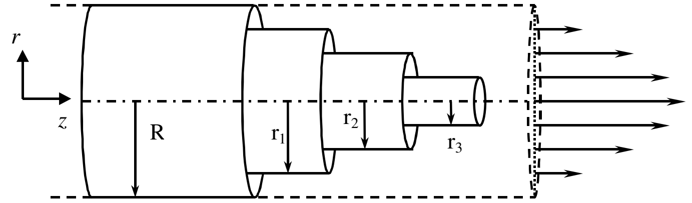
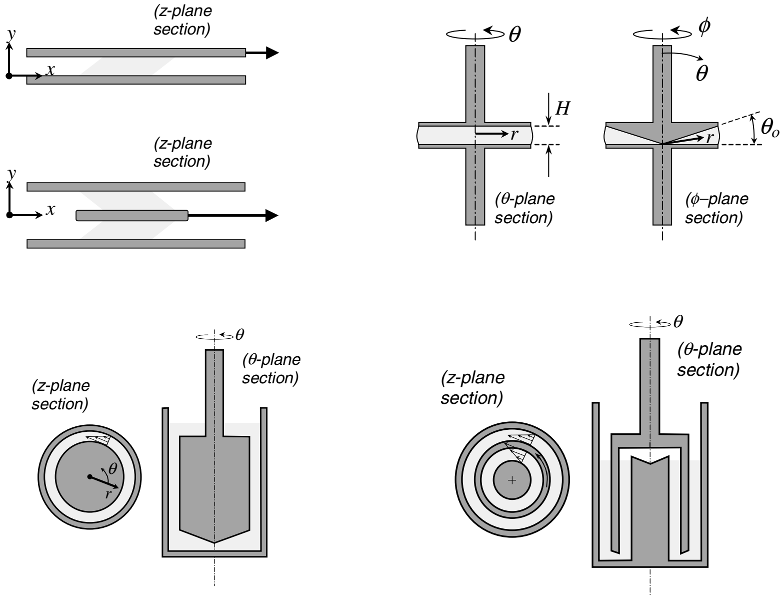
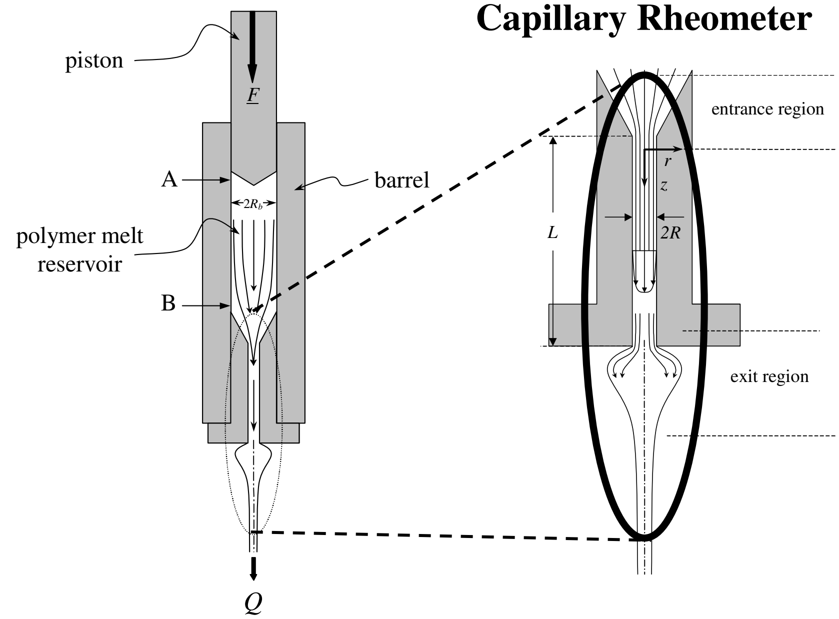
In Chapter 5 we defined material functions based on imposed flows; now we must consider, *how exactly can we impose these flows on materials with unknown rheological properties?*



