CM 4710, Biochemical Processes **Fall 2007**
Homework #7 **Fri. 26 Oct., 2007**

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1. Batch Filtration: Cell Culture.

A Buchner funnel 8 cm in diameter is available for testing the filtration of a cell culture suspension, which has a viscosity of 3.0 cp. The following data were obtained at a vacuum pressure of 600 mm Hg applied to the funnel.

t (min)	26	96	197	342	537	692	989 (end)
V (ml)	100	200	300	400	500	600	692

The cell solids on the filter at the end of filtration were dried and found to weigh 14.0 g. Determine the specific cake resistance α and the medium Resistance r_m. Then estimate how long it would take to obtain 10,000 liters of filtrate from this cell culture on a filter with a surface area of 10 m^2 and vacuum pressure of 500 mm Hg.

2. Continuous Centrifugation using a Tubular Centrifuge.

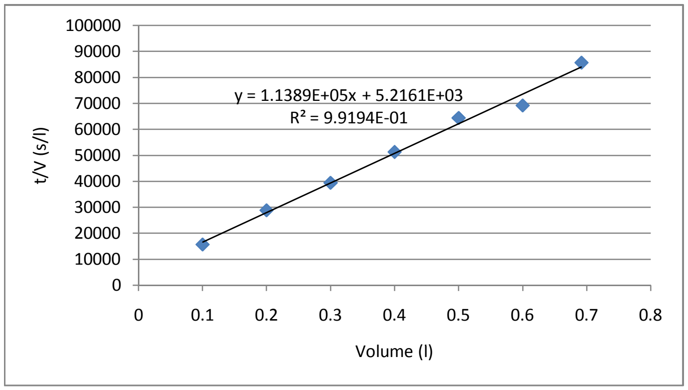
Yeast cells are recovered from a fermentation broth by using a tubular centrifuge. At a flow rate of 12 l/min, the centrifuge must be operated at a rotation rate of 4000 rpm to collect the cells. You are asked to predict operation under different conditions.

a) What rotation rate must you operate at if the flow rate is increased to 20 l/min?
b) If the length of the tubular portion of the centrifuge can be replaced, how much longer must the length be if the flow rate is increased to 20 l/min?
c) What rotation rate must the unit in part a) operate at if the particle diameter is doubled?

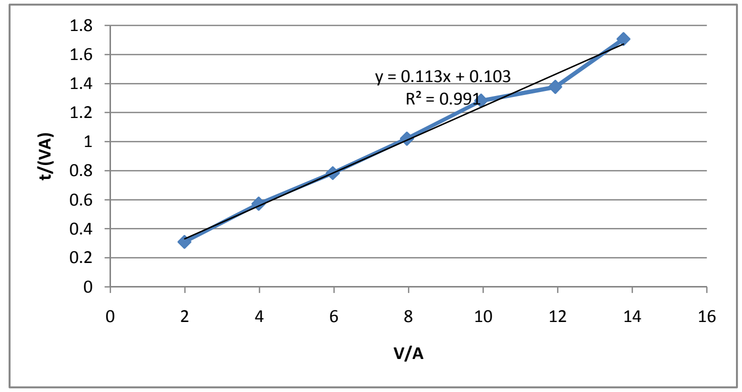
Due Fri. 2 Nov., 2007

Jill Jensen
HW7 (1)

time (s)	volume (l)	t/V s/l	time (min)
1560	0.1	15600	26
5760	0.2	28800	96
11820	0.3	39400	197
20520	0.4	51300	342
32220	0.5	64440	537
41520	0.6	69200	692
59340	0.692	85751.44509	989



volume cm3	t sec	A cm2	t/VA	V/A
100	1560	50.26544	0.3103524	1.989438
200	5760	50.26544	0.57295828	3.978877
300	11820	50.26544	0.78383876	5.968315
400	20520	50.26544	1.02058193	7.957754
500	32220	50.26544	1.28199415	9.947192
600	41520	50.26544	1.37669142	11.93663
692	59340	50.26544	1.70597224	13.76691



This is the graph plotted the way of the other book - I still don't get the slope that they do

