

Short Review of Basic Mathematics

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March 31, 2011

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Chapter 1

Review of Functions

1.1 Mathematical Identities

1. $e^{-b} = \frac{1}{e^b}$

2. $e^{a+b} = e^a e^b$

3. $e^{\ln(x)} = x$

4. $\ln(e^x) = x$

5. $\ln(ab) = \ln(a) + \ln(b)$

6. $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

7. $\ln(x^a) = a \ln(x)$

8. $\log(x) = \frac{\ln(x)}{\ln(10)} \approx \frac{\ln(x)}{2.303}$

9. $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

10. $\sin(a+b) = \cos(a)\sin(b) + \sin(a)\cos(b)$

11. $\cos^2(x) + \sin^2(x) = 1$

12. $\cos^2(x) - \sin^2(x) = \cos(2x)$

13. $2 \sin(x) \cos(x) = \sin(2x)$

14. $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

15. $(a + b)(a - b) = a^2 - b^2$

16. $(a + b)^2 = a^2 + 2ab + b^2$

17. $(a + b)^n = a^n + na^{n-1}b + \dots + \left(\frac{n!}{(n-k)!k!}\right) a^{n-k}b^k + \dots + b^n$

. where $n! = (n)(n-1) \cdots (3)(2)(1)$ and $0! = 1$

Define the imaginary number: $i = \sqrt{-1}$

18. $(a + ib)(a - ib) = a^2 + b^2$

19. $e^{it} = \cos(t) + i \sin(t)$

20. $e^{-it} = \cos(t) - i \sin(t)$

21. $\cos(t) = \frac{e^{it} + e^{-it}}{2}$

22. $\sin(t) = \frac{e^{it} - e^{-it}}{2i}$

23. $\frac{d}{dt}t^a = a t^{a-1}$

24. $\frac{d}{dt}a^t = a^t \ln(a)$

25. $\frac{d}{dt}e^{at} = ae^{at}$

$$26. \quad \frac{d}{dt} \ln(at) = \frac{1}{t}$$

$$27. \quad \frac{d}{dt} \sin(t) = \cos(t)$$

$$28. \quad \frac{d}{dt} \cos(t) = -\sin(t)$$

$$29. \text{ Derivative of Sums: } \quad \frac{d}{dt} (f(t) + g(t)) = \frac{df}{dt} + \frac{dg}{dt}$$

$$30. \text{ Derivative of Products: } \quad \frac{d}{dt} (f(t)g(t)) = \left(\frac{df}{dt}\right)g(t) + f(t)\left(\frac{dg}{dt}\right)$$

$$31. \text{ Derivative of Fractions: } \quad \frac{d}{dt} \left(\frac{f(t)}{g(t)}\right) = \frac{1}{f^2(t)} \left(f(t)\frac{dg}{dt} - g(t)\left(\frac{df}{dt}\right)\right)$$

$$32. \text{ Chain Rule: } \quad \frac{d}{dt} f(u(t)) = \frac{df}{du} \frac{du}{dt}$$

$$33. \quad \int_a^b (p f(t) + q g(t)) dt = p \int_a^b f(t) dt + q \int_a^b g(t) dt$$

$$34. \quad \int_a^b f(t) dt + \int_b^c f(t) dt = \int_a^c f(t) dt$$

$$35. \text{ Integration by parts: } \quad \int_{v(a)}^{v(b)} u dv = (u(b)v(b) - u(a)v(a)) - \int_{u(a)}^{u(b)} v du$$

1.2 Drills

$$1. \text{ Find } \quad \frac{d}{dt} (2e^{1+3t} - t^2 + 10)$$

$$2. \text{ Find } \quad \frac{d}{dt} (\ln(at) e^{t+it^2})$$

$$3. \text{ Find the roots of : } 3x^2 + 2x = 4$$

$$4. \text{ Find the solution of : } 2 \log(x + 3) = \log(x + 15)$$

$$5. \text{ Expand } (x + y)^4 \text{ into a polynomial in } x \text{ and } y$$

6. Prove the following identity:

$$(a + ib) e^{p+iq} + (a - ib) e^{p-iq} = e^p (2a \cos(q) - 2b \sin(q))$$

7. Find $\int_0^{10} (2 + t^2 e^{-2t}) dt$

8. Let r be a function of t . Show that the following is true:

$$\frac{d}{dt} (2\pi r^2 + 5r) = (4\pi r + 5) \frac{dr}{dt}$$

9. Find k and ϕ such that: $2 \cos(3t) + 4 \sin(3t) = k \sin(3t + \phi)$

10. Find $\int_0^2 t f(t) dt$, where $f(t) = \begin{cases} t & \text{if } t \leq 1 \\ 2 - t & \text{if } t > 1 \end{cases}$

