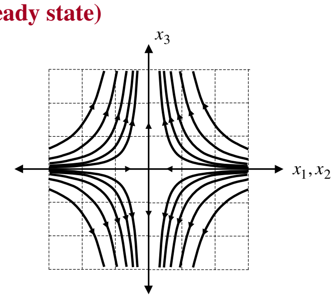


misses start-up effects

No matter what we pick for the function $\eta(\dot{\gamma})$, **we cannot predict** the time-dependence of shear start-up correctly with a GNF.

What does the GNF predict in steady elongational flow?

imposed deformation
(steady state)



elongational stress response

What the data show:

$$\lim_{\dot{\varepsilon}\to 0} \bar{\eta} = 3\eta_0$$ Trouton's Rule

(there is limited elongational viscosity data available)

What the GNF models predict:

$$\bar{\eta} = 3\eta$$ For all deformation rates

If a material shear-thins, GNF predicts it will tension-thin.

Summary: *Generalized Newtonian Fluid Constitutive Equations*

PRO:

- A first constitutive equation
- Can match steady shearing data very well
- Simple to calculate with
- Found to predict pressure-drop/flow rate relationships well

CON:

- Fails to predict shear normal stresses
- Fails to predict start-up or cessation effects (time-dependence, memory) – only a function of instantaneous velocity gradient
- Derived ad hoc from shear observations; unclear of validity in non-shear flows