Calculating and Plotting Size Distributions

An example size distribution from a set of test sieves is shown below in Table 1. Size fractions are normally identified by their top size and bottom size, as shown in column (A). The last size fraction, in this case -45 micrometers, is material that goes through all of the screens and is collected in the bottom pan (the final pan product).

Table 1: Example size distribution

(A) Sieve size range, um	(B) Nominal Sieve Size, um	(C) Individual weight retained, grams	(D) Individual % wt retained	(E) Cumulative % Passing	(F) Cumulative % Retained
+250	250	0.02	0.05	99.95	0.05
-250/+180	180	1.32	2.96	96.99	3.01
-180/+125	125	4.23	9.50	87.49	12.51
-125/+90	90	9.44	21.19	66.30	33.70
-90/+63	63	13.1	29.41	36.89	63.11
-63/+45	45	11.56	25.95	10.94	89.06
-45	0	4.87	10.94	0.00	100.00
Total		44.54	100		

The Nominal Sieve Size, column (B), is the size of the openings in the sieves.

The Individual weight retained, column (C), is the weight in grams that is retained on each sieve in the stack.

Column (D) is calculated from the values in column (C) by dividing each individual weight by the total weight and multiplying by 100.

The Cumulative % Passing, column (E), is the fraction of the total weight that passes through each individual screen. It is calculated for a given size by subtracting the material retained on all of the coarser sieves from 100. For example, in Table 1, the cumulative % passing 63 micrometers is 100 - 0.05 - 2.96 – 9.50 – 21.19 – 29.41 = 36.89 %. The last entry in column E (corresponding to the final pan product), must equal zero if no mistakes were made in the calculation.

The Cumulative % Retained, column (F), is the fraction of the total weight that has been retained by a given screen and all screens coarser than it is. Cumulative % Retained plus Cumulative % Passing at any given size must equal 100%.

Plotting data:

Individual % weight retained can be directly plotted against the sieve size, as shown in Figure 1. This is a frequency plot, and is sometimes plotted as a bar graph. This type of plot is used to determine what size fractions contain most of the material.