1.3 Answers to Drills

1. $6e^{1+3t} - 2t$

2. $\left(\frac{1}{t} + \ln(at)(1 + 2it) \right) e^{t+it^2}$

3. $x_1 = \frac{1 + \sqrt{13}}{3}, x_2 = \frac{1 - \sqrt{13}}{3}$

4. $x = 1$

5. $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

6.

$$\begin{aligned} (a + ib) e^{p+iq} + (a - ib) e^{p-iq} &= e^p \left((a + ib) (\cos(q) + i \sin(q)) \right. \\ &\quad \left. (a - ib) (\cos(q) - i \sin(q)) \right) \\ &= e^p (2a \cos(q) - 2b \sin(q)) \end{aligned}$$

7. $\frac{81}{4} - \frac{221}{4}e^{-20}$

8. Hint: use chain rule

9.

$$\begin{aligned} 2 \cos(3t) + 4 \sin(3t) &= k \sin(3t + \phi) \\ &= (k \sin(\phi)) \cos(3t) + (k \cos(\phi)) \sin(3t) \end{aligned}$$

Comparing coefficients on both sides

$$\begin{aligned} k \cos(\phi) &= 4 & \rightarrow & k = \sqrt{20} \\ k \sin(\phi) &= 2 & & \phi = \text{Atan}\left(\frac{1}{2}\right) \end{aligned}$$

10. $\int_0^1 t^2 dt + \int_1^2 (2-t)t dt = 1$

Chapter 2

Review of ODE Solutions

2.1 First Order Differential Equations

1. Separable Type

- **Form :** $\frac{dx}{dt} = f(t)g(x)$ subject to $x(0) = x_0$

- **Solution:** $\int_{x_0}^x \frac{1}{g(x)} dx = \int_0^t f(t) dt$

- **Example:**

$$\frac{dx}{dt} = 5 \frac{x}{1+t} \quad x(0) = 2$$

then $f(t) = \frac{5}{1+t}$ and $g(x) = x$,

$$\int_{x_0}^x \frac{1}{x} dx = \int_0^t \frac{5}{1+t} dt$$

$$\ln(x) - \ln(x_0) = 5 (\ln(1+t) - \ln(1))$$

$$\ln\left(\frac{x}{x_0}\right) = 5 \ln(1+t)$$

$$\frac{x}{x_0} = (1+t)^5$$

$$x = x_0 (1+t)^5$$

2. Linear First Order

- **Form :** $\frac{dx}{dt} + p(t)x = q(t)$ subject to $x(0) = x_0$

- **Solution:** Let $\phi = \exp\left(\int p(t)dt\right)$ be an integrating factor, then

$$x = \frac{1}{\phi(t)} \left(x_0 + \int_0^t q(\tau)\phi(\tau)d\tau \right)$$

- **Example:**

$$\frac{dx}{dt} + 2tx = 4t \quad x(0) = x_0$$

then $p(t) = 2t$ and $q(t) = 4t$,

$$\phi(t) = e^{\int 2tdt} = e^{t^2}$$

and

$$\begin{aligned} x &= e^{-t^2} \left(x_0 + \int_0^t 4\tau e^{\tau^2} d\tau \right) \\ &= e^{-t^2} \left(x_0 - 2 + 2e^{t^2} \right) \\ &= 2 + (x_0 - 2)e^{-t^2} \end{aligned}$$

2.2 High Order Linear ODEs with Constant Coefficients

- **Form :** $a_n \frac{d^n x}{dt^n} + \dots + a_1 \frac{dx}{dt} + a_0 x = q(t)$

subject to $x(0) = x_{o,0}, \dots, \frac{d^{n-1}x}{dt^{n-1}}(0) = x_{o,n-1}$

- **Solution:** $x(t) = x_c(t) + x_p(t)$

where

- x_c is the complementary solution which solves

$$a_n \frac{d^n x_c}{dt^n} + \dots + a_1 \frac{dx_c}{dt} + a_0 x_c = 0$$

- x_p is the particular solution which solves

$$a_n \frac{d^n x_p}{dt^n} + \dots + a_1 \frac{dx_p}{dt} + a_0 x_p = q(t)$$

• **Example: Second Order**

$$a_2 \frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0x = q(t)$$

Step 1: Find x_c :

