Chemical Engineering 4310
Fall Semester, 2005
Homework Assignment #3
Chapters 4 and 5
(For return in marked box in hall of 2nd floor
Chemical Sciences by noon on Friday, October 13)

Work problems 4-12, 4-13, 4-15, 4-33, 5-4, and 5-12 in the text

1. A tank truck hauling liquid benzene (C_6H_6) has overturned on I-94 in Detroit and a pool of benzene 30 m in diameter has formed. The terrain is fairly flat. It is 1-pm in the afternoon on a clear, sunny day. The wind is blowing at 7 m/s. The ambient temperature is 30°C.
   a. Estimate the evaporation rate of the benzene in kg/s.
   b. Use a dispersion model to estimate the downwind distance, in meters, to the ERPG-1 concentration.

   All physical properties required are contained within the texbook.

2. Liquid chlorine is supplied to a process from a regulated pressure source at 20 barg and supplied through 300 m of horizontal commercial steel pipe of actual inside diameter of 0.02 m. The ambient pressure is 1 atm and the ambient temperature is 30°C.
   A. If the pipe breaks off at the regulated source, estimate the flow rate through the leak, in kg/s.
   B. If the pipe breaks off at the end of the 300 m length, estimate the flow in kg/s.

   For liquid chlorine, the following properties are available at these conditions:

Density:	1380 kg/m^3
Viscosity:	0.328×10^{-3} Pa·s

4-12 : 15
4-13 : 15
4-15 : 15
4-33 : 10
5-4 : 10

5-12: 10
1: 15
2: 10
100

$\boxed{4\text{-}12}$ a) Physical properties of crude:

$$\rho = 928 \ kg/m^3$$

$$\mu = 0.004 \ kg/m\text{-}sec$$

Use mechanical energy balance, equation 4-28, along with 4-29, 4-30 and Table 4-2.
Have an entrance, a pipe length, and an exit

$$\therefore F \simeq \left(K_{ENT} + K_{PIPE} + K_{EXIT}\right)\left(\frac{u^2}{2g_c}\right)$$

$$K_{ENT} \simeq \frac{160}{Re} + 0.5$$

$$K_{PIPE} = \frac{4fL}{d}$$

$$K_{EXIT} \simeq 1.0$$

Assume commercial steel pipe, then from Table 4-1

$\epsilon \simeq 0.046 \text{ mm}$

and $\frac{\epsilon}{d} = \frac{0.046}{50} = 9.2 \times 10^{-4} = 0.00092$

Also $Re = \frac{du\rho}{\mu} = \frac{(0.05m)(u)(928 \text{ kg/m}^3)}{0.004 \text{ kg/m s}}$

$= 1.16 \times 10^4 u$

Procedure

1.) Guess u

2.) Compute Re

3.) Determine f from Figure 4-7 or equations

4.) Calculate K's

5.) Calculate u from ME balance

$$\frac{u^2}{2g_c} = -\left(F + \frac{g}{g_c}\Delta z\right)$$

6.) Iterate or continue guessing until converged

Spreadsheet solution shown on next page.

Problem 4-12a: Liquid Discharge through a Piping System

Input Data:

Guessed discharge velocity:	7.208 m/s

Fluid density:	928	kg/m**3
Fluid viscosity:	0.004	kg/m*s
Pipe diameter:	0.05	m
Pipe roughness:	0.046	mm
Point 1 pressure:	0	Pa
Point 2 pressure:	0	Pa
Point 1 velocity:	0	m/s
Point 1 height:	9	m
Point 2 height:	0	m
Pipe length:	2	m
Net pump energy:	0	kw

Fittings:

	Number	K1	K-inifinity
Elbows:	0	800	0.4
Valves:	0	300	0.1
Inlet:	1	160	0.5
Exit:	1	0	1

Calculated Results:

Reynolds No:	83617	
Friction factor:	0.0056	0.000373
Pipe area:	0.0020	m**2

Fittings and pipe K factors:

Elbows:	0.000
Valves:	0.000
Inlet:	0.502
Exit:	1.000
Pipe:	0.895
TOTAL:	2.397

Mechanical energy balance terms (m**2/s**2):

Pressure:	0.00
Height:	-88.26
Point 1 velocity:	0.00
Fittings/pipe:	62.28
Pump:	0.00
TOTAL:	-25.98

Calculated Discharge Velocity:	7.208 m/s
Velocity Difference:	-7.3E-06 m/s

Resulting mass discharge rate:	13.13 kg/s

b) For a hole in a tank, equation 4-12 applies, but
$P_g = 0$

$$Q_m = \rho A C_o \sqrt{2 g h_L}$$

$$= \left(\frac{928\ kg}{m^3}\right)\left(1.96\times10^{-3} m^2\right)(.61)\sqrt{(2)(9.8\ m/sec^2)(7\ m)}$$

$$= 13.0\ kg/sec$$

Can also use 2-K Method, shown on spreadsheet output on next page. Get 13.5 kg/s, which is close to answer above.

Problem 4-12b: Liquid Discharge through a Piping System

Input Data:

Guessed discharge velocity:	7.408 m/s

Fluid density:	928	kg/m**3
Fluid viscosity:	0.004	kg/m*s
Pipe diameter:	0.05	m
Pipe roughness:	0.046	mm
Point 1 pressure:	0	Pa
Point 2 pressure:	0	Pa
Point 1 velocity:	0	m/s
Point 1 height:	7	m
Point 2 height:	0	m
Pipe length:	0	m
Net pump energy:	0	kw

Fittings:

	Number	K1	K-inifinity
Elbows:	0	800	0.4
Valves:	0	300	0.1
Inlet:	1	160	0.5
Exit:	1	0	1

Calculated Results:

Reynolds No:	85933	
Friction factor:	0.0056	0.000368
Pipe area:	0.0020	m**2

Fittings and pipe K factors:

Elbows:	0.000
Valves:	0.000
Inlet:	0.502
Exit:	1.000
Pipe:	0.000
TOTAL:	1.502

Mechanical energy balance terms (m**2/s**2):

Pressure:	0.00
Height:	-68.65
Point 1 velocity:	0.00
Fittings/pipe:	41.21
Pump:	0.00
TOTAL:	-27.44

Calculated Discharge Velocity:	7.408 m/s
Velocity Difference:	-6E-11 m/s

Resulting mass discharge rate:	13.50 kg/s

$\boxed{4\text{-}13}$

$T = 80^\circ F = 540^\circ R$

$P_o = 2200$ psia

Assume adiabatic, choked flow, and an ideal gas

N_2 - 28 lbm/lbmole

$$A = \frac{\pi d^2}{4} = \frac{(3.14)(0.5 \text{ in})^2 (1 \text{ ft}^2/144 \text{ in}^2)}{4} = 1.36 \times 10^{-3} \text{ ft}^2$$

Assume $C_o = 1$ $\quad\quad$ $\gamma = 1.41$ (Table 4-3)

Equation 4-50 applies:

$$Q_{m\,(\text{CHOKED})} = C_o A P_o \sqrt{\frac{\gamma g_c M}{R_g T_o}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$$= (1)(1.36 \times 10^{-3} \text{ ft}^2)(2200 \text{ lb}_f/\text{in}^2)(144 \text{ in}^2/\text{ft}^2)$$

$$\times \sqrt{\frac{(1.4)(32.17 \text{ ft-lb}_m/\text{lb}_f\text{-sec}^2)(28 \text{ lb}_m/\text{lbmole})}{(1545 \text{ ft-lb}_f/\text{lbmole-}^\circ R)(540^\circ R)}\left(\frac{2}{2.4}\right)^{(2.4/0.4)}}$$

$= 9.69$ lbm/sec

Use equation 4-53 to compute the velocity of the flow:

$$a = \sqrt{\gamma g_c R_g T / M}$$

Need the temperature at choked conditions.

