Data Correlation for Friction Factor in Smooth Pipes

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The Moody Chart (Moody, 1944; Bird et. al, 2002; Denn, 1980; Geankoplis, 2003; White, 2006) is a staple of fluid flow calculations and fluid mechanics education. Engineers use the friction factors from this chart to calculate pressure drops and flow rates for flow in smooth and rough pipes. A correlation is presented that captures friction factor versus Reynolds number for smooth pipes for all values of Reynolds number (laminar, transitional, and turbulent).

There are various correlations for the data in the Moody chart. For laminar flow ($\mathrm{Re}<2100$), the analytical result $f=16/\mathrm{Re}$ is used; for turbulent flow in smooth and rough pipes, the Colebrook formula captures the data (Denn, 1980):

Colebrook equation, turbulent flow: $$\frac{1}{\sqrt{f}} = -4.0\log\left(\frac{\varepsilon}{D} + \frac{4.67}{\mathrm{Re}\sqrt{f}}\right) + 2.28 \qquad (1)$$

where f is the Fanning friction factor, ε is the pipe roughness, D is inner pipe diameter, and Re is the Reynolds number. The Colebrook equation is not explicit in friction factor and must be solved iteratively for some flow problems, as is discussed in most fluid mechanics textbooks. For smooth pipes, there is also a formula for the turbulent-flow regime that is explicit in friction factor (Denn, 1980):

Smooth pipes, turbulent flow:

$$f = \frac{1.02}{4}\log \mathrm{Re}^{-2.5} \qquad (2)$$

It is occasionally desirable to have a data correlation that spans the entire range of Reynolds number, from laminar flow, through transitional flow, and reaching the highest values of Reynolds number. For this purpose, we have developed a new data correlation for smooth pipes, one that is explicit in friction factor, and which is relatively simple in form.

Smooth pipes, all flow regimes: $$f = \left(\frac{0.0076\left(\frac{3170}{\mathrm{Re}}\right)^{0.165}}{1+\left(\frac{3170}{\mathrm{Re}}\right)^{7.0}}\right) + \frac{16}{\mathrm{Re}} \qquad (3)$$

A plot of equation 3 is shown in Figure 1 along with data for smooth pipes from Nikuradse (1933). At low Reynolds number, equation 3 becomes $f=16/\mathrm{Re}$. At high Reynolds numbers, equation 3 becomes the Prandtl correlation, the smooth-pipe equivalent and original source of the Colebrook equation (White, 2006).

Prandtl correlation, $4000 \le \mathrm{Re} \le 10^6$:

$$\frac{1}{\sqrt{f}} = 4.0 \log \mathrm{Re}\sqrt{f} - 4.0 \qquad (4)$$

Equation 3 captures the shape of Nikuradse's data through the highly variable transition region (Figure 1). Use of equation 3 beyond Re=10^6 is not recommended; Re=10^6 is the upper limit of the Prandtl equation (Denn, 1980).

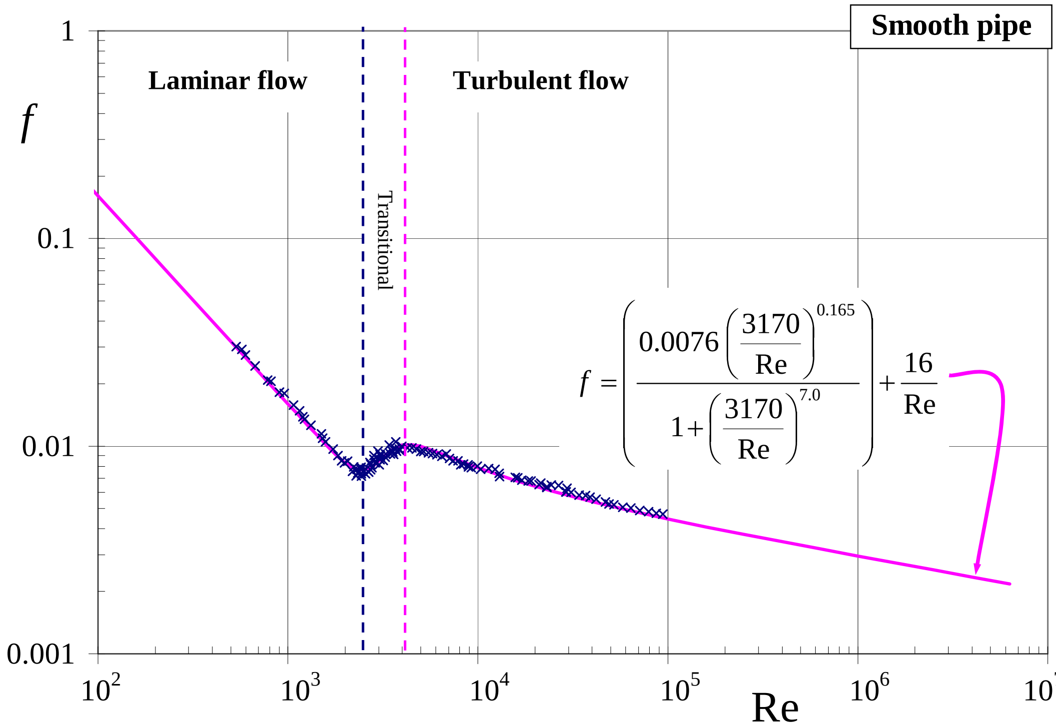

Figure 1: Equation 3 captures smooth pipe friction factor as a function of Reynolds number over the entire Reynolds-number range. Also shown are Nikuradse's experimental data for flow in smooth pipes (Nikuradse, 1933). Use beyond Re=10^6 is not recommended; for Re>*4000* equation 3 follows the Prandtl equation.

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