WORKING WITH THE MACROSCOPIC MOMENTUM BALANCE EQUATION

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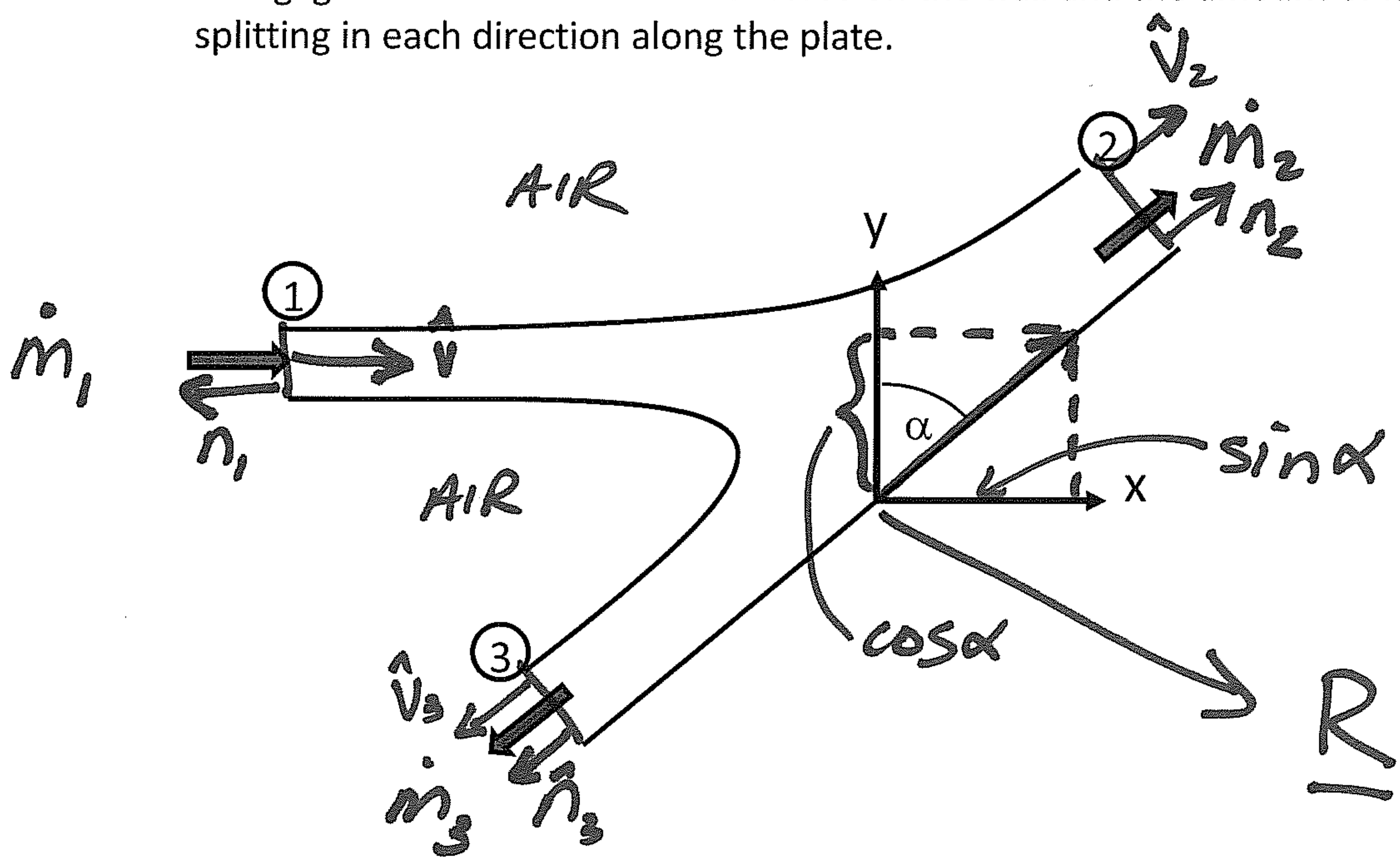
Water at 298 K discharges from a nozzle and travels horizontally, hitting a flat wall inclined 45° to the vertical. The nozzle has a diameter of 12mm, and the water leaves the nozzle with a flat velocity profile at a velocity of 6.0 m/s. You may assume that the sliding friction between the fluid and the wall is negligible. Calculate the vector force on the wall and the amount of fluid splitting in each direction along the plate.