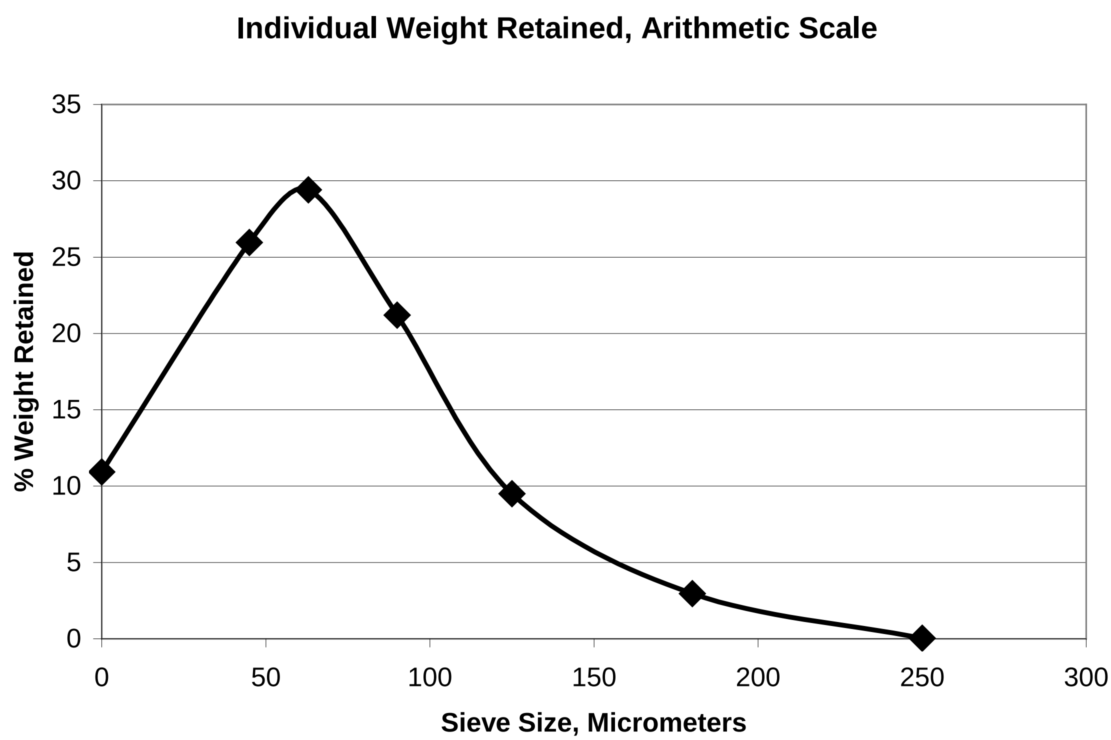

Figure 1: Plot of individual percent retained versus the nominal particle size of each individual size fraction for the values in Table 1.

The cumulative % passing and the cumulative % retained values can also be plotted versus size, as shown in Figure 2. Normally, only the cumulative % passing or cumulative % retained is plotted, not both, since one is the inverse of the other.

Arithmetic and Logarithmic Axes:

In a normal arithmetic plot, the numbers are uniformly spaced on the axis. In a logarithmic plot, the numbers are spaced according to their logarithms, so the larger numbers are closer together than the smaller numbers. The logarithmic plot is preferred for size distributions, because it allows a wide range of particle sizes to be plotted without crowding together the points for the finer size fractions. However, the final pan product should not be included in a logarithmic plot, because it would correspond to a size of 0.0 microns, which would plot at infinity. The example data from Table 1 is plotted on a semi-logarithmic plot (size is logarithmic, cumulative % passing is arithmetic) in Figure 3.

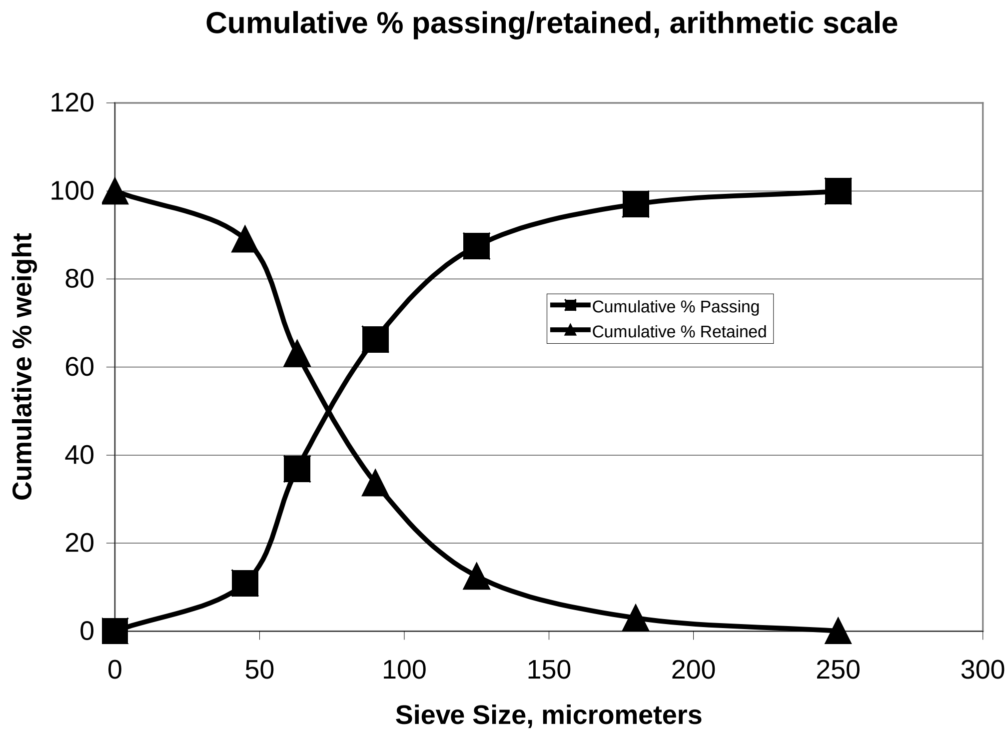

Figure 2: Cumulative % Passing and Cumulative % Retained versus size for the values in Table 1.

