Predictions of the (single-mode) Maxwell Model

$$\underline{\underline{\tau}} + \frac{\eta_0}{G}\frac{\partial \underline{\underline{\tau}}}{\partial t} = -\eta_0 \underline{\underline{\dot{\gamma}}}$$

$$\underline{\underline{\tau}}(t) = -\int_{-\infty}^{t} \left(\frac{\eta_0}{\lambda}\right) e^{-(t-t')/\lambda}\, \underline{\underline{\dot{\gamma}}}(t')\, dt'$$

Steady shear	$\eta = \eta_0$ $\Psi_1 = \Psi_2 = 0$	Fails to predict shear normal stresses. Fails to predict shear-thinning.
Steady elongation	$\bar{\eta} = 3\eta_0$	Trouton's rule

Steady shear viscosity and first normal stress coefficient

BOGER FLUIDS

There are some systems with a constant viscosity but still start-up effects.

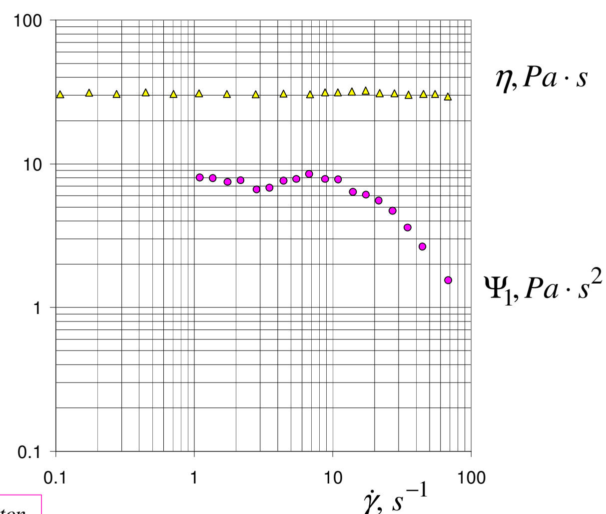


Figure 6.5, p. 173 Binnington and Boger; PIB soln

Steady shear viscosity and first and ***second*** normal stress coefficient

BOGER FLUIDS

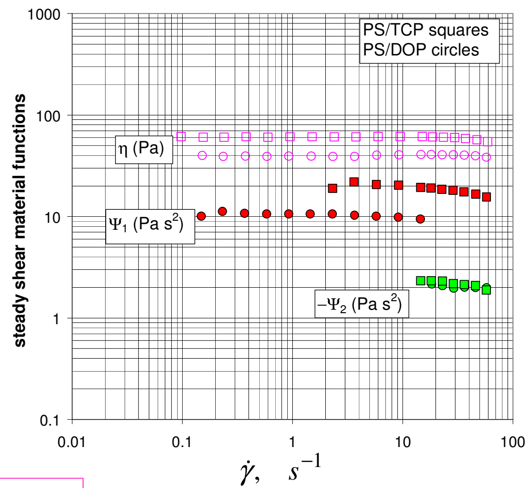


Figure 6.6, p. 174 Magda et al.; PS solns

Step Shear Strain Material Functions

Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\varsigma}(t) = \lim_{\varepsilon \to 0} \begin{cases} 0 & t < 0 \\ \dot{\gamma} & 0 \le t < \varepsilon \\ 0 & t \ge \varepsilon \end{cases}$$

$$\dot{\gamma}\varepsilon = \text{constant} = \gamma_0$$

Material Functions:

$$G(t, \gamma_0) \equiv \frac{-\tau_{21}(t, \gamma_0)}{\gamma_0}$$

Relaxation modulus

First normal-stress relaxation modulus

$$G_{\Psi_1} \equiv \frac{-(\tau_{11} - \tau_{22})}{\gamma_0^2}$$

Second normal-stress relaxation modulus

$$G_{\Psi_2} \equiv \frac{-(\tau_{22} - \tau_{33})}{\gamma_0^2}$$

Predictions of the (single-mode) Maxwell Model

$$\underline{\underline{\tau}} + \frac{\eta_0}{G}\frac{\partial \underline{\underline{\tau}}}{\partial t} = -\eta_0 \underline{\underline{\dot{\gamma}}}$$

$$\underline{\underline{\tau}}(t) = -\int_{-\infty}^{t} \left(\frac{\eta_0}{\lambda}\right) e^{-(t-t')/\lambda}\, \underline{\underline{\dot{\gamma}}}(t')\, dt'$$

Shear start-up

$$\eta^+(t) = \eta_0\left(1 - e^{-t/\lambda}\right)$$

$$\Psi_1^+(t) = \Psi_2^+(t) = 0$$

Does predict a gradual build-up of stresses on start-up.

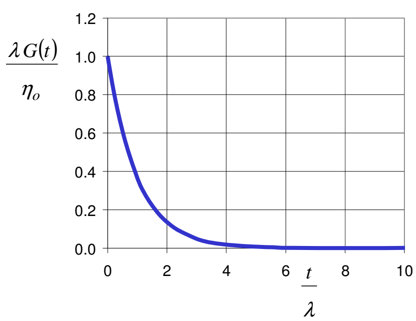
Step shear strain

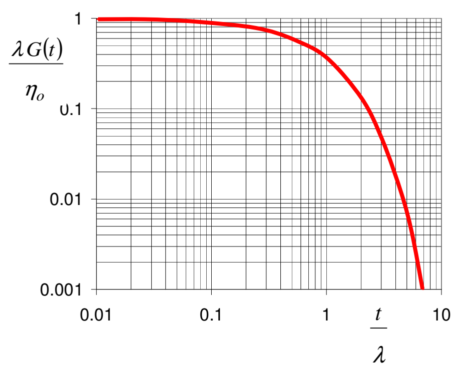
$$G(t) = \frac{\eta_0}{\lambda} e^{-t/\lambda}$$

$$G_{\Psi 1} = G_{\Psi 2} = 0$$

Does predict a reasonable relaxation function in step strain (but no normal stresses again).

Step-Shear-Strain Material Function *G(t)* for Maxwell Model





Comparison to experimental data

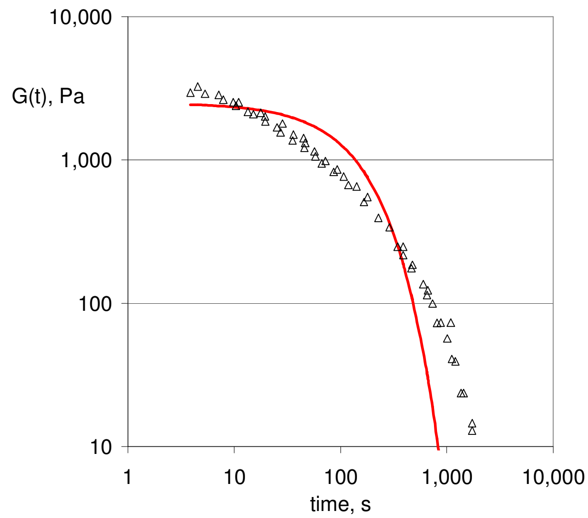


Figure 8.4, p. 274 data from Einaga et al., PS 20% soln in chlorinated diphenyl