Instructions:
i. Closed book, closed notes. One 8.5” by 11” study sheet allowed. All work on the exam must be your own.
ii. Write your solution work on one side of the page only. Do not write on the back of any pages.
iii. Please be neat. Only neat answers will be granted partial credit.
iv. Significant figures count.
v. Please box your final answers.
vi. If you use a calculator, please write your steps if you wish partial credit.
vii. No cell phones or internet-capable instruments allowed.

There are five problems on the test.
There are four pages to the test, plus 2 pages of attachments.
There is an additional handout of resources (4 pages).

1. (20 points) What are the frictional losses in 200.0 meters of smooth horizontal copper tubing of inner diameter 1.5 cm = 0.015 m? Water at 25°C is flowing at 1.31 \times 10^{-2} m/s average velocity. Please give your answer in meters (head loss units).
2. (20 points) To calculate the fluid force on a surface in contact with a flow, we need to evaluate the following equation from knowledge of the velocity and pressure fields:

\[ F = \int_S \left[ \hat{n} \cdot \Pi \right]_{surface} dS \]

where \( S \) is the surface of interest, and \( \hat{n} \) is the unit normal vector to the differential surface \( dS \) in contact with the fluid. The differential surface can be integrated over the appropriate limits to get the macroscopic surface of interest, \( S \). \( \Pi \) is the total stress tensor, a \( 3 \times 3 \) matrix of stress components.

For pressure-driven, laminar flow in a tube (see accompanying figure), the velocity and pressure fields are given below (\( p_0 \) is the upstream pressure; \( p_L \) is the pressure a distance \( L \) downstream; \( \rho \) is fluid density; \( \mu \) is fluid viscosity; \( R \) is the tube radius; \( g \) is the acceleration due to gravity. All are constants.). We seek to calculate the fluid force on the walls of the tube. What is the quantity \( \left[ \hat{n} \cdot \Pi \right]_{surface} \) for this flow? What is the differential surface \( dS \) equal to for the calculation of the total force on the walls of the tube?

\[
\vec{v} = \begin{pmatrix} 0 \\ 0 \\ \left( \frac{(p_0 - p_L + \rho g L)R^2}{4\mu L} \right) \left[ 1 - \left(\frac{r}{R}\right)^2 \right]_r \end{pmatrix}
\]

\[
p = -\left( \frac{p_0 - p_L}{L} \right) z + p_0
\]
3. (20 points) In wire coating, a metal wire is drawn through a long cylindrical liquid bath, pulling fluid along with it (see accompanying figure). Solving the microscopic mass and momentum balances in the flow region far from the ends of the bath, we can calculate the following steady state velocity and pressure fields:

\[
v = \begin{pmatrix}
0 \\
0 \\
V \left( \frac{\ln(r/R)}{\ln \kappa} \right)_{r \theta z}
\end{pmatrix}
\]

\[p = p_0\]

where \(V\) is the speed at which the wire is pulled, \(\kappa R\) is the radius of the wire, \(R\) is the radius of the bath, and \(p_0\) is the constant pressure in the fluid bath. What is the volumetric flow rate \(Q\) in the flow direction for this wire coating flow? Set up the equation to calculate \(Q\), including the limits of integration; you do not have to carry out the integration.
4. (20 points) What are the z-direction velocity boundary conditions for torsional flow between circular parallel plates? The upper plate turns counterclockwise at an angular speed of $\Omega$, the gap between the plates is $H$, and the radius of the plates is $R$. See additional information in the accompanying graphic. Give your answer relative to the coordinate system provided.

**Torsional flow between parallel plates**

- Newtonian
- Steady state
- Incompressible fluid

5. (20 points) Calculate the steady state velocity profile for a Newtonian, incompressible fluid flowing slowly down a long, wide inclined plane (see accompanying figure). We are interested in the flow away from the inlet, outlet, and the edges of the flow. The film thickness is $H$, and the incline makes an angle $\alpha$ with the horizontal. Calculate your answer in the coordinate system shown.