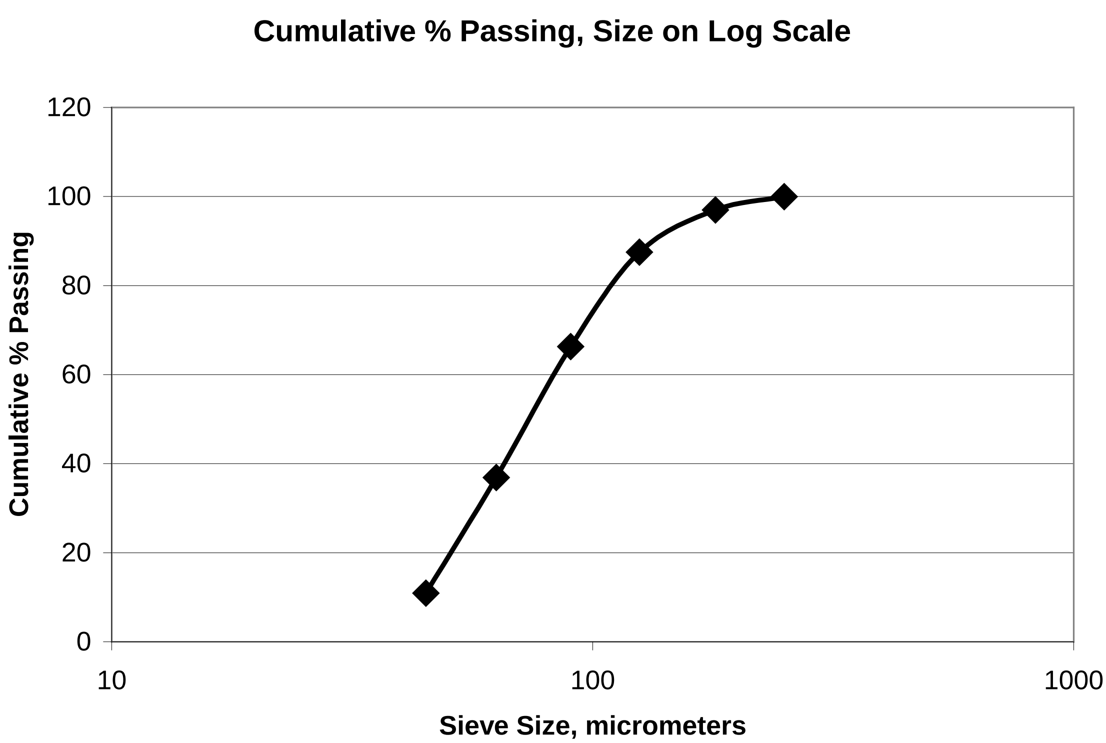

Figure 3: Semi-logarithmic plot of the cumulative % passing data from Table 1.

Gates/Gaudin/Schumann Plot

The Gates-Gaudin-Schumann plot is a graph of cumulative % passing versus nominal sieve size, with both the X and Y axes being logarithmic plots. In this type of plot, most of the data points (except for the two or three coarsest sizes measured) should lie nearly in a straight line. An example plot, from the data given in Table 2, is shown in Figure 4.

Table 2: Example size distribution for Gates/Gaudin/Schumann plot

Sieve size range, um	Nominal Sieve Size, um	Individual weight retained, grams	Individual % wt retained	Cumulative % Passing
+250	250	0.02	0.04	99.96
-250/+180	180	1.32	2.76	97.20
-180/+125	125	12.23	25.58	71.62
-125/+90	90	10.86	22.71	48.91
-90/+63	63	9.13	19.09	29.82
-63/+45	45	5.55	11.61	18.21
-45	0	8.71	18.21	0.00
Total		47.82	100	

The equation for the straight-line portion of the graph shown in Figure 4 is:

$y = 100*(x/k)^a$
where y = cumulative % passing,
x = particle size,
k = size modulus, and
a = distribution modulus.

If we take logs of each side of this equation, it converts to the equation of a straight line:

$\log(y) = a*\log(x) + (2 - a*\log(k))$
where a = slope of the line, and (2 - a*log(k)) = y-intercept of the line.

If the size distribution of particles from a crushing or grinding operation does not approximate a straight line, it suggests that there may have been a problem with the data collection, or there is something unusual happening in the comminution process.
The size modulus is a measure of how coarse the size distribution is, and the distribution modulus is a measure of how broad the size distribution is. Size modulus for a size distribution can be determined from a graph by extrapolating the straight-line portion up to 100% passing and finding the corresponding size value. The distribution modulus can be calculated by choosing two points in the linear portion of the graph, calculating the logs of the sizes and % passing values, and calculating the slope.