A = ∏*r^2 =	0.0050265 m^2		
r =	4 cm	0.04 m	
C = 14g/692ml =	0.02023 kg/l		
μ =	3 cp	0.18 kg/(m*min)	
ΔP =	600 mm Hg	79993.2 N/m^2	
slope = 1/K	113890 sec/l^2		
K = 1/slope	8.7804E-06 l^2/sec		
g_c =	1 (kg*m)/(s^2*N)		
intercept =	5216 s/l		
α = slope*(2A^2/Cμ)*ΔP*g_c	7.58515E+12 m/kg	note: 1000 converts l to m3, 60 converts min to s	
r_m = y int*K/2*a*C/A	6.9910E+11 m^{-1}		

α = slope*(2A^2/Cμ)*ΔP*g_c	7.58515E+12 m/kg	A =	10 m^2
ΔP =	500 mm Hg	66661 N/m^2	
t=(α*C*μ/2/A^2/ΔP/g_c)[V^2+2V(r_m*A/α/C)]	5.81E+04 min	note: 1000 converts l to m3, 60 converts min to s	
	967.93 hr		

HW7

P1. Filtration of a Cell Culture Suspension

data: dia. = 8 cm – $A = \pi r^2 = \pi(4\,cm)^2 = 50.26\,cm^2 = .005026\,m^2$

$\mu = 3.0\,cp = .003\,kg/(m \cdot s)$

$\Delta P = 600\,mmHg$ (vacuum pressure) $= 79{,}993.4\,N/m^2$

$C = 14g/.692\,l = 20.23\,g/l = .02023\,kg/l$

plot t/V (s/liter) vs V (liters)

slope $= 1/K = 113890\ sec/l^2$

$K = 1/\text{slope} = 8.78 \times 10^{-6}\ l^2/sec$

y-intercept $= 5{,}216\ s/l$

$$\alpha = \text{slope} \cdot \frac{2A^2 \Delta P\, g_c}{C\mu}$$

$$= \left(113890\,\frac{sec}{l^2}\right) \cdot \frac{2(.005026\,m^2)^2 (79{,}993.2\,\frac{N}{m^2})\left(1\,\frac{kg \cdot m}{s^2 \cdot N}\right)\left(10^3\,\frac{l}{m^3}\right)}{(.02023\,\frac{kg}{l})(.003\,kg/(m \cdot s))}$$

$$\boxed{= 7.59 \times 10^{12}\ m/kg}$$

$$r_m = \text{y-intercept} \cdot \frac{K}{2} \cdot \frac{\alpha C}{A}$$

$$= \left(5{,}216\,\frac{s}{l}\right)\left(\frac{8.78 \times 10^{-6}\,l^2/s}{2}\right)\left(\frac{(7.59 \times 10^{12}\,\frac{m}{kg})(.02023\,\frac{kg}{l})}{(.00503\,m^2)}\right)$$

$$\boxed{= 6.99 \times 10^{11}\ m^{-1}}$$

t (hr) for $V = 10^4 \ell$ filtered

$A = 10\ m^2$

$\Delta P = 500\ mmHg = 66{,}661\ N/m^2$

$\alpha = 7.59 \times 10^{12}\ m/kg$ (assume not a $f(\Delta P)$!)

$$t = \frac{\alpha C \mu}{2 A^2 \Delta P g_c}\left(V^2 + 2V\left(\frac{r_m A}{\alpha C}\right)\right)$$

$$= \frac{\left(7.59\times10^{12}\frac{m}{kg}\right)\left(\frac{1\ m^3}{10^3\ \ell}\right)\left(.02023\frac{kg}{\ell}\right)\left(.003\frac{kg}{m\cdot s}\right)}{2\left(10\ m^2\right)^2\left(66{,}661\frac{N}{m^2}\right)\left(1\frac{kg\cdot m}{s^2\cdot N}\right)}$$

$$\left(\left(10^4\ell\right)^2 + 2\left(10^4\ell\right)\frac{\left(6.99\times10^{11}\frac{1}{m}\right)\left(10\ m^2\right)}{\left(7.59\times10^{12}\frac{m}{kg}\right)\left(.02023\frac{kg}{\ell}\right)}\right)$$

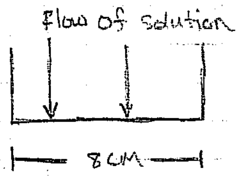
$= 3.48 \times 10^6$ sec

$= 5.81 \times 10^4$ min

$= 968$ hr

comment: $A = 10\ m^2$ is definitely too low for this application!

