Advanced Constitutive Modeling

Generalized Linear-Viscoelastic Model:

$$\underline{\underline{\tau}} = -\int_{-\infty}^{t} G(t-t')\,\underline{\underline{\dot{\gamma}}}(t')\,dt'$$

Good only for small strains, small strain-rates

$\underline{\underline{\dot{\gamma}}}$: strain-rate tensor

To develop constitutive equations for large strain, large strain-rate flows, the strain and strain history are important.

What is the strain measure that is used in the GLVE model?

Generalized Linear-Viscoelastic Model: (strain version)

$$\underline{\underline{\tau}} = +\int_{-\infty}^{t} M(t-t')\,\underline{\underline{\gamma}}(t,t')\,dt'$$

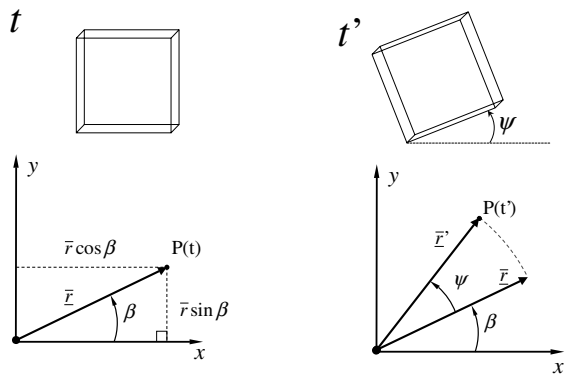
$\underline{\underline{\gamma}}$: infinitesimal strain tensor

$$M(t-t') \equiv \frac{\partial G(t-t')}{\partial t'}$$

$M(t-t')$: memory function

It is the use of the infinitesimal strain tensor as the strain measure that causes the frame-variance in the GLVE model.

No stress is generated when a fluid is rotated;
what does the GLVE predict?



GLVE Prediction for Rigid-Body Rotation around the z-axis

$$\underline{\underline{\tau}} = + \int_{-\infty}^{t} M(t-t') \begin{pmatrix} 2(\cos\psi - 1) & 0 & 0 \\ 0 & 2(\cos\psi - 1) & 0 \\ 0 & 0 & 0 \end{pmatrix}_{xyz} dt'$$

Why does GLVE make this erroneous prediction?

$$\underline{\underline{\gamma}}(t,t') = \nabla \underline{u}(t,t') + \left[\nabla \underline{u}(t,t')\right]^T$$

$$\underline{u}(t,t') = \underline{r}(t') - \underline{r}(t)$$

Because this vector, while accounting for deformation, ***also accounts for changes in orientation***.