Small-Amplitude Oscillatory Shear Material Functions

Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\dot{\varsigma}(t) = \dot{\gamma}_0 \cos \omega t$$

$$\gamma_0 \equiv \frac{\dot{\gamma}_0}{\omega}$$

Material Functions:

$$\frac{-\tau_{21}(t, \gamma_0)}{\gamma_0} = G' \sin \omega t + G'' \cos \omega t$$

$$G'(\omega) \equiv \frac{\tau_0}{\gamma_0} \cos \delta$$

Storage modulus

(δ is the phase difference between stress and strain)

$$G''(\omega) \equiv \frac{\tau_0}{\gamma_0} \sin \delta$$

Loss modulus

Predictions of the Generalized Maxwell Model (GMM) and Generalized Linear-Viscoelastic Model (GLVE)

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} G(t - t')\, \underline{\underline{\dot{\gamma}}}(t')\, dt'$$

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} \left[\sum_{k=1}^{3} \frac{\eta_k}{\lambda_k} e^{-(t-t')/\lambda_k} \right] \underline{\underline{\dot{\gamma}}}(t')\, dt'$$

Small-amplitude oscillatory shear

GLVE

$$G'(\omega) = \omega \int_0^{\infty} G(s) \cos \omega s \, ds$$

$$G''(\omega) = \omega \int_0^{\infty} G(s) \sin \omega s \, ds$$

GMM

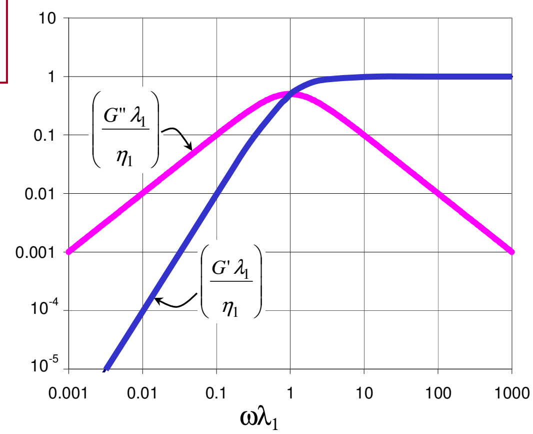
$$G'(\omega) = \sum_{k=1}^{N} \frac{\eta_k \lambda_k \omega^2}{1 + (\lambda_k \omega)^2}$$

$$G''(\omega) = \sum_{k=1}^{N} \frac{\eta_k \omega}{1 + (\lambda_k \omega)^2}$$

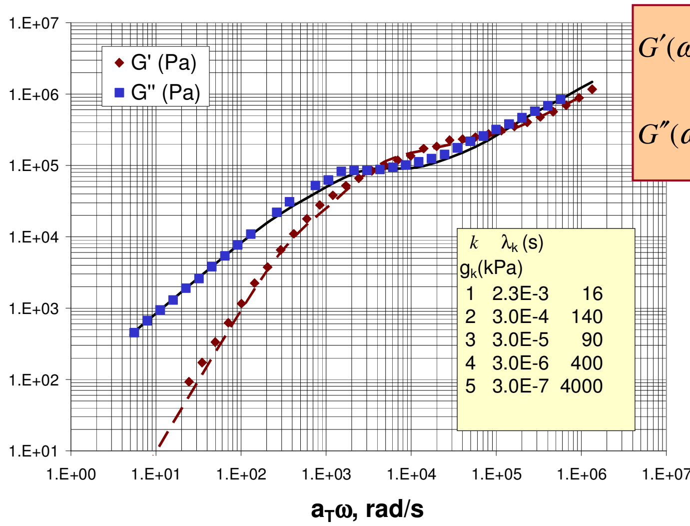
Predictions of (single-mode) Maxwell Model in SAOS

$$G'(\omega) = \frac{g_1 \lambda_1^2 \omega^2}{1+(\lambda_1 \omega)^2} = \frac{\eta_1 \lambda_1 \omega^2}{1+(\lambda_1 \omega)^2}$$

$$G''(\omega) = \frac{g_1 \lambda_1 \omega}{1+(\lambda_1 \omega)^2} = \frac{\eta_1 \omega}{1+(\lambda_1 \omega)^2}$$



