CM 4710, Biochemical Processes **Fall 2007**
Homework #5 **Fri. 12 Oct., '07**

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1. Continuous Culture with Recycle for Industrial Wastewater Treatment.

Problem 9.11 of the text.

2. Secondary Metabolite (for example penicillin) Production in a 2-Stage Chemostat

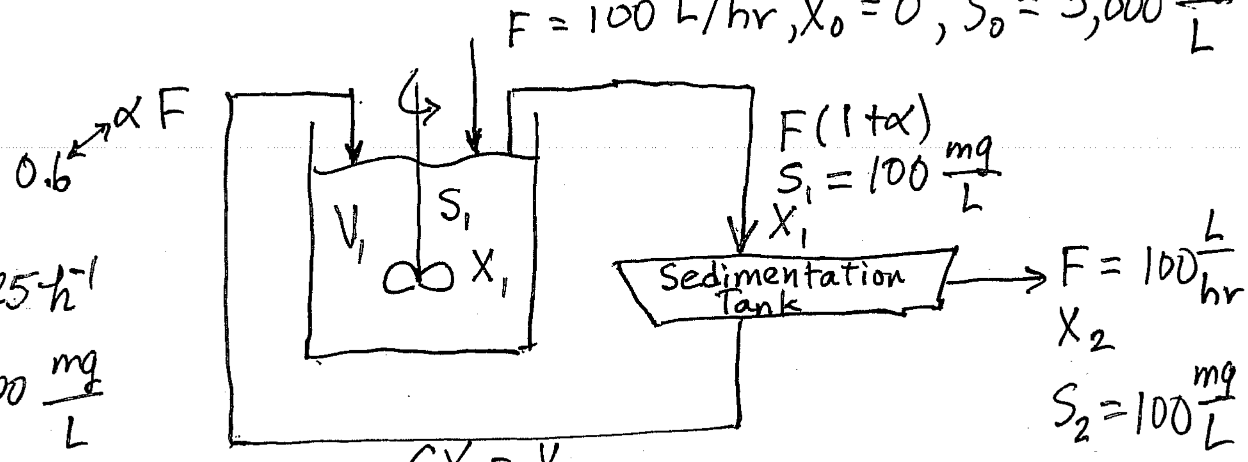
Problem 9.2 of the text.

3. Fed-Batch Bioreactor for Production of Penicillin

Problem 9.4 of the text. Note that “quasi-steady state” means $dX/dt = 0$ in the cell mass balance, which is the assumption we used to derive the equations in lecture.

Due Fri. 19 Oct., '07

9.11 Industrial Wastewater Treatment
Continuous Culture w/ Recycle.



$\mu_m = .25\ h^{-1}$

$K_s = 200 \frac{mg}{L}$

$Y^M_{X/S} = .4$

$CX_1 = X_R$

$C = 2$

a) Volume of Reactor, V_1

$$S_1 = \frac{K_s D_1 (1+\alpha-\alpha C)}{\mu_m - D_1(1+\alpha-\alpha C)} \leftrightarrow \text{eqn. } 9.13 : D_1 \equiv \frac{F}{V_1}$$

$$100 = \frac{(200) D_1 (1.6 - .6(2))}{.25 - D_1(1.6 - .6(2))} \Rightarrow 0.5 = \frac{0.4 D_1}{.25 - .4 D_1}$$

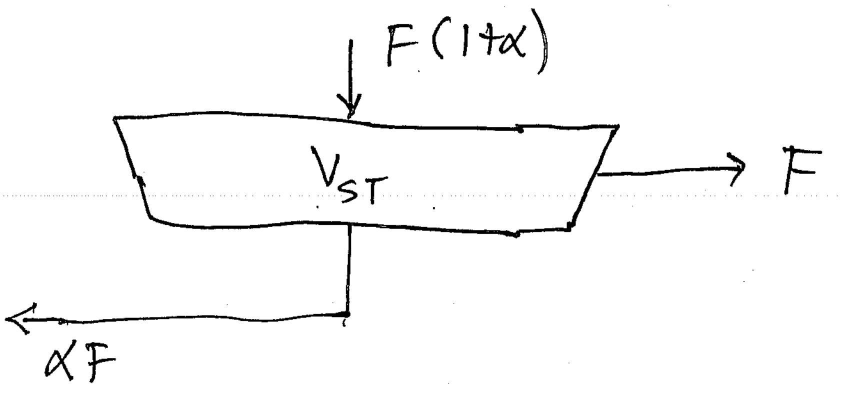
$$D_1 = .208\ h^{-1} \rightarrow V_1 = F/D_1 = \frac{100\ L/\cancel{h}}{.208\ \cancel{h}} = \boxed{481 L}$$

