PARALLEL PLATE RHEOMETER

Measureable: torque vs. rate-of-rotation

$\mathcal{T}$ (torque) $\qquad \Omega$ (rate-of-rotation)

can show $\dot{\gamma} = \dfrac{r\Omega}{H} = \dot{\gamma}_R \dfrac{r}{R}$

(see text)

$$\mathcal{T} = \int_{AREA} (\text{stress})(\text{Lever Arm})\, dA$$

replace w/ $r = \dfrac{\dot{\gamma} R}{\dot{\gamma}_R}$

$dr = \dfrac{R}{\dot{\gamma}_R}\, d\dot{\gamma}$

$$\begin{cases} r = 0 & \dot{\gamma} = 0 \\ r = R & \dot{\gamma} = \dot{\gamma}_R \end{cases}$$

$$= \int_0^R \underbrace{-\tau_{z\theta}\Big|_{z=H}}_{} \; r \; (2\pi r\, dr)$$

$$\eta = \frac{-\tau_{z\theta}(r)}{\dot{\gamma}(r)} = \eta(r)\Big|_{z=H}$$

$$\frac{\mathcal{J}}{2\pi} = \int_0^{\dot\gamma_R} \eta\dot\gamma \; \frac{\dot\gamma^2 R^2}{\dot\gamma_R^2} \; \frac{R}{\dot\gamma_R} \; d\dot\gamma$$

$$\frac{\mathcal{J}}{2\pi R^3}\,\dot\gamma_R^3 = \int_0^{\dot\gamma_R} \eta\dot\gamma^3 \, d\dot\gamma$$

Now, following Weissenberg-Rabinowitsch logic, get rid of integral by differentiating wrt $\dot\gamma_R$:

$$\frac{\mathcal{J}}{2\pi R^3}\left(3\dot\gamma_R^2\right) + \dot\gamma_R^3 \, \frac{d}{d\dot\gamma_R}\left(\frac{\mathcal{J}}{2\pi R^3}\right) = \int_0^{\dot\gamma_R} \underbrace{\frac{\partial}{\partial\dot\gamma_R}\left(\eta\dot\gamma^3\right)}_{0} d\dot\gamma + \underbrace{(1)\,\eta\dot\gamma^3\Big|_{\dot\gamma_R}}_{\eta(\dot\gamma_R)\,\dot\gamma_R^3}$$

solve for $\eta(\dot\gamma_R)$

$$\eta(\dot\gamma_R)\,\dot\gamma_R^{\,3} = \frac{\mathcal{T}\,\dot\gamma_R^{\,2}}{2\pi R^3}\left[3 + \frac{d(\mathcal{T}/2\pi R^3)}{d\dot\gamma_R}\,\frac{\dot\gamma_R}{\mathcal{T}/(2\pi R^3)}\right]$$

$$\frac{dx}{x} = d\ln x$$

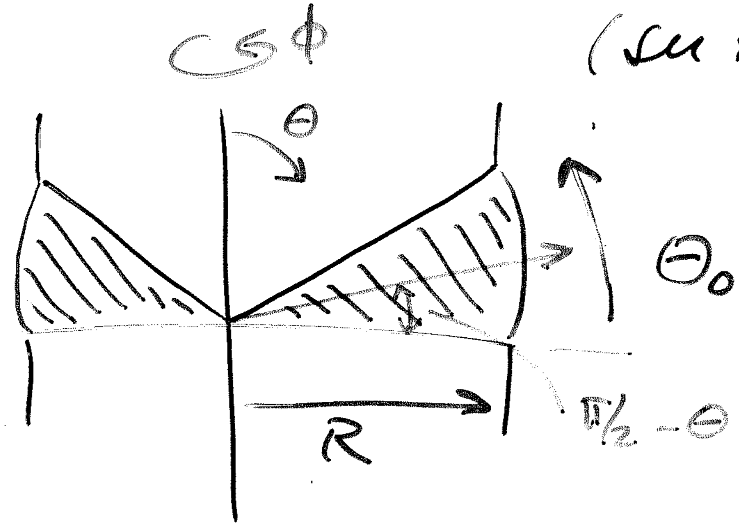
$$\boxed{\eta(\dot\gamma_R) = \frac{\mathcal{T}/(2\pi R^3)}{\dot\gamma_R}\left[3 + \underbrace{\frac{d\ln(\mathcal{T}/2\pi R^3)}{d\ln\dot\gamma_R}}_{\text{get this slope from plot of } \dot\gamma_R \text{ vs } \mathcal{T}/2\pi R^3 \text{ (slope}^{-1})}\right]}$$

$\frac{R\Omega}{H}$

measure $\mathcal{T}$ vs Ω

Cone & Plate Rheometer

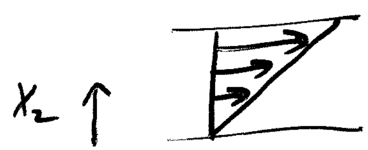
(see text for details)



$$\underline{v} = \begin{pmatrix} 0 \\ 0 \\ v_\phi \end{pmatrix}_{r\theta\phi}$$

Linear velocity profile in these coordinates: $v_\phi = r \underbrace{\frac{\Omega}{\Theta_0}}_{\dot{\gamma}_0} \left(\frac{\pi}{2} - \theta \right)$

NOTE: $\dot{\gamma}$ is independent of coord. position!



$v_1 = \dot{\gamma}_0 x_2$

$x_2 \sim (\pi/2 - \theta) r$

Measure: $\mathcal{T}$ vs Ω

$$\mathcal{T} = \int_{AREA} (\text{stress})(\text{lever arm})\, dA$$

$$= \int_0^{2\pi}\int_0^R \underbrace{\tau_{\theta\phi}\Big|_{\theta=\pi/2}}_{\tau_{\theta\phi} = \tau_{\theta\phi}(\dot{\gamma})} (r)\, r\, d\phi\, dr$$

$\uparrow$ but $\dot{\gamma} = \underline{\underline{\text{constant}}}$

$\Rightarrow \tau_{\theta\phi} = \text{constant}$

see text $\left\{ \begin{array}{l} \text{due to geometry} \end{array} \right.$

$$\eta = \frac{+\,\overbrace{\tau_{\theta\phi}}}{\dot{\gamma}_0}$$

$$\mathcal{T} = \overbrace{\tau_{\theta\phi}\Big|_{\theta=\pi/2}}\; 2\pi \underbrace{\int_0^R r^2\, dr}_{\frac{r^3}{3}\Big|_0^R} = \frac{\eta\, \overset{\overset{\Omega/\Theta_0}{\shortparallel}}{\dot{\gamma}_0}\, 2\pi R^3}{3}$$

$$\boxed{\eta = \frac{3\mathcal{T}\,\Theta_0}{2\pi R^3 \Omega}}$$

No corrections needed!