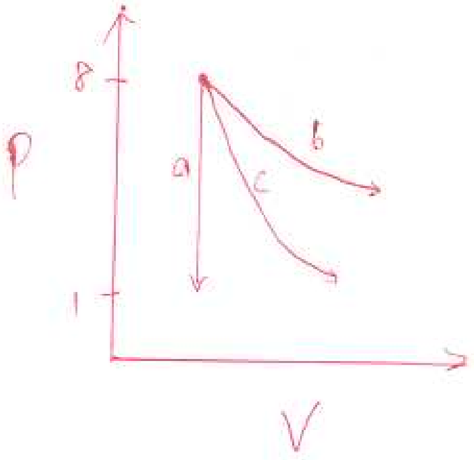
Homework 2 solutions

3.8

Given : $P_1 = 8 \text{ bar}$ $P_2 = 1 \text{ bar}$ $T_1 = 600 \text{ K}$

$C_p = \frac{7}{2} R$ $C_v = \frac{5}{2} R$

Required : W , Q , ΔU , ΔH



a) Constant Volume

$\boxed{W = 0}$

$\Delta U = Q = C_v \cdot \Delta T$

$T_2 = T_1 \frac{P_2}{P_1}$ $\Delta T = T_2 - T_1$, $\Delta T = -525 \text{ K}$

$\Delta U = C_v \cdot \Delta T$ $\boxed{Q = \Delta U = -10.91 \frac{kJ}{mol}}$

$\Delta H = C_p \cdot \Delta T$ $\boxed{\Delta H = -15.28 \frac{kJ}{mol}}$

b) Constant Temperature

$\boxed{\Delta U = \Delta H = 0}$ $Q = W$

$W = R T_1 \ln\left(\frac{P_2}{P_1}\right)$ $\boxed{Q = W = -10.37 \frac{kJ}{mol}}$

c) Adiabatic

$\boxed{Q = 0}$ $\Delta U = W = C_v \cdot \Delta T$

$\gamma = \frac{C_p}{C_v}$ $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$, $T_2 = 331.227 \text{ K}$ $\Delta T = T_2 - T_1$

$\boxed{\Delta U = W = C_v \cdot \Delta T = -5.586 \frac{kJ}{mol}}$

$\boxed{\Delta H = C_p \cdot \Delta T = -7.821 \frac{kJ}{mol}}$

3.11

Given: $T P^{(1-\delta)/\delta}$ = constant

$$\frac{dP}{dz} = -M\rho g$$

Required: $\frac{dT}{dz}$

$$\frac{dT}{dz} = \frac{dT}{dP} \times \frac{dP}{dz}$$

$$T = C P^{(\delta-1)/\delta}$$

let $n = \delta - 1/\delta$

$$\frac{\partial T}{\partial P} = C n P^{n-1}$$

$$\frac{\partial T}{\partial P} = C \frac{(\delta-1)}{\delta} P^{\frac{(\delta-1)}{\delta}-1}$$

$$= C \frac{(\delta-1)}{\delta} P^{-1/\delta}$$

$$\rho = \frac{n}{V}$$

$$\frac{\partial P}{\partial z} = -M\left(\frac{n}{V}\right)g$$

$$PV = nRT$$

$$\frac{n}{V} = \frac{P}{RT}$$

$$= \frac{P}{R} \frac{1}{C P^{(\delta-1)/\delta}}$$

$$\frac{\partial T}{\partial z} = \frac{\partial T}{\partial P} \times \frac{\partial P}{\partial z}$$

$$= \left(\cancel{C} \frac{(\delta-1)}{\delta} P^{-1/\delta}\right)\left(-Mg\frac{P}{R \cancel{C} P^{(\delta-1)/\delta}}\right)$$

$$= -\frac{Mg}{R} \frac{(\delta-1)}{\delta} \left[\frac{P^{-1/\delta} \cdot P^{\delta/\delta}}{P^{(\delta-1)/\delta}}\right]$$

$$= -\frac{Mg}{R} \frac{(\delta-1)}{\delta} \left[\frac{\cancel{P^{(\delta-1)/\delta}}}{\cancel{P^{(\delta-1)/\delta}}}\right]$$

$$= \boxed{-\frac{Mg}{R} \frac{(\delta-1)}{\delta}}$$

3.17

a) No work is done ; no heat is transferred

$\Delta U^t = \Delta T = 0$

$T_2 = T_1 = 100\ ^\circ C$

Not reversible

b) The gas is returned to its initial state by isothermal compression.