b) X_1 and X_R

$$X_1 = \frac{Y^M_{X/S}(S_0 - S_1)}{(1+\alpha-\alpha C)} = \frac{(.4)(5{,}000 - 100)}{(1.6 - .6(2))} = \boxed{4{,}900 \frac{mg}{L}}$$

$$X_R = CX_1 = 2(4{,}900) = \boxed{9{,}800\ mg/L}$$

c) Sedimentation Tank.



Residence Time, $\theta_{ST} = \frac{V_{ST}}{(1+\alpha)F} = 2\ hr$

$V_{ST} = (2\ hr)(1+.6)\ 100\ L/h = \boxed{320\ L}$

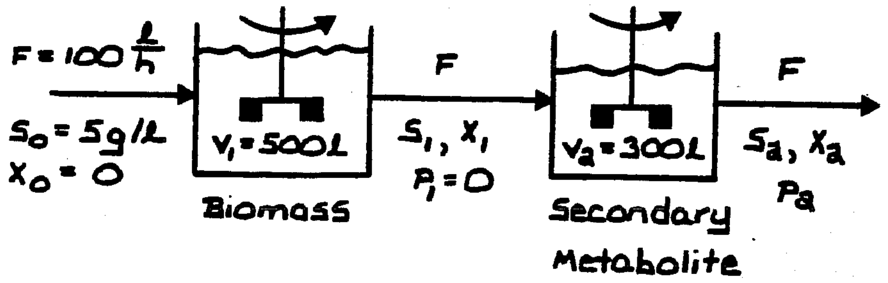
$X_2 = (1+\alpha)X_1 - \alpha C X_1$; from Example 9.1

$= (1.6)X_1 - (1.2)X_1$

$= 0.4X_1 = 0.4(4{,}900\ mg/L)$

$\boxed{= 1{,}960\ mg/L}$

Problem 9.2 $\mu_m = 0.3\,h^{-1}$ $K_s = 0.1\,g/L$ $Y_{X/S} = 0.4\,g/g$



a) $D_1 = \frac{F}{V_1} = \frac{100\,L/h}{500\,L} = 0.2\,h^{-1} = \mu_1$ at steady state

$\therefore 0.2\,h^{-1} = \mu_m \frac{S_1}{K_s + S_1} = 0.3\,h^{-1}\left(\frac{S_1}{0.1\,g/L + S_1}\right)$

Solving for S_1: $S_1 = 0.2\,g/L$

$X_1 = Y_{X/S}(S_o - S_1) = 0.4\,g/g\,(5 - 0.2\,g/L) = 1.92\,g/L$

b) $\mu_2 = 0$ $\therefore X_2 = X_1$ $q_P = 0.02\,g/(g\text{ cell}\cdot h)$
$Y_{P/S} = 0.6\,g/g$

$D_2 = \frac{F}{V_2} = \frac{100\,L/h}{300\,L} = 0.333\,h^{-1}$

Substrate balance around second stage:

$F(S_1 - S_2) - \frac{q_P X_2 V_2}{Y_{P/S}} = V_2 \frac{dS_2}{dt} = 0$ at steady state

Solving for S_2: $S_2 = S_1 - \frac{q_P X_2 V_2}{F Y_{P/S}} = S_1 - \frac{q_P X_2}{D_2 Y_{P/S}}$

$\therefore S_2 = 0.2\,g/L - \frac{0.02\,g\,g^{-1}\,h^{-1}\,(1.92\,g/L)}{0.333\,h^{-1}\,(0.6\,g/g)} = 0.008\,g/L$