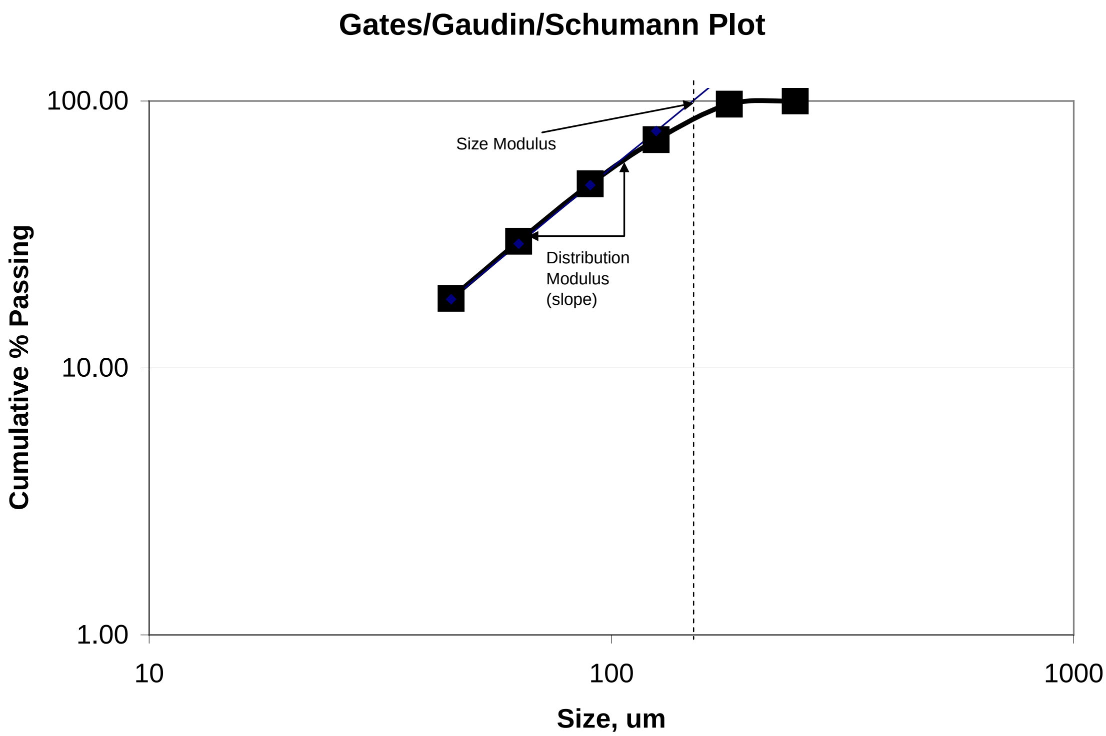

Figure 4: The Gates/Gaudin/Schumann plot of the data in Table 2. Both sieve size and cumulative % passing are plotted on a logarithmic scale. Except for the coarsest fractions, the distribution should approximate a straight line.

For example, in Figure 4 the size modulus can be seen from the graph to be approximately 150 micrometers. The distribution modulus, calculated from values in the linear portion of the graph (% passing 90 micrometers and % passing 45 micrometers), is:

$$a = (\log(48.91)-\log(18.21))/(\log(90)-\log(45)) = 1.42$$

These values can also be calculated more accurately using any curve-fitting software.

Name ________________________

CM2200, Fall 2008, Homework 2

A) Complete the following size analysis table (25 pts)

Sieve Size, Micrometers	Individual Wt. Retained, g	Individual % Wt. Retained	Cumulative % Wt. Retained	Cumulative % Wt. Passing
600	1.25			
424	15.52			
300	45.1			
212	160.4			
150	125.2			
106	97.7			
75	64.3			
53	50.6			
0 (Pan)	70.5			
Total:				

B) Plot Cumulative % Wt. Passing versus size, and estimate the 80% passing size. (10 points)

C) Make a Gates/Gaudin/Schumann plot, and determine the values for the size modulus and the distribution modulus in the Schumann equation. (15 points)