Use Equation 4-63

$$\frac{T_{choked}}{T_1} = \frac{2Y_1}{\gamma+1}$$

Assume $Y_1 = 1$ since $Ma_1 \to 0$.

Then $$\frac{T_{choked}}{T_1} = \frac{2(1)}{2.41} = 0.829$$

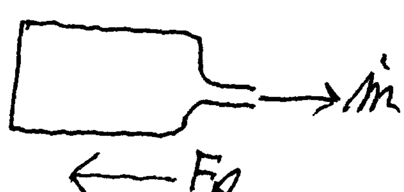
$$T_{choked} = (0.829)(540°R) = 448°R$$

Then

$$a = \sqrt{(1.41)(32.17\ ft\ lb_m/lb_f\ s^2)\left(1545\ \frac{ft\ lb_f}{lb\text{-}mole\ °R}\right)(448°R)\Big/\left(28\ \frac{lb_m}{lb\text{-}mole}\right)}$$

$= 1059$ ft/sec

Use momentum balance to determine force



$$\sum F_x = +F_p - A(P_E - P_A) = \frac{Q_m u}{g_c}$$

Need choking pressure, from Equation 4-49

$$\frac{P_{choked}}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} = \left(\frac{2}{2.41}\right)^{1.41/0.41} \simeq 0.526$$

$$\therefore P_{choked} = (0.526)(2200\,psia) = 1158\,psia$$

Then,

$$F_p = \frac{(9.69\,lb_m/sec)(1059\,ft/sec)}{32.17\,ft\,lb_m/lb_f\,sec^2} +$$

$$(1.36\times10^{-3}\,ft^2)\left(\frac{144\,in^2}{ft^2}\right)(1158-14.7)\frac{lb_f}{in^2}$$

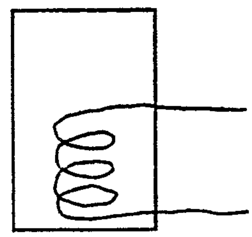
$$\simeq (319 + 224)\,lb_f$$

$$\simeq 542\,lb_f$$

This is a huge force. Cylinder may rocket away.

4-15

Reactor Vessel



Coils are ½" schedule 80, ID = 0.546"
Equivalent length of pipe = 53 ft
Coil length = 20 ft

a) Oriface

Assume choked flow through a 0.546" diameter hole. From equation 4-50:

$$(Q_m)_{CHOKED} = C_o A P_o \sqrt{\left(\frac{\gamma g_c M}{R_g T_o}\right)\left(\frac{2}{\gamma+1}\right)^{(\gamma+1/\gamma-1)}}$$

For H_2O $\gamma \approx 1.32$

$$A = \frac{\pi d^2}{4} = \frac{(3.14)(0.546\ \text{in})^2\left(\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right)}{4} = 1.625 \times 10^{-3}\ \text{ft}^2$$

$M = 18$

Assuming saturated steam, $T_o = 353°F = 813°R$ (from steam tables)

Assume $C_o = 1.0$, and

$$\left(\frac{2}{\gamma+1}\right)^{(\gamma+1/\gamma-1)} = \left(\frac{2}{2.32}\right)^{(2.32/.32)} = 0.341$$

$$(Q_m)_{CHOKED} = (1.0)(1.625 \times 10^{-3}\ \text{ft}^2)(125+14.7)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{144\ \text{in}^2}{\text{ft}^2}\right)$$
$$\times \sqrt{\frac{(1.32)(32.12\ \text{ft-lbm/lbf-sec}^2)(18\ \text{lbm/lbmole})}{(1545\ \text{ft-lbf/lbmole-°R})(813\ °\text{R})(.341)^{-1}}}$$

$$= 32.69 \sqrt{2.075 \times 10^{-4}}$$

$$(Q_m)_{CHOKED} = \underline{\underline{0.471 \ lb_m/sec}}$$

b) For the adiabatic case, worse case occurs when coil breaks off a the <u>beginning of coil</u>. Thus, coil length = 0 and 125 psig steam flows only through a 53 foot (equivalent) pipe. Assume choked, adiabatic flow. Then equations 4-63 through 4-67 are valid. Solve for Ma from equation 4-67:

$L = 53 \text{ ft}$

$d = 0.546'' = 0.0455 \text{ ft.} = 13.87 \text{ mm}$

Determine f assuming fully developed turbulent flow. From Table 4-1, $\epsilon = 0.046$ mm (commercial steel pipe).

$$\frac{1}{\sqrt{f}} = 4 \log\left(3.7 \frac{d}{\epsilon}\right)$$

$$= 4 \log\left[(3.7)\left(\frac{13.87 \text{ mm}}{0.046 \text{ mm}}\right)\right] = 12.19$$

$$\sqrt{f} = 0.0820 \Rightarrow f = 6.73 \times 10^{-3}$$

$$\left(\frac{4fL}{d}\right)\gamma = \left[\frac{(4)(6.73 \times 10^{-3})(53 \text{ ft})}{0.0455 \text{ ft}}\right](1.32) = 41.39$$

(part of equation 4-67)

$$Y_1 = 1 + \frac{1.32-1}{2} Ma^2 = 1 + .16\, Ma^2 \quad \text{(part of 4-67)}$$

$$\left(\frac{\gamma+1}{2}\right) = \frac{1.32+1}{2} = 1.16 \quad \text{"}$$

Then:

$$1.16 \ln\left(\frac{2 + 0.32\, Ma^2}{2.32\, Ma^2}\right) - \left(\frac{1}{Ma^2} - 1\right) + 41.39 = 0$$

Guessed Ma	LHS of Equation
0.3	2.64 − 10.11 + 41.39 = 33.9
0.1	5.17 − 99.0 + 41.39 = −52.44
0.2	21.69 − 24.0 + 41.39 = 39.08
0.15	4.23 − 43.44 + 41.39 = 2.18
0.145	4.32 − 46.56 + 41.39 = −.858
0.146	4.30 − 45.9 + 41.39 = −.223

Close enough!

From equation 4-66:

$$Q_m = Ma_1 P_1 A \sqrt{\frac{\gamma g_c M}{R_g T_1}}$$

$$= (0.141)(125 + 14.7)\left(\frac{lb_f}{in^2}\right)\left(\frac{144\, in^2}{ft^2}\right)(1.625 \times 10^{-3}\, ft^2)$$

$$\times \sqrt{\frac{(1.32)(32.17\ ft\text{-}lb_m/lb_f\text{-}sec^2)(18\ lb_m/lbmole)}{(1545\ ft\text{-}lb_f/lbmole\text{-}{}^\circ R)(813\,{}^\circ R)}}$$

$$= (8.75)\sqrt{6.085 \times 10^{-4}} = \underline{0.117\ lb_m/sec}$$

Can also solve problem using Equation 4-68 and correlations in Figure 4-13, 4-14 and Table 4-4. Solution is direct. Results are shown below.

Problem 4-15: Gas Discharge through a Piping System

Input Data:

Heat capacity ratio, k:	1.32	
Temperature:	452	K = 813°R
Molecular weight of gas:	18	
Point 1 pressure:	962932	Pa ≃ (125+14.7) psia
Point 2 pressure:	101325	Pa
Pipe diameter:	0.0139	m ≃ 0.546"
Pipe length:	16.1544	m ≃ 53 ft
Pipe roughness:	0.046	mm

Fittings:

	Number	K infinite
Elbows:	0	0.4
Valves:	0	0.1
Inlet:	0	0.5
Exit:	1	1

Calculated Results:

Pipe area:	0.000152	m**2
Initial gas density:	4.61	kg/m**3
Pipe friction factor:	0.006725	

Equation 4-37

Fittings and pipe K factors:

Elbows:	0.00
Valves:	0.00
Inlet:	0.00
Exit:	1.00
Pipe:	31.26
TOTAL:	32.26

Ln(K): 3.47

Expansion factor: 0.717

Equation in Table 4-4 or Fig. 4-14

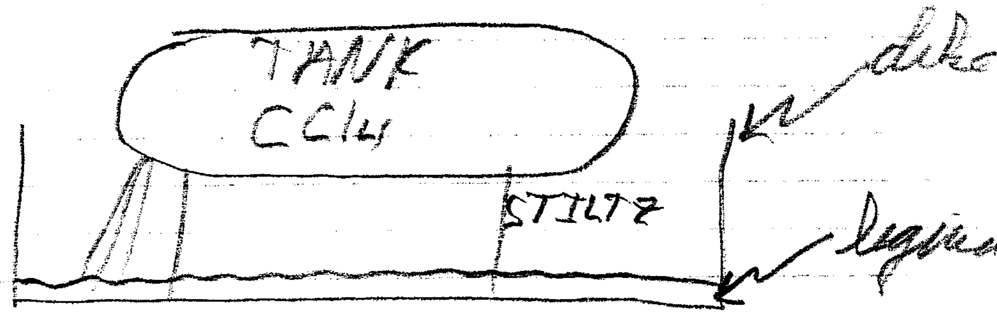
Heat capacity ratio,k:	1.2	1.4	1.67	
(P1 - P2)/P1:	0.859	0.873	0.891	
P-choked:	135841.9	122153.7	104783.7	Pa
Mass flow:	0.052938	0.053374	0.053922	kg/s

Table 4-4 or Figure 4-13

Equation 4-68

$\boxed{\text{Interpolated mass flow: } 0.053199 \text{ kg/s}}$ = 0.117 lb_m/sec

4-33



$T = 35°C = 273+35 = 308\,K$

$M = 12 + (4)(35.4) = 153.6$

a.) Use equation (3-12) to calculate Q_m.

At steady state evaporation rate = spill rate.

$$Q_m = \frac{MKAP^{sat}}{R_g T_L}$$

From equation (3-18)

$$K = K_o\left(\frac{M_o}{M}\right)^{1/3} = \left(0.83\ \frac{cm}{s}\right)\left(\frac{18}{153.6}\right)^{1/3} = 0.406\, cm/s$$

$$= 0.00406\, m/s$$

p^{sat} provided in Appendix E

$$\ln p^{sat} = 15.8742 - \frac{2808.19}{T - 45.99}$$

$$= 15.8742 - \frac{2808.19}{308 - 45.99} = 5.156$$

$p^{sat} = e^{5.156} = 173.5\ mm\,Hg = 0.228\ atm$

$A = 100\, m^2$

Substituting

$$Q_m = \frac{\left(153.6 \frac{kg}{kg\text{-}mole}\right)(0.00406\, m/s)(100\, m^2)(0.2285\, atm)}{\left(0.082057 \frac{m^3\, atm}{kg\text{-}mole\, K}\right)(308\, K)}$$

$$\boxed{Q_m = 0.564\, kg/s}$$

b.) Use equation (3-9) to estimate concentration

$$Q_V = (300\, ft^3/min)\left(\frac{2.832 \times 10^{-2}\, m^3}{1\, ft^3}\right)\left(\frac{1\, min}{60\, s}\right)$$

$Q_V = 1.42\, m^3/s$

Use $Q_m = 0.564\, kg/s$ since the leak rate given is greater than the result of part a

$$C_{ppm} = \frac{Q_m R_g T}{k Q_V P M} \times 10^6$$

$$= \frac{(0.564\, kg/s)\left(0.08205 \frac{m^3\, atm}{kg\text{-}mole\, K}\right)(308\, K)}{k(1.42\, m^3/s)(1\, atm)(153.6\, kg/kg\text{-}mole)} \times 10^6$$