1. Get the **characteristic polynomial**,

$$a_2s^2 + a_1s + a_0 = 0$$

2. Solve for the roots, also known as **eigenvalues**,

$$r_1 = \frac{-a_1 + \sqrt{a_1^2 - 4a_2a_0}}{2a_2} \quad r_2 = \frac{-a_1 - \sqrt{a_1^2 - 4a_2a_0}}{2a_2}$$

3. Form the complementary solution to be

$$x_c = C_1y_1(t) + C_2y_2(t)$$

where

$$y_1(t) = e^{r_1t}$$

$$y_2(t) = \begin{cases} e^{r_2t} & \text{if } r_1 \neq r_2 \\ te^{r_1t} & \text{if } r_1 = r_2 \end{cases}$$

where C_1 and C_2 are constants, evaluated later to fit the initial conditions

Step 2: Find x_p :

Method : Variation of Parameters

1. Evaluate the **Wronskian**: $W(t) = y_1 \frac{dy_2}{dt} - y_2 \frac{dy_1}{dt}$
2. Calculate u_1 and u_2 :

$$u_1(t) = - \int \frac{y_2(t)q(t)}{a_2W(t)} dt \quad u_2(t) = \int \frac{y_1(t)q(t)}{a_2W(t)} dt$$

3. Form the particular solution: $x_p(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$

Step 3: Use initial conditions to determine the values of C_1 and C_2 .

• **Numerical Example:**

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 6x = e^{-t} \sin(2t) \quad \text{subject to} \quad x(0) = 1, \quad \frac{dx}{dt}(0) = 0$$

– Step 1: complementary solution

$$s^2 + s - 6 = 0 \quad \rightarrow r_1 = -3 ; r_2 = 2 \quad \rightarrow y_1 = e^{-3t} ; y_2 = e^{2t}$$

$$x_c = C_1 e^{-3t} + C_2 e^{2t}$$

– Step 2: particular solution

$$\begin{aligned} W(t) &= 5e^{-t} \\ u_1(t) &= \frac{1}{20} e^{2t} (\cos(2t) - \sin(2t)) \\ u_2(t) &= \frac{1}{65} e^{-3t} (-2 \cos(2t) - 3 \sin(2t)) \\ \rightarrow x_p(t) &= \frac{1}{52} e^{-t} (\cos(2t) - 5 \sin(2t)) \end{aligned}$$

– Step 3: determine integration constant

$$\begin{aligned} x &= x_c + x_p \\ &= C_1 e^{-3t} + C_2 e^{2t} + \frac{1}{52} e^{-t} (\cos(2t) - 5 \sin(2t)) \\ x(0) &= 1 = C_1 + C_2 + \frac{1}{52} \\ \frac{dx}{dt}(0) &= 0 = -3C_1 + 2C_2 - \frac{11}{52} \end{aligned}$$

$$\rightarrow C_1 = \frac{7}{20} \quad C_2 = \frac{41}{65}$$

$$\text{Solution:} \quad x(t) = \frac{7}{20} e^{-3t} + \frac{41}{65} e^{2t} + \frac{1}{52} e^{-t} (\cos(2t) - 5 \sin(2t))$$

2.3 Drills

Solve the following differential equations

1. $\frac{dx}{dt} = e^{-t}(x + 2)$ subject to $x(0) = 1$

$$2. \frac{dx}{dt} + (1+t)x = \sin(2t) \quad \text{subject to } x(0) = 1$$

$$3. \frac{d}{dt}(e^{2t}x) = e^{-t} \quad \text{subject to } x(0) = 2$$

$$4. \frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 3 + e^{-t}\sin(t) \quad \text{subject to } x(0) = 2 \text{ and } \frac{dx}{dt}(0) = 0$$

$$5. \frac{d^2x}{dt^2} - e^{-2t} = -5\frac{dx}{dt} - 6x \quad \text{subject to } x(0) = 0 \text{ and } \frac{dx}{dt}(0) = 0$$