(S4)

$P_2 = Y_{P/S}(S_1 - S_2) = 0.6\,g/g\,(.2 - .008)\,g/L = .115\,g/L$

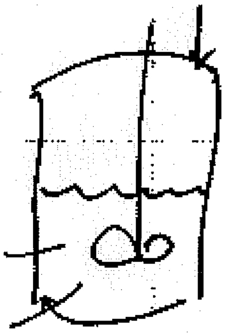
2. P9.4 Penicillin production in Fed-Batch

$\mu_m = 0.2\ h^{-1}$

$K_s = .5\ g/l$

$Y^M_{X/S} = 0.3\ g$ dw cells/g glucose

$F = 50\ l/h$ $S_0 = 300\ g$ gluc./l



$V_0 = 500\ l$

$X_0 = 20\ \frac{g}{l}$

a) V at t = 10 hr.

$$V = V_0 + Ft = 500\ l + 50\ l/hr\ (10\ hr)$$
$$\boxed{= 1000\ l}$$

b) S @ t = 10 hr (quasi-steady state)

$$\mu = \frac{D_0}{1+D_0 t} = \frac{F/V_0}{1+F/V_0\, t} = \frac{50/500\ hr^{-1}}{1+\frac{50}{500}hr^{-1}(10\ hr)}$$
$$= .05\ hr^{-1}$$
$$0.05\ hr^{-1} = \frac{\mu_m S}{K_s + S}$$
$$S = \frac{K_s\ (0.05\ hr^{-1})}{\mu_m - (.05\ hr^{-1})} = \frac{0.5\frac{g}{l}\ (.05\ hr^{-1})}{(0.2 - .05)\ hr^{-1}}$$
$$\boxed{= .167\ g/l}$$

c) X_t and X @ $t = 10$ hr

$$X_t = X_0^t + FY_{x/s}S_0 t$$

$$= X_0V_0 + FY_{x/s}S_0 t$$

$$= \left(20\frac{g}{l}\right)(500 l) + \left(50\frac{l}{h}\right)\left(.3\frac{g\ cells}{g\ gluc.}\right)\left(300\frac{g\ gluc}{l}\right)(10\ hr)$$

$$= 10{,}000\ g + 45{,}000 = \boxed{55{,}000\ \frac{g\ cells}{l}}$$

$$X(10\ hrs) = \frac{X_t}{V_0} = \frac{55{,}000\ g\ cells/l}{1000\ l} = \boxed{55\ \frac{g\ cells}{l}}$$

d) P at $t = 10$ hrs.

$$q_p = 0.05\ \frac{g\ penicillin}{g\ cells \cdot hr}$$

$$P_0 = 0.1\ \frac{g\ penicillin}{l}$$

$$\frac{dP_t}{dt} = q_p X_t = q_p\left(X_0^t + FY_{x/s}S_0 t\right)$$

$$\int_{P_0^t}^{P_t} dP_t = \int_0^{10hr} q_p\left(X_0^t + FY_{x/s}S_0 t\right)dt$$

$$P_t - P_0^t = q_p\left(X_0^t t + FY_{x/s}S_0 \frac{t^2}{2}\right)\Big|_0^{10hr}$$

$$P_t = \underbrace{P_o V_o}_{P_o^t} + q_p (10^4 g)(10 hr) + (50 \tfrac{l}{h})(.3 \tfrac{gX}{gS})(300 \tfrac{gS}{l})(\tfrac{10^2 hr^2}{2}$$

$$= (.1 \tfrac{g}{l}) 500 l + .05 \tfrac{gP}{gX \cdot hr}(10^5 gX \cdot hr + 225{,}000\, gX \cdot hr)$$

$$= 50\, gP + .05 \tfrac{gP}{gX \cdot hr}(325{,}000\, gX \cdot hr)$$

$$= 50 + 16{,}250 = \boxed{16{,}300\, g \text{ Penicillin}}$$

$$P = \frac{P_t}{V} = \frac{16{,}300\, gP}{1000\, l} = \boxed{16.3 \frac{gP}{l}}$$