HEAT TRANSFER

\[ \rho c P \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = k \nabla^2 T + S \]

Fouier's Law \( \frac{q_x}{A} = -k \frac{dT}{dx} \)

* One Dimensional Heat Conduction in Rectangular Slab

** CASE 1 **

\[ T = Ax + B \]

\[ \frac{q_x}{A} = \text{constant} \]

- linear temp profile
- constant heat flux
Newton's Law of Cooling Boundary Conditions

\[ \frac{Q_x}{A} = \frac{T_{b1} - T_{b2}}{\frac{1}{h_1} + \frac{B}{K} + \frac{1}{h_2}} \]

What happens as \( h_1, h_2 \) increase?
Example: Radial conduction in a pipe wall

Equation of energy for Newtonian fluids of constant density, $\rho$, and thermal conductivity, $k$, with source term (source could be viscous dissipation, electrical energy, chemical energy, etc., with units of energy/(volume time)).

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Gibbs notation (vector notation)

$$\left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = -\frac{k}{\rho C_p} \nabla^2 T + \frac{S}{\rho C_p}$$

Cartesian (xyz) coordinates:

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = k \frac{1}{\rho C_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{S}{\rho C_p}$$

Cylindrical (rθz) coordinates:

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \frac{k}{\rho C_p} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{S}{\rho C_p}$$

Spherical (rθφ) coordinates:

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \frac{k}{\rho C_p} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial^2 T}{\partial \phi^2} \right) + \frac{S}{\rho C_p}$$

\[ \frac{d}{dr} \left( r \frac{df}{dr} \right) = 0 \]
\[ \Rightarrow \quad \frac{df}{dr} = \frac{C_1}{r} \]
\[ r \frac{df}{dr} = C_1 \]
\[ T = C_1 \ln r + C_2 \]

\[ \begin{array}{l}
  r = R_1, \quad T = T_1 \\
  r = R_2, \quad T = T_2 \\
\end{array} \]
Flux: Fourier's law:

\[ \frac{q_r}{A} = -k \frac{dT}{dr} \]

\[ \Rightarrow \frac{q_r}{A} = -k \frac{q}{r} \]

\[ \frac{dT}{dr} = \frac{q}{r} \]

One dimensional heat conduction in an annulus

**Case 2**

\[ T = A \ln r + B \]

- Logarithmic temp profile

\[ \frac{q_r}{A} = - \frac{q}{r} \]

- Heat flux \( \sim \frac{1}{r} \)
How can we apply Newton's Law of cooling BC?
\[
\left\{ \frac{9c}{A} \right\} = h \left( T_b - T(\text{in}) \right)
\]

\[
\frac{c}{R_1} = \frac{9r}{A} \bigg|_{r=R_2}
\]

\[
\text{Case 2:} \quad \frac{9r}{A} = \frac{c}{r}
\]

\[
T = -\frac{c}{k} \ln r + c_2
\]

\[
R_1 < r < R_2
\]
BC1: \[ \frac{c_i}{R_1} = h_1(T_b, -\left(-\frac{c_i}{k}ln R_1 + c_2\right)) \]

BC2: \[ \frac{\partial r}{\partial t} \bigg|_{r=R_e} = h_2(T(R_e) - T_b) \]

BC2: \[ \frac{c_i}{R_2} = h_2\left(1-\frac{c_i}{k}ln R_2 + c_2\right) - T_b \]

Solve for \(c_1, c_2\)

(See printed notes)
Equation of energy for Newtonian fluids of constant density, \( \rho \), and thermal conductivity, \( k \), with source term (source could be viscous dissipation, electrical energy, chemical energy, etc., with units of energy/(volume time)).

http://www.chem.mtu.edu/~fmrorriso/cm310/energy_eqn.pdf
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Gibbs notation (vector notation)

\[
\left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = \frac{k}{\rho \mathcal{C}_p} \nabla^2 T + \frac{S}{\rho \mathcal{C}_p}
\]

Cartesian (xyz) coordinates:

\[
\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} = \frac{k}{\rho \mathcal{C}_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{S}{\rho \mathcal{C}_p}
\]

Cylindrical (r\( \theta \)z) coordinates:

\[
\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_z}{r \sin \theta} \frac{\partial T}{\partial z} = \frac{k}{\rho \mathcal{C}_p} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{S}{\rho \mathcal{C}_p}
\]

Spherical (r\( \theta \)\( \phi \)) coordinates:

\[
\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} = \frac{k}{\rho \mathcal{C}_p} \left( \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{\partial^2 T}{\partial \phi^2} \right) + \frac{S}{\rho \mathcal{C}_p}
\]

\[ k \frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + \frac{Se}{r} = 0 \]

\[ \frac{d\psi}{dr} = -\frac{Se}{k} r \]

\[ r \frac{d\psi}{dr} = \psi = -\frac{Se}{k} \frac{r^2}{2} + C_1 \]

\[ \frac{dT}{dr} = -\frac{Se}{2k} r + \frac{C_1}{r} \]

**Bc 1:**
\[ r = 0 \quad \frac{dT}{dr} = \text{finite} \Rightarrow C_1 = 0 \]
\[ \frac{dT}{dr} \text{ cannot go to } \infty \text{ at } r = 0 \]

\[ T = -\frac{Se}{2k} \frac{r^2}{2} + C_2 \]

\[ r = R \quad T = T_w \]

Solve (see slides)