Homework 4 Solutions

5.2

Let the symbols Q and work represent rates in kJ/s.

Then by Eq. (5.8)

$$\eta = \frac{|work|}{|Q_H|} = 1 - \frac{T_C}{T_H}$$

$T_C = 323.15$ K

$T_H = 798.15$ K

$Q_H = 250 \frac{kJ}{s}$

$$Work = \left| Q_H \cdot \left(1 - \frac{T_C}{T_H}\right) \right|$$

$$\boxed{\text{Power Developed} = 148.78 \frac{kJ}{s} \quad \text{or} \quad 148.78 \text{ kW}}$$

By eq. (5.1)

$$Q_C = |Q_H| - |work|$$

$$\boxed{Q_C = 101.22 \frac{kJ}{s}}$$ (Heat rejected)

5.9

$P_1 = 1 \text{ bar}$ $\quad T_1 = 500 K$ $\quad V = 0.06 \, m^3$

$n = \frac{P_1 V}{R T_1}$ $\quad n = 1.443 \text{mol}$ $\quad C_V = \frac{5}{2} R$

$Q = 15000 J$

a) constant V heating

$$\Delta U = Q + W$$
$$= Q$$
$$= n \, C_V (T_2 - T_1)$$

$T_2 = T_1 + \frac{Q}{n C_V}$ $\quad T_2 = 1 \times 10^3 K$

By Eq. (5.18) $\quad \Delta S = n\left(C_P \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{P_2}{P_1}\right)\right)$

But $\frac{P_2}{P_1} = \frac{T_2}{T_1}$ whence $\Delta S = n \, C_V \ln\left(\frac{T_2}{T_1}\right)$

$$\boxed{\Delta S = 20.794 \frac{J}{K}}$$

b) The entropy change of the gas is the same as in (a).
The entropy change of the surroundings is zero. whence

$$\boxed{\Delta S_{total} = 20.794 \frac{J}{K}}$$

The stirring process is irreversible

5.25

$P = 4 \qquad T = 800$

Step 1-2 Volume decreases at constant P.
Heat flows out of the system. Work is done on the system.

$$W_{12} = -\left[P(V_2 - V_1) \right] = -\left[R(T_2 - T_1) \right]$$

Step 2-3 Isothermal compression. Work is done on the system.
Heat flows out of the system

$$W_{23} = RT_2 \ln\left(\frac{P_3}{P_2}\right) = RT_2 \ln\left(\frac{P_3}{P_1}\right)$$

Step 3-1 Expansion process that produces work.
Heat flow into the system. Since the PT product is constant.

$$P \cdot dT + T\,dP = 0 \qquad T \cdot \frac{dP}{P} = -dT \quad \text{------ (A)}$$

$$PV = RT \qquad P\,dV + V\,dP = R\,dT$$

$$P\,dV = R\,dT - V\,dP = R\,dT - RT\frac{dP}{P}$$

In combination with (A) this becomes

$$P\,dV = R\,dT + R\,dT = 2R\,dT$$

Moreover, $\quad P_3 = P_1 \dfrac{T_1}{T_3} = P_1 \dfrac{T_1}{T_2}$

5.25 cont.

$$W_{31} = -\int_{V_3}^{V_1} P\,dV = -2R(T_1 - T_3) = -2R(T_1 - T_2)$$

$$Q_{31} = \Delta U_{31} - W_{31} = C_v(T_1 - T_3) + 2R(T_1 - T_3)$$

$$Q_{31} = (C_v + 2R)(T_1 - T_3) = (C_p + R)(T_1 - T_2)$$

$$\eta = \frac{|W_{net}|}{Q_{in}} = \frac{|W_{12} + W_{23} + W_{31}|}{|Q_{31}|}$$

$C_p = \frac{7}{2}R$ $\quad T_1 = 700K$ $\quad T_2 = 350K$

$P_1 = 1.5$ bar $\quad P_3 = P_1 \frac{T_1}{T_2}$

$W_{12} = -[R(T_2 - T_1)]$ $\quad W_{12} = 2.91 \times 10^3 \frac{J}{mol}$

$W_{23} = R\,T_2 \ln\left(\frac{P_3}{P_1}\right)$ $\quad W_{23} = 2.017 \times 10^3 \frac{J}{mol}$

$W_{31} = -2R(T_1 - T_2)$ $\quad W_{31} = -5.82 \times 10^3 \frac{J}{mol}$

$Q_{31} = (C_p + R)(T_1 - T_2)$ $\quad Q_{31} = 1.309 \times 10^4 \frac{J}{mol}$

$$\eta = \frac{|W_{21} + W_{23} + W_{31}|}{Q_{31}}$$

$$\boxed{\eta = 0.068}$$

5.30

$T_1 = 523.15\ K$ $\qquad T_2 = 353.15\ K$ $\qquad C_p = \frac{7}{2}R$

$P_1 = 3\ bar$ $\qquad P_2 = 1\ bar$ $\qquad C_v = C_p - R$

$T_{res} = 303.15\ K$ $\qquad Work = -1800\ \frac{J}{mol}$

$$Q = \Delta U - Work$$

$$Q = C_v(T_2 - T_1) - Work$$

$$\Delta S_{res} = \frac{-Q}{T_{res}} \qquad \Delta S_{res} = 5.718\ \frac{J}{mol \cdot K}$$

$$Q = -1.733 \times 10^3\ \frac{J}{mol}$$

$$\Delta S = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$\Delta S = -2.301\ \frac{J}{mol \cdot K}$$

$$\Delta S_{total} = \Delta S + \Delta S_{res}$$

$$\boxed{\Delta S_{total} = 3.42\ \frac{J}{mol \cdot K}}$$ (Process is possible)

5.42

$Q_H = 1 \text{ kJ}$

$W = 0.45 \text{ kJ}$

$T_H = (250 + 273.15)\text{ K}$ $\quad T_H = 523.15 \text{ K}$

$T_c = (25 + 273.15)\text{ K}$ $\quad T_c = 298.15 \text{ K}$

$\eta_{actual} = \dfrac{|W|}{|Q_H|}$ $\quad \eta_{actual} = 0.45$

$\eta_{max} = 1 - \dfrac{T_c}{T_H}$ $\quad \eta_{max} = 0.43$

since $\boxed{\eta_{actual} > \eta_{max}\text{, the process is impossible}}$