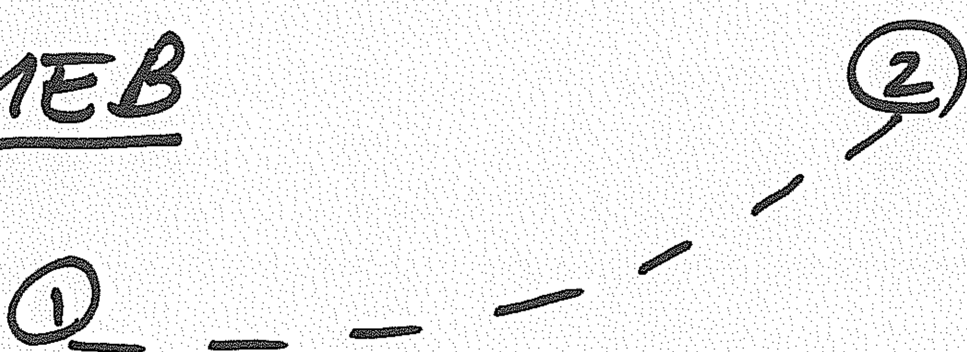


$$\hat{v}^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

MASS BALANCE

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

MEB



Friction neglected

$$\cancel{\frac{\Delta P}{\rho}} + \frac{\Delta \langle v \rangle^2}{2\alpha} + \cancel{g \Delta z} + \cancel{F} = \cancel{\frac{W_{s,m}}{\dot{m}}}$$

$$V_2^2 - V_1^2 = 0$$

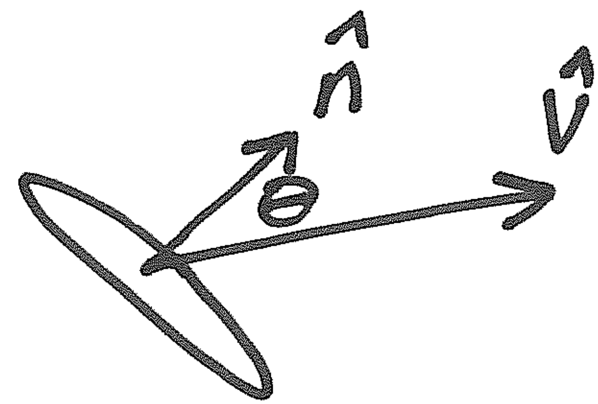
$$\boxed{V_1 = V_2} \qquad \boxed{V_1 = V_3}$$

Steady-State Macroscopic Momentum Balance

$$0 = \sum_{i=1}^{N} \left[\frac{-\rho A \langle v \rangle^2 \cos\theta}{\beta} \hat{v} \right]_i + \cancel{F_p} + \overset{\underline{R}}{F_v} + \cancel{F_g}$$

$$0 = \sum_{i=1}^{N} \left[\frac{-\dot{m} \langle v \rangle \cos\theta}{\beta} \hat{v} \right]_i + \cancel{F_p} + F_v + \cancel{F_g}$$

$$\dot{m} = \rho A \langle v \rangle$$



$$\left. \begin{array}{l} \beta = 1 \\ \alpha = 1 \end{array} \right\} \text{turbulent flow}$$

FROM MEB, not the angle!

$$- \rho_1 A_1 V_{(1)}^2 \overbrace{\cos\theta_1}^{-1} \hat{V}_{(1)}$$

$$- \rho_1 A_2 V_{(2)}^2 \overbrace{\cos\theta_2}^{1} \hat{V}_{(2)}$$

$$- \rho_3 A_3 V_{(3)}^2 \overbrace{\cos\theta_3}^{1} \hat{V}_{(3)}$$

$$+ \underline{R} = 0$$

$$\hat{n}_1 = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}_{xyz} \qquad \hat{n}_2 = \begin{pmatrix} \sin\alpha \\ \cos\alpha \\ 0 \end{pmatrix}_{xyz} \qquad \hat{n}_3 = \begin{pmatrix} -\sin\alpha \\ -\cos\alpha \\ 0 \end{pmatrix}_{xyz}$$

$$\hat{V}^{(1)} = -\hat{n}_1 \qquad \hat{V}^{(2)} = \hat{n}_2 \qquad \hat{V}^{(3)} = \hat{n}_3$$