Σ1]



$\mu = 3.0$ cp

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a) See plot attached for t/V vs. V

- Calculate α and r_m ⇒ The cell solids are dried and found to be 14.0 g
- $\Delta P = 600$ mm Hg

$$\text{Slope} = \frac{1}{K} \quad \therefore \quad \frac{1}{K} = 0.0019 \Rightarrow K = 526.3 \ \frac{1\,\text{mL}^2}{\text{min}} \cdot \frac{1\,\text{cm}^6}{1\,\text{mL}^2} \cdot \frac{1\,\text{m}^6}{(100\,\text{cm})^6} = 5\times10^{-16}\,\frac{\text{m}^6}{\text{min}}$$

For V_0, $y = 0.0019x + (0.0869)$ ← y intercept $\therefore \frac{2V_0}{K} = 0.0869$

$$V_0 = \frac{0.0869 \cdot K}{2}$$

$\therefore$ we know $W = CV \Rightarrow C = \frac{W}{V} = \frac{14.0\,g}{692\,\text{cm}^3} = 0.020\,\frac{g}{\text{cm}^3}$ $\quad V_0 = 22.86\ \text{cm}^3$

and $V_0 = \frac{r_m}{\alpha C} A$ and $K = \left(\frac{2A^2}{\alpha C \mu}\right) \cdot \Delta P \cdot g_c$

$$A = \pi \left(4\,\text{cm} \cdot \frac{1\,\text{m}}{100\,\text{cm}}\right)^2 = 0.005027\,\text{m}^2$$

$$\Delta P = 600\,\text{mmHg} \cdot \frac{1\,\text{atm}}{760\,\text{mmHg}} \cdot \frac{1.01\times10^5\,\text{pa}}{1\,\text{atm}} = 7.97\times10^4\,\text{pa} = 7.97\times10^4\,\frac{N}{m^2}$$

$$C = 0.020\,\frac{g}{\text{cm}^3} \cdot \frac{(100\,\text{cm})^3}{(1\,\text{m}^3)} \cdot \frac{1\,\text{kg}}{1000\,g} = 20.2\,\frac{\text{kg}}{\text{m}^3}$$

$$\mu = 3.0\,\text{cp} \cdot \frac{0.001\,N\cdot s/m^2}{1\,\text{cp}} = 0.003\,\frac{N\cdot s}{m^2}$$

Solve for α ⇒ $\frac{K}{\Delta P \cdot g_c} = \frac{2A^2}{\alpha C \mu} \Rightarrow \alpha C \mu = 2A^2 \cdot \frac{\Delta P \cdot g_c}{K}$

$$\alpha = \frac{2A^2 \cdot \Delta P \cdot g_c}{C \mu \cdot K} = \frac{2(0.005027\,m^2)^2 \left(7.97\times10^4\,\frac{N}{m^2}\right)\left(1\,\frac{kg\cdot m}{s^2\cdot N}\right)}{\left(20.2\,\frac{kg}{m^3}\right)\left(0.003\,\frac{N\cdot s}{m^2}\right)\left(5\times10^{-16}\,\frac{m^6}{min}\cdot\frac{1\,min}{60\,s}\right)}$$

$$\Rightarrow \frac{m^4 \cdot \frac{N}{m^2} \cdot \frac{kg\cdot m}{s^2\cdot N}}{\frac{kg}{m^3}\cdot\frac{N\cdot s}{m^2}\cdot\frac{m^6}{s}} = \frac{\frac{kg\cdot m^3}{s^2}}{kg\cdot N\cdot m} = \frac{m^2}{s^2\cdot N} = \frac{m^2}{\frac{kg\cdot m}{s^2}\cdot s^2} = \frac{m}{kg}$$

7.59×10^{12} m/kg

$\alpha = 7.97\times10^{12}\,\frac{m}{kg}$ ✓ v. close

$$V_0 = \frac{r_m}{\alpha C} A \Rightarrow r_m = \frac{\alpha V_0 C}{A} = \frac{\left(7.97\times10^{12}\,\frac{m}{kg}\right)\left(22.86\,\text{cm}^3 \cdot \frac{1\,m^3}{(100\,\text{cm})^3}\right)\left(20.2\,\frac{kg}{m^3}\right)}{0.005027\,m^2}$$

$= 7.32\times10^{11}\,m^{-1}$ v. close again

~~$1.17\times10^{10}\,m^{-1}$~~

$6.99\times10^{11}\,m^{-1}$

[1] b) How long would it take to obtain 10,000 L of filtrate with a $10\ m^2$ surface area and a vacuum pressure of 500 mm Hg.

