We can improve this fit by adjusting the Maxwell model to allow multiple relaxation modes

$$\underline{\underline{\tau}}_{(k)} = -\int_{-\infty}^{t} \left(\frac{\eta_k}{\lambda_k} \right) e^{-(t-t')/\lambda_k} \, \underline{\underline{\dot{\gamma}}}(t') \, dt'$$

$$\underline{\underline{\tau}}(t) = \sum_{k=1}^{N} \underline{\underline{\tau}}_{(k)}$$

Generalized Maxwell Model

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} \left[\sum_{k=1}^{3} \frac{\eta_k}{\lambda_k} e^{-(t-t')/\lambda_k} \right] \underline{\underline{\dot{\gamma}}}(t') \, dt'$$

2N parameters (can fit *anything*)

Predictions of the Generalized Maxwell Model

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} \left[\sum_{k=1}^{3} \frac{\eta_k}{\lambda_k} e^{-(t-t')/\lambda_k} \right] \underline{\underline{\dot{\gamma}}}(t') \, dt'$$

Steady shear	$\eta = \sum_{k=1}^{N} \eta_k$ $\Psi_1 = \Psi_2 = 0$	Fails to predict shear normal stresses Fails to predict shear-thinning
Step shear strain	$G(t) = \sum_{k=1}^{N} \frac{\eta_k}{\lambda_k} e^{-t/\lambda_k}$ $G_{\Psi 1} = G_{\Psi 2} = 0$	This function can fit ***any*** observed data; note that the GMM does not predict shear normal stresses.

Fitting G(t) to Generalized Maxwell Model

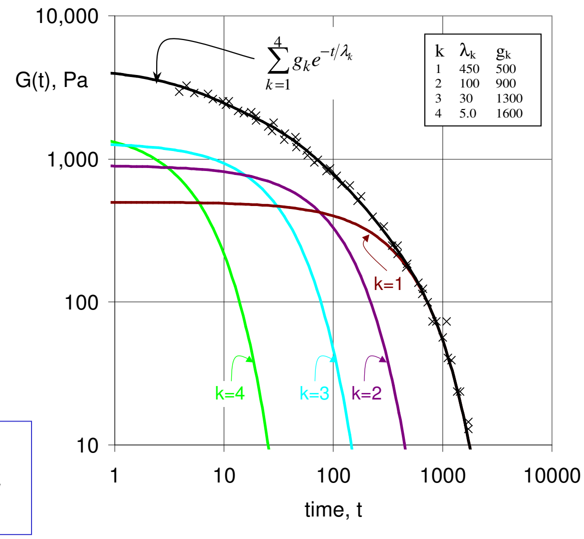

Figure 8.4, p. 274 data from Einaga et al., PS 20% soln in chlorinated diphenyl

The Linear-Viscoelastic Models

Differential Maxwell (one mode):

$$\underline{\underline{\tau}} + \frac{\eta_0}{G}\frac{\partial \underline{\underline{\tau}}}{\partial t} = -\eta_0 \underline{\underline{\dot{\gamma}}}$$

Integral Maxwell (one mode):

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} \frac{\eta_0}{\lambda} e^{-\frac{(t-t')}{\lambda}} \underline{\underline{\dot{\gamma}}}(t')\,dt'$$

Generalized Maxwell model (N modes):

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} \left[\sum_{k=1}^{N} \frac{\eta_k}{\lambda_k} e^{-\frac{(t-t')}{\lambda_k}}\right] \underline{\underline{\dot{\gamma}}}(t')\,dt'$$

The Linear-Viscoelastic Models

Differential Maxwell (one mode):

$$\underline{\underline{\tau}} + \frac{\eta_0}{G}\frac{\partial \underline{\underline{\tau}}}{\partial t} = -\eta_0 \dot{\underline{\underline{\gamma}}}$$

Integral Maxwell (one mode):

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} \frac{\eta_0}{\lambda} e^{-\frac{(t-t')}{\lambda}} \dot{\underline{\underline{\gamma}}}(t')\, dt'$$

Generalized Maxwell model (N modes):

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} \left[\sum_{k=1}^{N} \frac{\eta_k}{\lambda_k} e^{-\frac{(t-t')}{\lambda_k}} \right] \dot{\underline{\underline{\gamma}}}(t')\, dt'$$

Since the term in brackets is just the predicted relaxation modulus *G(t)*, we can write an even more ***general linear viscoelastic model*** by leaving this function unspecified.

The Linear-Viscoelastic Models

Differential Maxwell (one mode):

$$\underline{\underline{\tau}} + \frac{\eta_0}{G}\frac{\partial \underline{\underline{\tau}}}{\partial t} = -\eta_0 \dot{\underline{\underline{\gamma}}}$$

Integral Maxwell (one mode):

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} \frac{\eta_0}{\lambda} e^{-\frac{(t-t')}{\lambda}} \dot{\underline{\underline{\gamma}}}(t')\, dt'$$

Generalized Maxwell model (N modes):

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} \left[\sum_{k=1}^{N} \frac{\eta_k}{\lambda_k} e^{-\frac{(t-t')}{\lambda_k}} \right] \dot{\underline{\underline{\gamma}}}(t')\, dt'$$

Generalized Linear-Viscoelastic Model:

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} G(t-t')\, \dot{\underline{\underline{\gamma}}}(t')\, dt'$$