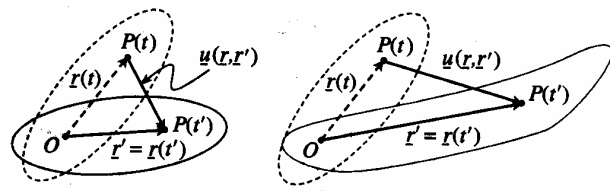


$$\underline{\gamma}(t, t') = \nabla \underline{u}(t, t') + [\nabla \underline{u}(t, t')]^T$$

$$\underline{u}(t, t') = \underline{r}(t') - \underline{r}(t)$$

Accounts for changes in shape and orientation.

$\underline{u}(\underline{r}, t') = \underline{r}' - \underline{r}$
Origin O
fixed in space



Orientation changes
(\underline{r} changes direction)
Shape does not change
(length of \underline{r} does not change)

Orientation changes
Shape changes

© Faith A. Morrison, Michigan Tech U.

We desire a strain tensor that accurately captures large-strain deformation without being affected by rigid-body rotation.

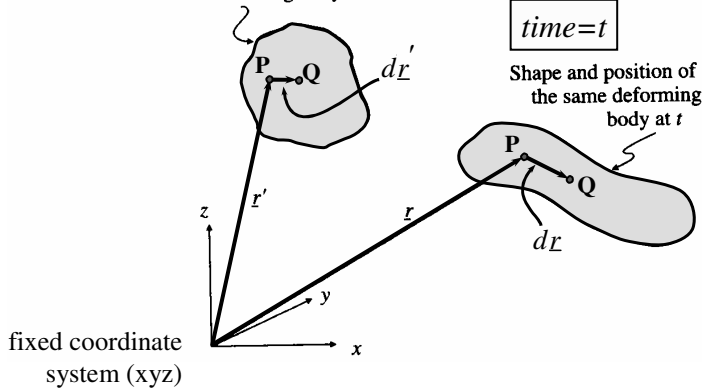
Consider:

$time = t'$

Shape and position of a deforming body at t'

$time = t$

Shape and position of the same deforming body at t



© Faith A. Morrison, Michigan Tech U.

Deformation-
gradient tensor

$$\underline{\underline{F}}(t, t') \equiv \frac{\partial \underline{r}'}{\partial \underline{r}} = \frac{\partial r'_i}{\partial r_p} \hat{e}_p \hat{e}_i = \begin{pmatrix} \frac{\partial x'}{\partial x} & \frac{\partial y'}{\partial x} & \frac{\partial z'}{\partial x} \\ \frac{\partial x'}{\partial y} & \frac{\partial y'}{\partial y} & \frac{\partial z'}{\partial y} \\ \frac{\partial x'}{\partial z} & \frac{\partial y'}{\partial z} & \frac{\partial z'}{\partial z} \end{pmatrix}_{xyz}$$

Inverse deformation-
gradient tensor

$$\underline{\underline{F}}^{-1}(t', t) \equiv \frac{\partial \underline{r}}{\partial \underline{r}'} = \frac{\partial r_m}{\partial r'_j} \hat{e}_j \hat{e}_m = \begin{pmatrix} \frac{\partial x}{\partial x'} & \frac{\partial y}{\partial x'} & \frac{\partial z}{\partial x'} \\ \frac{\partial x}{\partial y'} & \frac{\partial y}{\partial y'} & \frac{\partial z}{\partial y'} \\ \frac{\partial x}{\partial z'} & \frac{\partial y}{\partial z'} & \frac{\partial z}{\partial z'} \end{pmatrix}_{xyz}$$

© Faith A. Morrison, Michigan Tech U.

EXAMPLE: What is the inverse-deformation gradient tensor in steady shear flow?

$$\underline{v} = \begin{pmatrix} \gamma_0 y \\ 0 \\ 0 \end{pmatrix}_{xyz} \quad \underline{r} = \begin{pmatrix} x' + (t - t') \gamma_0 y' \\ y' \\ z' \end{pmatrix}_{xyz}$$

© Faith A. Morrison, Michigan Tech U.

EXAMPLE: What is $\frac{\partial F^{-1}}{\partial t}$?

We can answer by writing the definition of the deformation gradient tensor in Einstein notation and expanding. We will also need the chain rule of differentiation.

© Faith A. Morrison, Michigan Tech U.