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$$500\ mm\ Hg \cdot \frac{1\ atm}{760\ mm\ Hg} \cdot \frac{1.01\times10^5\ N/m^2}{atm} = 6.64\times10^4\ \frac{N}{m^2}$$

$$K = \left(\frac{2A^2}{\alpha c \mu}\right) \cdot \Delta P \cdot g_c$$

$$= \frac{2\,(10\ m^2)^2}{\left(7.97\times10^{12}\ \frac{m}{kg}\right) 20.2\ \frac{kg}{m^3} \left(0.003\ \frac{N\cdot s}{m^2}\right)} \cdot \left(6.64\times10^4\ \frac{N}{m^2}\right) \cdot \left(1\ \frac{kg\cdot m}{s^2\cdot N}\right)$$

$$= \frac{m^4 \cdot \frac{N}{m^2} \cdot \frac{kg\ m}{s^2\ N}}{\frac{m}{kg} \cdot \frac{kg}{m^3} \cdot \frac{N\cdot s}{m^2}} = \frac{\frac{kg\cdot m^3}{s^2}}{\frac{N\cdot s}{m^4}} = \frac{\frac{kg\cdot m^7}{s^2}}{N\cdot s} = \frac{\frac{kg\cdot m^7}{1\cdot s^2}}{\frac{kg\cdot m}{s^2}\cdot s} = \frac{m^6}{s}$$

$$= 2.75\times10^{-5}\ \frac{m^6}{s}$$

$$V_0 = \frac{r_m A}{\alpha c} = \frac{\left(7.32\times10^{11}\ \frac{1}{m}\right)(10\ m^2)}{\left(7.97\times10^{12}\ \frac{m}{kg}\right)\left(20.2\ \frac{kg}{m^3}\right)} = 0.0454\ m^3$$

$$\therefore\ V = 10000\ L \cdot \frac{1\ m^3}{1000\ L} = 10\ m^3$$

$$\Rightarrow V^2 + 2VV_0 = Kt \Rightarrow t = \frac{V^2 + 2VV_0}{K} = \frac{(10\ m^3)^2 + 2(10\ m^3)(0.0454\ m^3)}{2.75\times10^{-5}\ \frac{m^6}{s}}$$

$= 3.67\times10^6$ seconds
$= 1019.27$ hrs ✓ **v. close 960 hr**
$\cong$ 42 days

[2] Centrifuge Problem

$F = 12\ \frac{l}{min}$

rpm = 4000 rpm

a) What rpm should be set for $F = 20\ \frac{l}{min}$

$$F_c \propto \omega^2 \therefore$$

$$\frac{F_{c2}}{F_{c1}} = \frac{(RPM_2)^2}{(RPM_1)^2} \Rightarrow (R_{pm2})^2 = \frac{F_{c2}}{F_{c1}}(RPM_1)^2 = \frac{20\ \frac{l}{min}}{12\ \frac{l}{min}} \cdot (4000)^2\ rpm = 5163.9\ RPM$$ ✓

b) $F_c \propto L \therefore$

$$\frac{F_{c2}}{F_{c1}} = \frac{L_2}{L_1} \quad \therefore\ L_2 = \frac{20}{12} L_1 = 1.67\ L_1$$ ✓