$W = nRT \ln\left(\frac{V_1}{V_2}\right)$ but $nRT = P_2 V_2$

$V_1 = 4 m^3$ $V_2 = \frac{4}{3} m^3$ $P_2 = 6$ bar

$$\boxed{W = P_2 V_2 \ln\left(\frac{V_1}{V_2}\right) = 878.9 \text{ kJ}}$$

3.22

Given : $C_p = \frac{7}{2}R \quad C_v = \frac{5}{2}R$ (set 1)

$C_p = \frac{5}{2}R \quad C_v = \frac{3}{2}R$ (set 2)

$T_1 = 303.15\ K \quad T_3 = 403.15\ K \quad P_1 = 1\ bar \quad P_3 = 10\ bar$

Required: ΔU , ΔH , Q , W

$\Delta U = C_v(T_3 - T_1) \qquad \Delta H = C_p(T_3 - T_1)$

$\boxed{\Delta U = 2.079 \frac{kJ}{mol}} \qquad \boxed{\Delta H = 2.91 \frac{kJ}{mol}}$

Each part consists of two steps , 12 & 23

a) $T_2 = T_3 \qquad P_2 = P_1 \frac{T_2}{T_1}$

$W_{23} = RT_2 \ln\left(\frac{P_3}{P_2}\right) \qquad \boxed{Work = W_{23} = 6.762 \frac{kJ}{mol}}$

$Q = \Delta U - Work \qquad \boxed{Q = -4.684 \frac{kJ}{mol}}$

b) $P_2 = P_1 \qquad T_2 = T_3 \qquad \Delta U_{12} = C_v(T_2 - T_1)$

$Q_{12} = \Delta H_{12} \qquad \Delta H_{12} = C_p(T_2 - T_1)$

$W_{12} = \Delta U_{12} - Q_{12} = -0.831 \frac{kJ}{mol}$

$W_{23} = RT_2 \ln\left(\frac{P_3}{P_2}\right) = 7.718 \frac{kJ}{mol}$

$\boxed{Work = W_{12} + W_{23} = 6.886 \frac{kJ}{mol}}$

$\boxed{Q = \Delta U - Work = -4.808 \frac{kJ}{mol}}$

3.22 cont

c) $T_2 = T_1$ $P_2 = P_3$ $W_{12} = RT_1 \ln\left(\frac{P_2}{P_1}\right)$

$\Delta H_{23} = C_p(T_3 - T_2)$ $Q_{23} = \Delta H_{23}$

$\Delta U_{23} = C_v(T_3 - T_2)$ $W_{23} = \Delta U_{23} - Q_{23}$

$\boxed{\text{Work} = W_{12} + W_{23} = 4.972\ \frac{kJ}{mol}}$

$\boxed{Q = \Delta U - \text{work} = -2.894\ \frac{kJ}{mol}}$

For second set of heat capacity, answer in (kJ/mol)

$\boxed{\Delta U = 1.247}$ $\boxed{\Delta H = 2.079}$

a) $\boxed{\text{Work} = 6.762}$ $\boxed{Q = -5.515}$

b) $\boxed{\text{Work} = 6.886}$ $\boxed{Q = -5.639}$

c) $\boxed{\text{Work} = 4.972}$ $\boxed{Q = -3.725}$

3.28

Given : from P_1 to P_2

$$Z = 1 + B'P$$

Compare to ideal gas equation

$$Z = \frac{PV}{RT}$$

$$1 + B'P = \frac{PV}{RT}$$

$$V = \frac{RT(1+B'P)}{P} = \frac{RT}{P} + RTB'$$

$$dV = -\frac{RT}{P^2} dP$$

$$W = -\int P \, dV$$

$$W = \int_{P_1}^{P_2} P \left(\frac{RT}{P^2} \right. dP$$

$$W = RT \int_{P_1}^{P_2} \frac{1}{P} \, dP$$

$$\boxed{W = RT \ln \left(\frac{P_2}{P_1} \right)}$$

∴ same as equation for ideal Gas