2.4 Answers to Drills

$$1. \quad x(t) = \exp\left(1 + \ln(3) - e^{-t}\right) - 2$$

$$2. \quad x(t) = \frac{1}{\phi(t)} \left(1 + \int_0^t \phi(\tau) \sin(2\tau) d\tau\right) \quad \text{where } \phi(t) = \exp\left(\frac{t^2 + 2t}{2}\right)$$

$$3. \quad x(t) = \frac{3 - e^{-t}}{e^{2t}}$$

$$4. \quad x(t) = \frac{3}{4} + \left(3t + \frac{7}{4}\right) e^{-2t} - \frac{1}{2} e^{-t} \cos(t)$$

$$5. \quad x(t) = e^{-3t} + (t-1)e^{-2t}$$

Chapter 3

Review of Matrix Operations

3.1 Basic Operations

1. Addition:

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} + \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{m1} & \cdots & b_{mn} \end{pmatrix} = \begin{pmatrix} (a_{11} + b_{11}) & \cdots & (a_{1n} + b_{1n}) \\ \vdots & \ddots & \vdots \\ (a_{m1} + b_{m1}) & \cdots & (a_{mn} + b_{mn}) \end{pmatrix}$$

Example:
$$\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \end{pmatrix} + \begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 1 \\ 4 & 3 & 3 \end{pmatrix}$$

2. Scalar Product:

$$\gamma \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} = \begin{pmatrix} \gamma a_{11} & \cdots & \gamma a_{1n} \\ \vdots & \ddots & \vdots \\ \gamma a_{m1} & \cdots & \gamma a_{mn} \end{pmatrix}$$

Example:
$$3 \begin{pmatrix} 2 & 4 \\ 2 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 6 & 12 \\ 6 & 3 \\ 3 & -3 \end{pmatrix}$$

3. Matrix Product:

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} b_{11} & \cdots & b_{1p} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{np} \end{pmatrix} = \begin{pmatrix} c_{11} & \cdots & c_{1p} \\ \vdots & \ddots & \vdots \\ c_{m1} & \cdots & c_{mp} \end{pmatrix}$$

where,

$$c_{ij} = \sum_{k=1}^n a_{ik} b_{kj} \quad i = 1, \dots, m \quad j = 1, \dots, p$$

Example:

$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & 2 & 3 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} (-1+2-1) & (2+2+0) \\ (-2+4+3) & (4+4+0) \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ 5 & 8 \end{pmatrix}$$

4. Transpose:

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}^T = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix}$$

Example: $\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}^T = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$

3.2 Determinants

Notation: $\det(A) = |A|$

1. 2 × 2 Matrices:

$$\left| \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \right| = a_{11}a_{22} - a_{12}a_{21}$$

Example: $\left| \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \right| = 1 \cdot 2 - 2 \cdot 3 = -4$

2. n × n Matrices:

Method 1: row expansion

- Given matrix A , choose any row, say row i .
- Let $A_{ij\downarrow}$ be matrix A with row i and column j removed.
- The determinant can be determined recursively by

$$|A| = \sum_{j=1}^n (-1)^{i+j} |A_{ij\downarrow}|$$

Example: Expand along row 2,

$$\begin{aligned} \left| \begin{pmatrix} 1 & 2 & -1 \\ 3 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix} \right| &= 3 \cdot (-1)^{2+1} \left| \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \right| + 2 \cdot (-1)^{2+2} \left| \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \right| \\ &\quad + 0 \cdot (-1)^{2+3} \left| \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \right| \\ &= -9 \end{aligned}$$

Method 2: column expansion

- (a) Given matrix A , choose any column, say column j .
- (b) Let $A_{ij\downarrow}$ be matrix A with row i and column j removed.
- (c) The determinant can be recursively determined by

$$|A| = \sum_{i=1}^n (-1)^{i+j} |A_{ij\downarrow}|$$

Example: Expand along column 2,

$$\begin{aligned} \left| \begin{pmatrix} 1 & 2 & -1 \\ 3 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix} \right| &= 2 \cdot (-1)^{1+2} \left| \begin{pmatrix} 3 & 0 \\ -1 & 1 \end{pmatrix} \right| + 2 \cdot (-1)^{2+2} \left| \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \right| \\ &\quad + 1 \cdot (-1)^{3+2} \left| \begin{pmatrix} 1 & -1 \\ 3 & 0 \end{pmatrix} \right| \\ &= -9 \end{aligned}$$