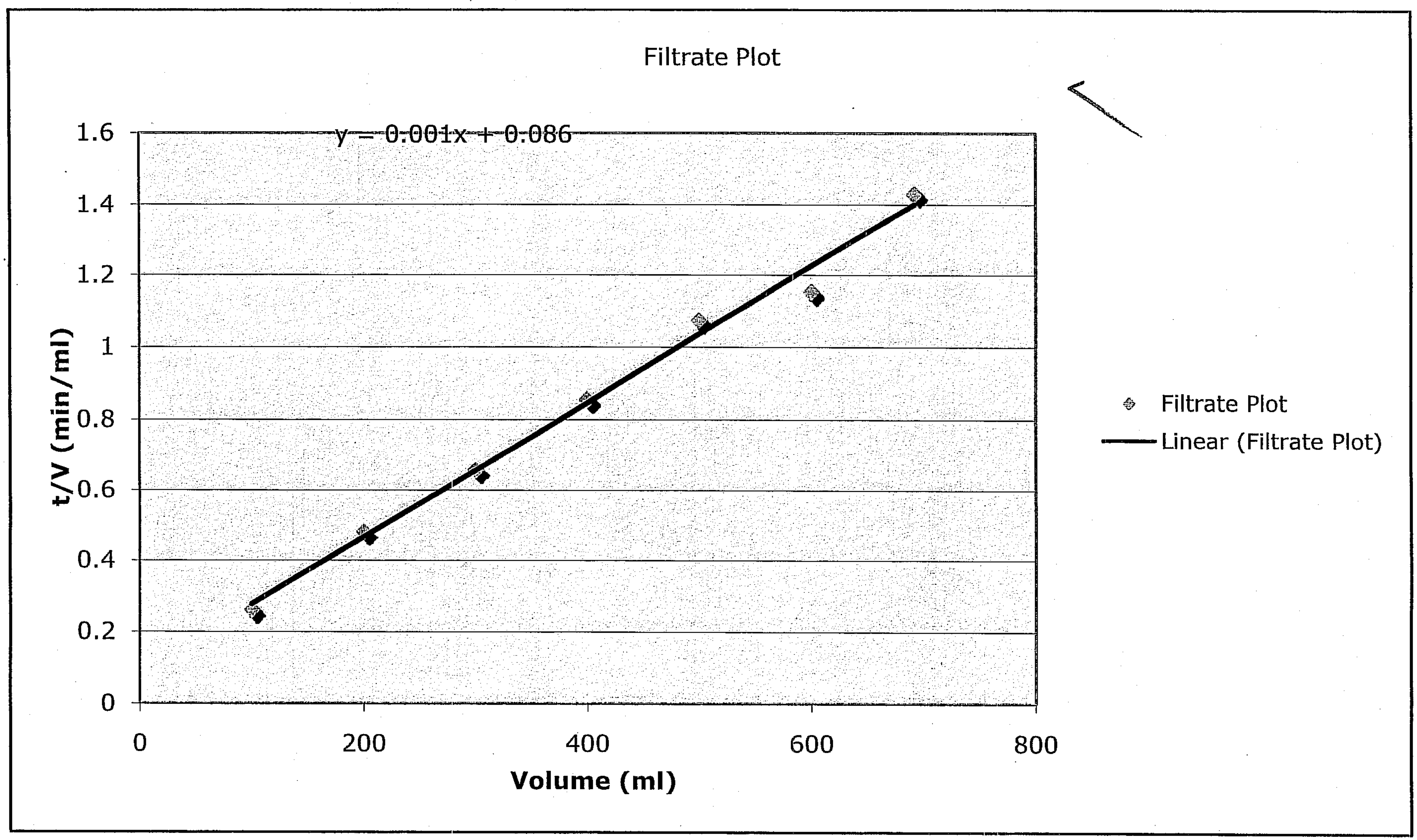
c)

$$F_c = 2\left[\frac{g D_p^2 (\rho_p - \rho_f)}{18\mu}\right] \frac{r_e \omega^2 V_c}{g L_e}$$

If this is doubled (D_p^2) → this (ω^2) must be $\frac{1}{2}$ of what it originally was if the flow rate is the same

$D_p^2 \propto \omega^2 \therefore$ if $D_p \uparrow$, $\omega^2 \downarrow$

$$\frac{5163.9\ RPM}{2} = 2581.95\ RPM$$ ✓


Filtrate Plot
y = 0.001x + 0.086
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
t/V (min/ml)
0
200
400
600
800
Volume (ml)
Filtrate Plot
Linear (Filtrate Plot)