HEAT TRANSFER - summary

(What have we learned?)

homogeneous materials

\[ \frac{q_x}{A} = -k \frac{dT}{dx} \]

(Brownian motion)

inhomogeneous materials

\[ h(T_s - T_b) = \frac{q_x}{A} \]

[at boundaries where physics is complex]

[Legendre polynomials]
fluid in motion (gas, liquid)

Microscopic E-BAR (homogeneous mat.)

\[ \rho C_p \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{T} \right) = k \nabla^2 T + S \]

- time rate of change
- convection
- conduction
- current rxn
"Lumped" parameters (used in B.C.)

\[
h = \text{heat transfer coefficient}
\]

\[
\begin{align*}
  h (R_e, Pr, \frac{L}{D}) \\
  h (Gr, Pr, \frac{L}{D}) \\
  h (\text{regime of system}) \\
  \Delta T
\end{align*}
\]

\[
T_{\text{bulk}} \quad h \quad T_{\text{surface}}
\]
Radiative Heat Transfer

Stefan-Boltzmann Law

\[ \text{Emitted black body} = \text{Stefan-Boltzmann constant} \times T^4 \]

\[ P\nu = nRT \]

Also absolute temp

In \( K \) or \( \text{(absolute temp)} \)
Emission
\[ \text{Emission} = \frac{\text{Emissivity}}{\text{Area}} \]
\[ \varepsilon = \frac{\text{Emissivity}}{\text{Temperature}} \]
\[ \varepsilon > 0 \]
Absorption
\[ \text{Absorption} = \alpha \times \text{Incident} \]
\[ \alpha < 1 \]
Kirchhoff's Law
\[ \alpha = \varepsilon \] (Nice Simplification)
Radiation net heat xfer:

\[ \text{Energy transfer to } = A \varepsilon \sigma \left( T_s^4 - T_b^4 \right) \]

(\text{Can write as heat xfer coeff, if desired})

\[ \text{Use: See notes on heat shield} \]

\[ \text{See example w/ hot pipe} \]
Final Topic:

Designing a Heat Exchanger

Double Pipe

How can I estimate the overall heat transfer coefficient $U$ for a heat exchanger I am designing?
\[ T_1 \rightarrow A \rightarrow T_2 \]

Outside diameter of inner pipe:

\[ A = \pi D L \]

\[ Q = A U \Delta T_{\text{em}} \]

E-Ba2 inside:

\[ \Delta H = Q_{in} \]

\[ Q_{in} = m C_p \Delta T \]

or \[ m \Delta H \text{ for condensing} \]

Which physics is it? Answer: up to you.
- Forced convection: pump
  - fan

- Natural (free) convection: vertical design
  - natural connection "cells"

- Phase change: boiling condensation
What determines performance (how much Q) of a heat exchanger?

- material of construction (k steel)
- what fluid is on inside outside
We have solved this geometry before! See lecture 5.
Example 4: Heat flux in a cylindrical shell, Newton's law of cooling boundary Conditions

Results: Radial Heat flux in an Annulus

\[
T - T_{b2} = \frac{(T_{b1} - T_{b2}) \left( \ln \left( \frac{R_2}{r} \right) + \frac{k}{h_2R_2} \right)}{k \frac{1}{h_2R_2} + \ln \left( \frac{R_2}{R_1} \right) + \frac{k}{h_1R_1}}
\]

\[
\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\left( \frac{1}{h_2R_2} + \frac{1}{k} \ln \left( \frac{R_2}{R_1} \right) + \frac{1}{h_1R_1} \right) \left( \frac{1}{r} \right)}
\]

Evaluate at \( r = R_2 \) the outer radius (heat flux into surface)

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Heat flux through an annular pipe with Newton's law of cooling BC.

\[ \frac{q_r}{A} = \frac{T_{b_1} - T_{b_2}}{\left( \frac{1}{h_2 R_2} + \frac{1}{k} \ln \frac{R_2}{R_1} + \frac{1}{h_1 R_1} \right)^\frac{1}{n}} \]

\[ q_r \bigg|_{R = R_2} = A (T_{b_1} - T_{b_2}) \left[ \frac{1}{h_2 R_2} + \frac{1}{k} \ln \frac{R_2}{R_1} + \frac{1}{h_1 R_1} \right]^\frac{1}{n} \]
• This is the result for a cross section of a long pipe.

• For a double-pipe heat exchanger, \( \Delta T \) driving varies along the length from 0 to the tube.

• We replace \( T_1 - T_2 \) with \( \Delta T_{\text{log mean}} \) or correct average driving force for heat transfer for double-pipe H.E.
\[ Q = A \Delta T_{\text{m}} \left( \frac{1}{R_2} \right) \]

This is called

Sizing a heat exchanger

get \( h_1, h_2 \) from the appropriate correlation depending on the design (laminar forced convective, turbulent forced convective, free convective, etc.)