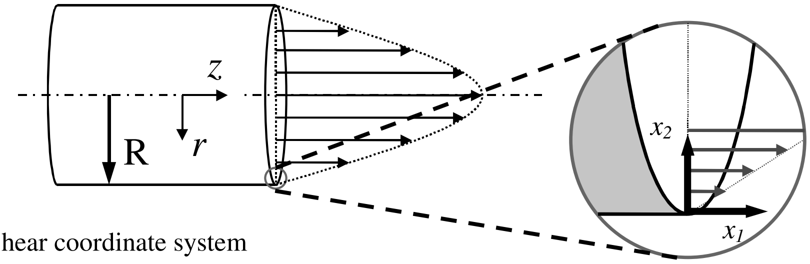
Experimental Shear Geometries



Capillary Rheometer



Viscosity from capillary flow



Shear coordinate system
near wall:

$$\hat{e}_1 = \hat{e}_z$$
$$\hat{e}_2 = -\hat{e}_r$$
$$\hat{e}_3 = -\hat{e}_\theta$$

$$\tau_{21} = -\tau_{rz}\big|_{r=R} \equiv -\tau_R$$

$$\dot{\gamma}_0 = \frac{\partial v_z}{\partial(-r)} = -\frac{\partial v_z}{\partial r} = \dot{\gamma}\big|_{r=R} \equiv \dot{\gamma}_R$$

$$\boxed{\eta = \frac{-\tau_{21}}{\dot{\gamma}_0} = \frac{\tau_R}{\dot{\gamma}_R}}$$

wall shear stress

wall shear rate

To calculate wall shear rate, wall shear stress, look at EOM

$$\eta = \frac{\tau_R}{\dot{\gamma}_R}$$

$$P \equiv p - \rho g z$$

$$\rho\left(\cancel{\frac{\partial \underline{v}}{\partial t}} + \cancel{\underline{v}\cdot\nabla\underline{v}}\right) = -\nabla P - \nabla\cdot\underline{\underline{\tau}}$$

steady state

unidirectional

Assume:
- no θ-dependence
- long tube
- symmetric stress tensor

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{r\theta z} = \begin{pmatrix} -\frac{\partial P}{\partial r} \\ 0 \\ -\frac{\partial P}{\partial z} \end{pmatrix}_{r\theta z} - \begin{pmatrix} \frac{1}{r}\frac{\partial r\tau_{rr}}{\partial r} - \frac{\tau_{\theta\theta}}{r} \\ \frac{1}{r^2}\frac{\partial r^2\tau_{r\theta}}{\partial r} \\ \frac{1}{r}\frac{\partial r\tau_{rz}}{\partial r} \end{pmatrix}_{r\theta z}$$

Shear stress in capillary flow:

$$\boxed{\tau_{rz} = \frac{(P_0 - P_L)r}{2L} = \tau_R \frac{r}{R}} \qquad \left(\frac{\partial P}{\partial z} = \text{constant}\right)$$

What is shear rate at the wall in capillary flow?

$$\dot{\gamma}_0 = \frac{\partial v_z}{\partial(-r)} = -\frac{\partial v_z}{\partial r} = \dot{\gamma}\big|_{r=R} \equiv \dot{\gamma}_R$$

If $v_z(r)$ is known, it is easy to calculate.

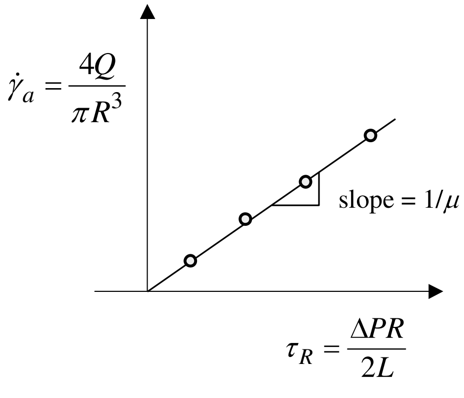
Velocity fields, Flow in a Capillary

Newtonian fluid: $v_z(r) = \frac{2Q}{\pi R^2}\left[1-\left(\frac{r}{R}\right)^2\right]$

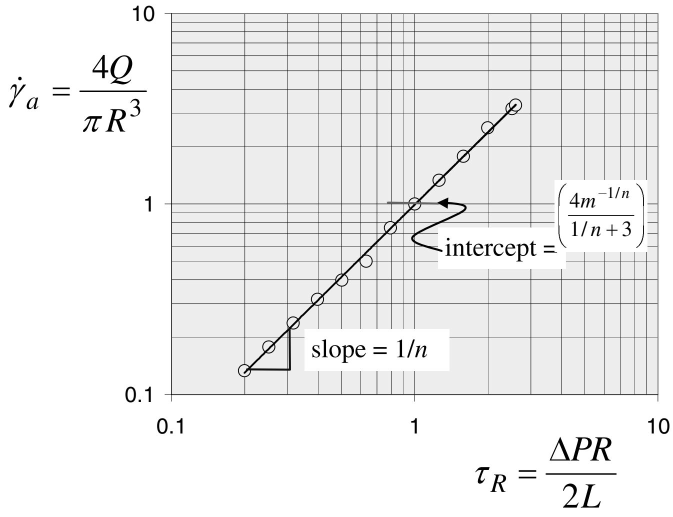
Power-law GNF fluid:

$$v_z(r) = R^{\frac{1}{n}+1}\left(\frac{P_0 - P_L}{2mL}\right)^{\frac{1}{n}}\left(\frac{1}{1/n+1}\right)\left[1-\left(\frac{r}{R}\right)^{\frac{1}{n}+1}\right]$$

Wall shear-rate for a Newtonian fluid

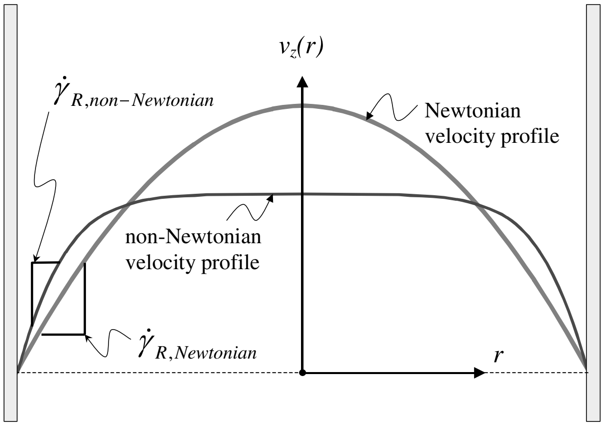


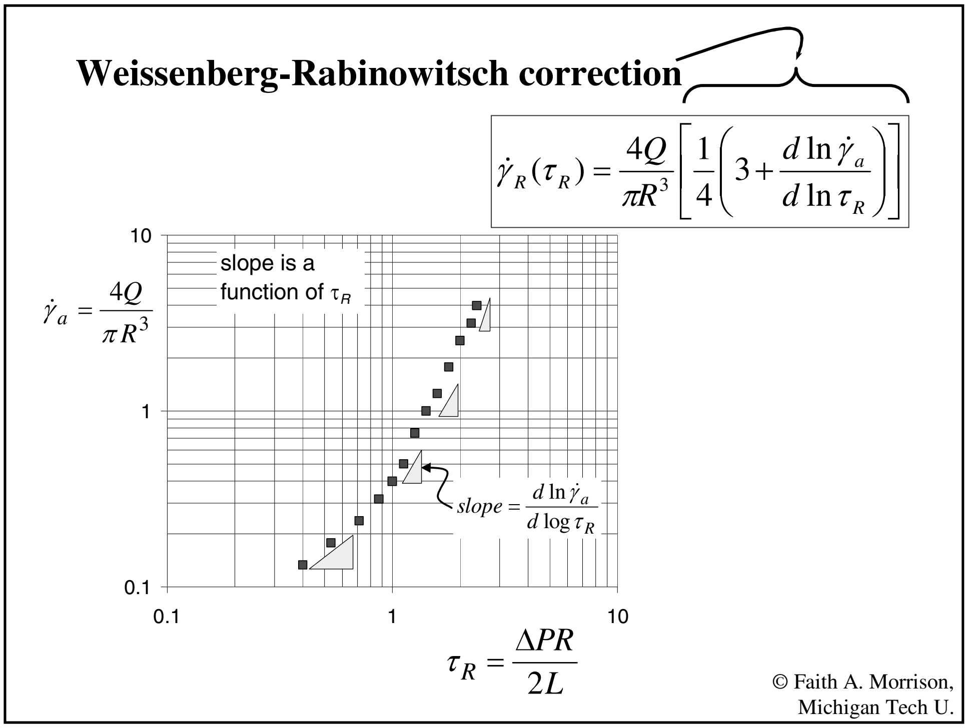
Wall shear-rate for a Power-law GNF



For an unknown, non-Newtonian fluid, we need to take special steps to determine the wall shear rate

The wall shear rate is generally greater than for a Newtonian fluid.





Weissenberg-Rabinowitsch correction

$$\dot{\gamma}_R(\tau_R) = \frac{4Q}{\pi R^3}\overbrace{\left[\frac{1}{4}\left(3+\frac{d\ln\dot{\gamma}_a}{d\ln\tau_R}\right)\right]}$$

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