3.3 Matrix Inverse

1. 2 × 2 Matrices:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} = \frac{1}{|A|} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$$

Example:

$$\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}^{-1} = \frac{1}{1 \cdot 2 - 2 \cdot 3} \begin{pmatrix} 2 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -1/2 & 1/2 \\ 3/4 & -1/4 \end{pmatrix}$$

Checking: $A^{-1}A = \begin{pmatrix} -1/2 & 1/2 \\ 3/4 & -1/4 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2. $n \times n$ Matrices:

- (a) Given matrix A , let $A_{ij\downarrow}$ be matrix A with row i and column j removed.
- (b) Build a new matrix B ,

$$B = \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{pmatrix}$$

where $b_{ij} = (-1)^{i+j} |A_{ij\downarrow}|$.

(c) The inverse of A is given by

$$A^{-1} = \frac{1}{|A|} B^T$$

Example: Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 0 \\ 0 & 1 & -2 \end{pmatrix}$ then

$$b_{11} = \begin{vmatrix} 2 & 0 \\ 1 & -2 \end{vmatrix} ; b_{12} = - \begin{vmatrix} 3 & 0 \\ 0 & -2 \end{vmatrix} ; b_{13} = \begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix}$$

$$b_{21} = - \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} ; b_{22} = \begin{vmatrix} 1 & 3 \\ 0 & -2 \end{vmatrix} ; b_{23} = - \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$$

$$b_{31} = \begin{vmatrix} 2 & 3 \\ 2 & 0 \end{vmatrix} ; b_{32} = - \begin{vmatrix} 1 & 3 \\ 3 & 0 \end{vmatrix} ; b_{33} = \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}$$

$$B = \begin{pmatrix} -4 & 6 & 3 \\ 7 & -2 & -1 \\ -6 & 9 & -4 \end{pmatrix}$$

Expanding along row 3,

$$|A| = 0 + 1 \cdot (-1)^{3+2} \cdot \begin{vmatrix} 1 & 3 \\ 3 & 0 \end{vmatrix} - 2 \cdot (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = 17$$

Thus, the inverse is given by

$$A^{-1} = \frac{1}{17} \begin{pmatrix} -4 & 6 & 3 \\ 7 & -2 & -1 \\ -6 & 9 & -4 \end{pmatrix}^T = \begin{pmatrix} -4/17 & 7/17 & -6/17 \\ 6/17 & -2/17 & 9/17 \\ 3/17 & -1/17 & -4/17 \end{pmatrix}$$

Checking:

$$A^{-1}A = \begin{pmatrix} -4/17 & 7/17 & -6/17 \\ 6/17 & -2/17 & 9/17 \\ 3/17 & -1/17 & -4/17 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 0 \\ 0 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3.4 Drills

Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 & 1 \\ 3 & 2 & 0 \\ 1 & 2 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

and let s be a scalar variable.

Evaluate the following:

1. $AA^T - B$
2. ACA^T
3. B^{-1}
4. $(sI_2 - C)^{-1}$
5. BC

3.5 Answer to Drills

$$1. \quad AA^T = \begin{pmatrix} 5 & 11 & 3 \\ 11 & 25 & 7 \\ 3 & 7 & 2 \end{pmatrix} \rightarrow AA^T - B = \begin{pmatrix} 4 & 12 & 2 \\ 8 & 23 & 7 \\ 2 & 5 & 0 \end{pmatrix}$$

$$2. \quad ACA^T = \begin{pmatrix} 18 & 44 & 13 \\ 40 & 98 & 29 \\ 11 & 27 & 8 \end{pmatrix}$$

$$3. \quad B^{-1} = \begin{pmatrix} 4/14 & 4/14 & -1/14 \\ -6/14 & 1/14 & 3/14 \\ 4/14 & -3/14 & 5/14 \end{pmatrix}$$

$$4. \quad (sI_2 - C)^{-1} = \frac{1}{s^2 - 4s + 1} \begin{pmatrix} s - 2 & 1 \\ 3 & s - 2 \end{pmatrix}$$

5. Not applicable! Sizes of B and C are not conformable for the product BC .