$= 65{,}400/k$

If $k = 0.5$ $C_{ppm} = 130{,}800$ ppm

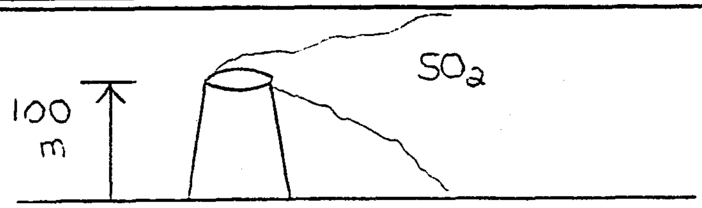
If $k = 0.1$ $C_{ppm} = 654{,}000$ ppm, which is greater than the saturation vapor pressure of 0.228 atm

Thus

$$\boxed{130{,}000\,\text{ppm} \le C_{ppm} \le 228{,}000\,\text{ppm}}$$

Can also use equation (3-14) to get same answer.

5-4



Sunny day with wind= 2 m/sec ($\bar{u}$)

Stability class "A" appropriate

$\langle C \rangle = 5 \times 10^{-5}$ gm/m^3

$x = 200$ m $\qquad y = 0$

Assume measurement is taken on the ground. Then $z = 0$

Equation 5-49 applies with $z = 0$

Since $y = 0$ also, Equation 5-51 applies

and

$$\frac{1}{Q_m} = \frac{1}{\pi \sigma_y \sigma_z u \langle C \rangle} \exp\left[-\frac{1}{2}\left(\frac{H_r}{\sigma_z}\right)^2\right]$$

Use Table 5-2 to compute σ_y and σ_z

Assume rural conditions

$$\sigma_y = 0.22x(1 + 0.0001x)^{-1/2}$$

$$= (0.22)(200\,m)\left[1 + (0.0001)(200\,m)\right]^{-1/2}$$

$$= 43.6\,m$$

$$\sigma_z = 0.20x = (0.20)(200\,m) = 40\,m$$

Then

$$\frac{1}{Q_m} = \left[\frac{1}{(3.14)(43.6\,m)(40\,m)(2\,m/s)(5 \times 10^{-5}\,gm/m^3)}\right]$$

$$\times \exp\left[-\left(\frac{1}{2}\right)\left(\frac{100\,m}{40\,m}\right)^2\right]$$

$$= (1.83)(4.39 \times 10^{-2})(gm/s)^{-1}$$

$$\boxed{Q_m = 12.4\,gm/s}$$

Use Equation 5-53 to determine location of max. conc.

$$\sigma_z = \frac{H_r}{\sqrt{2}} = \frac{100\,m}{\sqrt{2}} = 70.7\,m$$

From Table 5-2

$$\sigma_z = 0.20\,x = 70.7\,m$$

$$\therefore \underline{x \simeq 353\ m \text{ downwind}}$$

The max. conc. at this location is found from Equation 5-52

$$\langle C \rangle_{max} = \frac{2 Q_m}{e \pi u H_r^2}\left(\frac{\sigma_z}{\sigma_y}\right)$$

$$= \frac{(2)(12.4\,gm/s)}{(2.72)(3.14)(2\,m/s)(100\,m)^2}\left(\frac{40\,m}{43.6\,m}\right)$$

$$\underline{\langle c \rangle = 1.33 \times 10^{-4}\,gm/m^3}$$

$\boxed{5\text{-}12}$ 800 lbm Cl_2 Total release in 10 minutes. Assume that a plume can form in this time.

$$Q_m = \left(\frac{800 \text{ lbm}}{10 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)\left(\frac{453.6 \text{ gm}}{\text{lbm}}\right) = 604.8 \text{ gm/sec}$$
$$= 6.048 \times 10^5 \text{ mg/sec}$$

The largest plume will occur when the dispersion coefficients are the smallest and the wind speed is small. This occurs with stability class "F", and a rural release

$\therefore$ $u = 2$ m/sec

also assume a release at ground level. The maximum concentration will occur at the center line. Thus, equation 5-48 applies:

$$\langle C \rangle (x,0,0) = \frac{Q_m}{\pi \sigma_y \sigma_z u}$$

From Table 5-6, the ERPG-1 for chlorine is 1 ppm. From Table 2-8 this is 3.0 mg/m^3 at 25°C