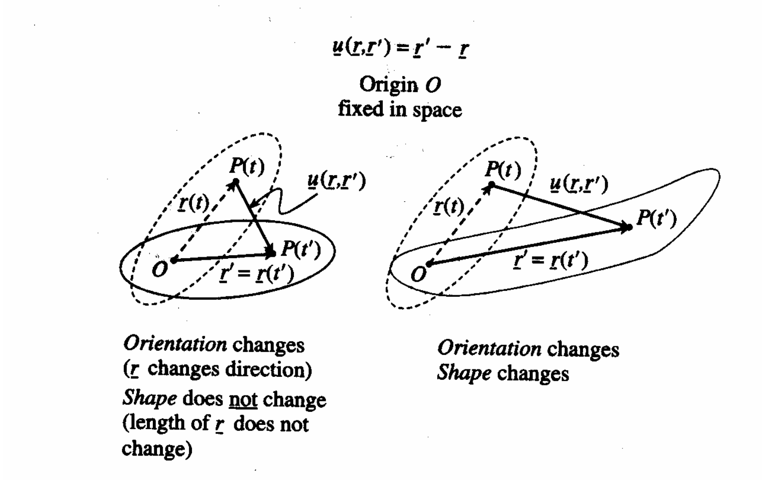
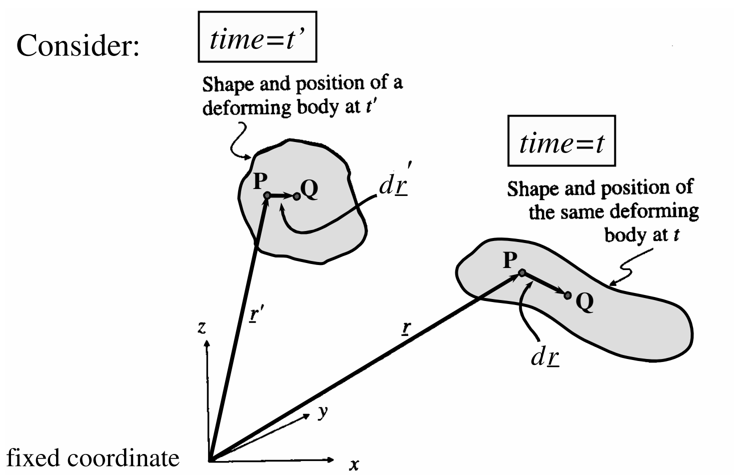
$$\underline{\underline{\gamma}}(t,t') = \nabla \underline{u}(t,t') + \left[\nabla \underline{u}(t,t')\right]^T$$
$$\underline{u}(t,t') = \underline{r}(t') - \underline{r}(t)$$

Accounts for changes in shape and orientation.



We desire a strain tensor that accurately captures large-strain deformation without being affected by rigid-body rotation.

Consider:



fixed coordinate system (xyz)

Deformation-gradient tensor

$$\underline{\underline{F}}(t,t') \equiv \frac{\partial \underline{r}'}{\partial \underline{r}} = \frac{\partial r_i'}{\partial r_p}\hat{e}_p\hat{e}_i = \begin{pmatrix} \frac{\partial x'}{\partial x} & \frac{\partial y'}{\partial x} & \frac{\partial z'}{\partial x} \\ \frac{\partial x'}{\partial y} & \frac{\partial y'}{\partial y} & \frac{\partial z'}{\partial y} \\ \frac{\partial x'}{\partial z} & \frac{\partial y'}{\partial z} & \frac{\partial z'}{\partial z} \end{pmatrix}_{xyz}$$

Inverse deformation-gradient tensor

$$\underline{\underline{F}}^{-1}(t',t) \equiv \frac{\partial \underline{r}}{\partial \underline{r}'} = \frac{\partial r_m}{\partial r_j'}\hat{e}_j\hat{e}_m = \begin{pmatrix} \frac{\partial x}{\partial x'} & \frac{\partial y}{\partial x'} & \frac{\partial z}{\partial x'} \\ \frac{\partial x}{\partial y'} & \frac{\partial y}{\partial y'} & \frac{\partial z}{\partial y'} \\ \frac{\partial x}{\partial z'} & \frac{\partial y}{\partial z'} & \frac{\partial z}{\partial z'} \end{pmatrix}_{xyz}$$

EXAMPLE: What is the inverse-deformation gradient tensor in steady shear flow?

$$\underline{v} = \begin{pmatrix} \dot{\gamma}_0 y \\ 0 \\ 0 \end{pmatrix}_{xyz} \qquad \underline{r} = \begin{pmatrix} x' + (t - t')\dot{\gamma}_0 y' \\ y' \\ z' \end{pmatrix}_{xyz}$$

EXAMPLE: What is $\frac{\partial \underline{\underline{F}}^{-1}}{\partial t}$?

We can answer by writing the definition of the deformation gradient tensor in Einstein notation and expanding. We will also need the chain rule of differentiation.