Predictions of (multi-mode) Maxwell Model in SAOS

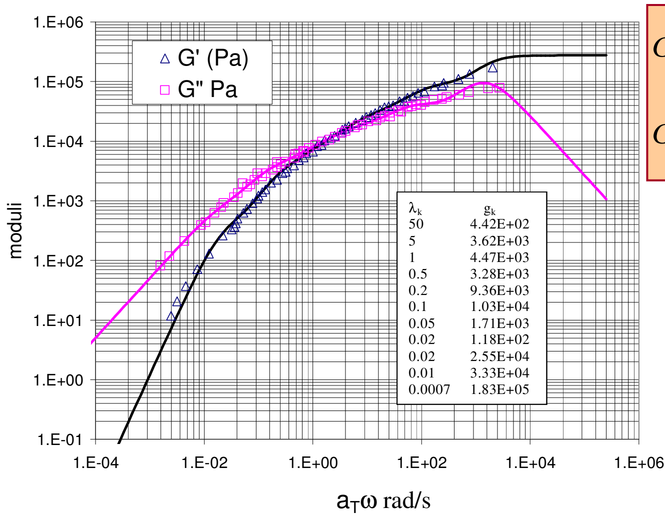


$$G'(\omega) = \sum_{k=1}^{N} \frac{\eta_k \lambda_k \omega^2}{1+(\lambda_k \omega)^2}$$

$$G''(\omega) = \sum_{k=1}^{N} \frac{\eta_k \omega}{1+(\lambda_k \omega)^2}$$

Figure 8.8, p. 284 data from Vinogradov, PS melt

Predictions of (multi-mode) Maxwell Model in SAOS



λ_k	g_k
50	4.42E+02
5	3.62E+03
1	4.47E+03
0.5	3.28E+03
0.2	9.36E+03
0.1	1.03E+04
0.05	1.71E+03
0.02	1.18E+02
0.02	2.55E+04
0.01	3.33E+04
0.0007	1.83E+05

$$G'(\omega) = \sum_{k=1}^{N} \frac{\eta_k \lambda_k \omega^2}{1+(\lambda_k \omega)^2}$$

$$G''(\omega) = \sum_{k=1}^{N} \frac{\eta_k \omega}{1+(\lambda_k \omega)^2}$$

Figure 8.10, p. 286 data from Laun, PE melt

Limitations of the GLVE Models

- Predicts constant shear viscosity
- Only valid in regime where strain is additive (small-strain, low rates)
- All stresses are proportional to the deformation-rate tensor; thus shear normal stresses cannot be predicted
- Cannot describe flows with a superposed rigid rotation (as we will now discuss; see Morrison p296)

Steady shear viscosity and first and ***second*** normal stress coefficient

BOGER FLUIDS

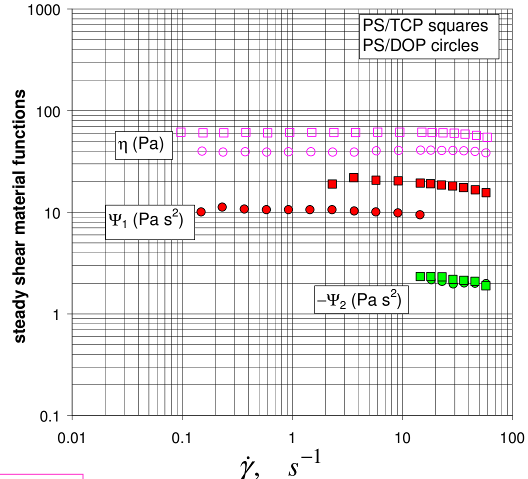


Figure 6.6, p. 174 Magda et al.; PS solns