b

$$m_1 \langle V\rangle_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz} - m_2 \langle V\rangle_2 \begin{pmatrix} \sin\alpha \\ \cos\alpha \\ 0 \end{pmatrix}_{xyz}$$

$$- m_3 \langle V\rangle_3 \begin{pmatrix} -\sin\alpha \\ -\cos\alpha \\ 0 \end{pmatrix}_{xyz} + \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} = 0$$

z-component: $R_z = 0$

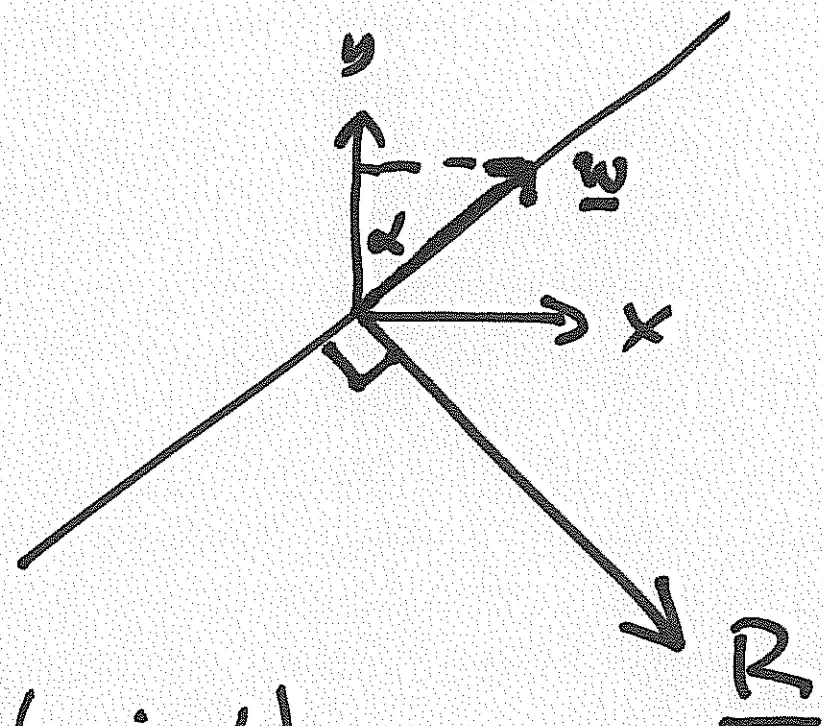
x-component:

$$\dot{m}_1 \langle V\rangle_1 - \dot{m}_2 \langle V\rangle_2 \sin\alpha + \dot{m}_3 \langle V\rangle_3 \sin\alpha = -R_x$$

y-component:

$$-\dot{m}_2 \langle V\rangle_2 \cos\alpha + \dot{m}_3 \langle V\rangle_3 \cos\alpha = -R_y$$

$$\underline{R} = \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix}$$



$$\hat{W} = \begin{pmatrix} \sin\alpha \\ \cos\alpha \\ 0 \end{pmatrix}_{xyz}$$

$$\underline{R} \cdot \hat{\underline{w}} = 0$$

$$\underline{R} \cdot \hat{w} = \boxed{R_x \sin\alpha + R_y \cos\alpha = 0}$$

since $\alpha = 45^\circ$, $\sin\alpha = \cos\alpha$ $\therefore$ $R_x + R_y = 0$

$\alpha = 45^\circ$

$\sin\alpha = \frac{1}{\sqrt{2}}$

$\cos\alpha = \frac{1}{\sqrt{2}}$

$$\boxed{\sin\alpha = \cos\alpha = \frac{1}{\sqrt{2}}}$$

$$\dot{m}_1 V - \dot{m}_2 V \sin\alpha + \dot{m}_3 V \sin\alpha = -R_x$$

$$-\dot{m}_2 V \sin\alpha + \dot{m}_3 V \sin\alpha = -R_y$$

$\therefore$ $m_3 = 0.171\ m_2$

$m_2 = 0.578$ kg/s

$m_3 = 0.099$ kg/s

$$\underline{R} = \begin{pmatrix} 2.03\,N \\ -2.03\,N \\ 0 \end{pmatrix}_{xyz}$$

$$=$$